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DOUBLE VARIATIONAL PRINCIPLE FOR THE CONSTRUCTION SYNTHESIS OF STEERABLE RADIOTELESCOPES*

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Abstract. A numerical variational method for synthesis of the maximum stiffness construction of radiotelescopes based on the double variational principle of potential energy of deformations is described. The results obtained show that the maximum stiffness construction considerably reduces the values of the λ_{min}/D ratio for steerable radiotelescopes.

Key words: Construction of radiotelescopes

Effective constructing of radiotelescopes for decreasing wavelengths demands to enlarge their construction stiffness. This is the main way which determines possibilities of observational ground-based radioastronomy. Let us discuss this problem in its different aspects.

1. Potentialities of classical structural mechanics

The theory of elasticity of deformed bodies taken as a whole and the structural mechanics as its specific technical branch have originated from the Lagrange minimum displacement principle or the variational potential energy principle: "Of displacements complying with a set of given boundary requirements, only those meeting the

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demands of equilibrium conditions impart stationary value to the potential energy. In the case of stable equilibrium the potential energy is minimal" (Chi-Teh Wang, 1955). This principle has been put in the most concise form by Donatti using the tensor of stress σ_{ij} and the variations of tensor of deformation ϵ_{ij} corresponding to the volume V of building material (Oravas and McLeon, 1966):

$$\delta U = \int_V \sigma_{ij} \delta\epsilon_{ij} dV = 0. \tag{1}$$

Writing (1) in components of stress and deformation we obtain the variational equation of the potential energy of deformation like this:

$$\delta U = \int_V (\sigma_x \delta\epsilon_x + \sigma_y \delta\epsilon_y + \sigma_z \delta\epsilon_z + \tau_{xy}\delta\gamma_{xy} + $$

$$+ \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz})dV = 0. \tag{2}$$

Variational equations (2) of the potential deformation energy of bar system, consisting of $r = 1, 2, ..., s$ bars, can be expressed in the following form (Filin, 1966):

$$\delta U_b = \int_V (M_x \delta\kappa_x + M_y \delta\kappa_y + M_z \delta v + Q_x \delta\gamma_x + $$

$$+ Q_y \delta\gamma_y + N_z \delta\epsilon_z)dz = 0 \tag{3}$$

and

$$\delta U_{b.s} = \sum_{r=1}^{s} \int_{l_r} (M_{xr} \delta\kappa_{xr} + M_{yr} \delta\kappa_{yr} + M_{zr} \delta v_r + Q_{xr} \delta\gamma_{xr} + $$

$$+ Q_{yr} \delta\gamma_{yr} + N_{zr} \delta\epsilon_{zr})dz = 0. \tag{4}$$

Here M_{xr}, M_{yr} , M_{zr} are the components of moments in sections of the bar r; Q_{xr}, Q_{yr} are the components of shear forces; N_{zr} is longitudinal force; κ_{xr}, κ_{yr} are the components of sections of the deviation angle; γ_{xr}, γ_{yr} are the components of shear deformations; v_r is torsion deformation and ϵ_{zr} is axial deformation.

Variational equations (2), (3) and (4) provide us with all the necessary compatibility conditions of displacements and equilibrium taking into account material stiffness qualities and giving a total description of the sample material, of the construction elements and of the construction as a whole. The stiffness qualities are due to electromagnetic forces of fundamental particle interaction in the structural material. Thus, the variational principle of the potential energy of elastic deformation in the material resisting mechanical force enables one to carry out an unequivocal analysis of the state of stress and deformation on the level of the material functioning (the level of microworld).

Creation of the load-bearing building constructions is essentially not an object of analysis but a typical task of synthesis. It is most obvious in the constructions of mirror antennae built with a limited level of deformability or the demanded law of deviation distribution of the reflecting surface. This task must be solved by filling in a particular manner a definite volume of space with the necessary quantity of bars having definite cross-section areas. It has been proved that the classical variational principle of the potential energy of material deformation is insufficient for synthesis tasks, since it does not provide variation of the total mass and its distribution in the volume of space occupied by the construction (Polyak and Bervalds, 1990). In other words, equation (4) can be regarded merely as the variational expression, which provides minimum potential energy of deformation of bars having given structural parameters of a bar and total mass of the material.

2. Construction stiffness criteria

The choice of a stiffness criterion is of importance when defining the task of building load-bearing constructions with required stiffness properties. The traditional criteria of stiffness of mirror antennae constructions are indices of statistical deviations of the marker points (maximum, rms or other) (Ruse, 1966). Their merit is their close links with radiotechnical characteristics of the antenna. Their drawback is that they are not direct characteristics of stiffness as a physical property of the construction, but only the probability indicators of these properties appearing through additional local characteristics, deviations of construction joints. Thus, their application is quite convenient in the analysis but is quite complicated in the synthesis of the maximum stiffness constructions.

On the other hand, the so-called matrixes of stiffness, composed of the maximum values of stiffness coefficient of separate bars are widely used in structural mechanics. However, expression of local stiffness properties of a stressed and deformed system in the matrix form, convenient in calculating the construction, is unsuitable for the synthesis of a construction of required stiffness, since it describes this construction property as a whole. Possibilities to use the sum of stiffness coefficients of separate bars for this aim are limited. To secure a single-valued result and not to lose the physical meaning of this mathematical operation, such summing up of coefficients is permitted only for bars coming together in joints.

The only integral stiffness criterion of a bar system to be used is the potential energy of its elastic deformation

$$U_{b.s} = \sum_{r=1}^{s} \int_{l_r} \left(\frac{M_{xr}^2}{2EI_{xr}} + \frac{M_{yr}^2}{2EI_{yr}} + \frac{M_{zr}^2}{2GI_{zr}} + \frac{Q_{xr}^2}{2GA_{xr}} + \frac{Q_{yr}^2}{2GA_{yr}} + \frac{N_{zr}^2}{2EA_r} \right) dz, \tag{5}$$

which is the most complete, general and sensitive criterion of construction stiffness (Bervalds, 1982). Here E and G are the modules of material elasticity (characteristics of stiffness); I_{xr}, I_{yr}, I_{zr} are the components of bar inertia momentum; A_{xr}, A_{yr} are the components of reduced cross section area; and A_r is the cross section area of the bar.

Consequently, the condition

$$U_{b.s} = \min \tag{6}$$

would mean that the construction of maximum possible stiffness has been created. Thus, creating of the mirror antenna of the maximum stiffness is directly related to the synthesis problem of a load-bearing building construction on the basis of energetic characteristics, the most fundamental characteristics of the material world. To have this problem solved, there is a need for a common principle to be formulated.

3. Double variational principle of the potential energy

It would be logical to expect that, in creating the engineering constructions, a designer would follow the demands of elaborating the building materials which accumulate minimum energy when deformed. Unfortunately, the principle followed both in theory and practice of engineering structural design is much like the principle "at any price". This is proved by the existing official building norms and regulations. It refers to both the ultimate conditions, i.e.

$$N_r/\varphi A_r \leq R_y \gamma \text{ or } \Delta_k \leq [\Delta]. \tag{7}$$

Here R_y is the design strength; Δ_k and $[\Delta]$ are actual and permissible deviations resulting from construction deformation; φ and γ are the normative coefficients.

To return to a natural rational approach, we shall submit the functioning of a load-bearing building construction as a whole (both on the material level and on the level of the construction itself) to the double variational principle of potential energy. This principle can be defined like this:

"Of all possible volumes of space occupied by a bar system with a possible material mass and their possible distribution which comply with given equilibrium boundary conditions, only those providing the minimum potential energy of the elastic deformation of the system should be valid".

Since the principle of potential energy of deformation on the level of material behaviour works automatically, the sign of deformation variation in equation (1) can be omitted and the variation of material space with the volume occupied by the construction must be set up as a condition. Then the principle of synthesis of load-bearing building construction can be expressed by the following variational equation:

$$\delta U_{b.s} = \int_{\delta V} \sigma_{ij} \epsilon_{ij} dV = 0. \tag{8}$$

In accordance with (5) the variational equation of synthesis of a bar system can be expressed in the form:

$$\delta U_{b.s} = \sum_{r=1}^{\delta s} \int_{\delta l} \left(\frac{M_{xr}^2}{2E\delta I_{xr}} + \frac{M_{yr}^2}{2E\delta I_{yr}} + \frac{M_{zr}^2}{2G\delta I_{zr}} + \frac{Q_{xr}^2}{2G\delta A_{xr}} + \right.$$
$$\left. + \frac{Q_{yr}^2}{2G\delta A_{yr}} + \frac{N_{zr}^2}{2E\delta A_r} \right) dz = 0. \tag{9}$$

Here variation of integral construction mass has been deciphered as a variation of the number of bars, their orientation and material characteristics (length, form and area of cross section). The variation limit of the volume of material space is not set up in advance but depends on the construction function. In a construction meant for the first limit state (carrying capacity and stability) it may be more restricted. A construction of maximum stiffness (having restrictions in calculating the second limit state) may demand a large volume of space and a great mass of the material.

4. Numerical variational method of synthesis of the maximum stiffness construction, its application and the main conclusions

A complete analytical solution of equation (9) with derivation of dependencies suitable for the application in a general case is confronted with difficulties of principal character. At present, a numerical variational method of synthesis which does not change directly the topology of the construction is worked out. To use this method, an algorithm and a software have been worked out and thoroughly tested (Fig. 1) (Bervalds, 1978, 1986; Bervalds, Laure and Mac, 1986). This algorithm uses two steps of computation. The first is iterational recomputing of distribution of the accepted mass (volume) of the material to achieve a stable solution, which would correspond to the maximum stiffness construction. In the second step we determine either the mass needed for the maximum stiffness construction or the minimum mass for a construction of fixed stiffness. This method is applied for calculating hinged bar constructions in terms of their resistance to a statical load and to the load of the construction itself. The results obtained using the variational method to the bar construction synthesis makes it possible:

1) to distribute the given building material of bars for achieving the maximum construction stiffness;

2) to determine the required mass of the material and to distribute it in bars to achieve the maximum construction stiffness;

3) to create a construction of the minimum mass satisfying the demands of carrying capacity and stability. Possibilities of this method, used for computation of 12, 30 and 70 m radiotelescope constructions, are shown on the well-known Hoerner graph (Fig. 2, Table 1) which demonstrates the evolution of radiotelescope building and overcoming of some natural restrictions (Hoerner, 1967). By comparing the results of synthesis calculations with those obtained

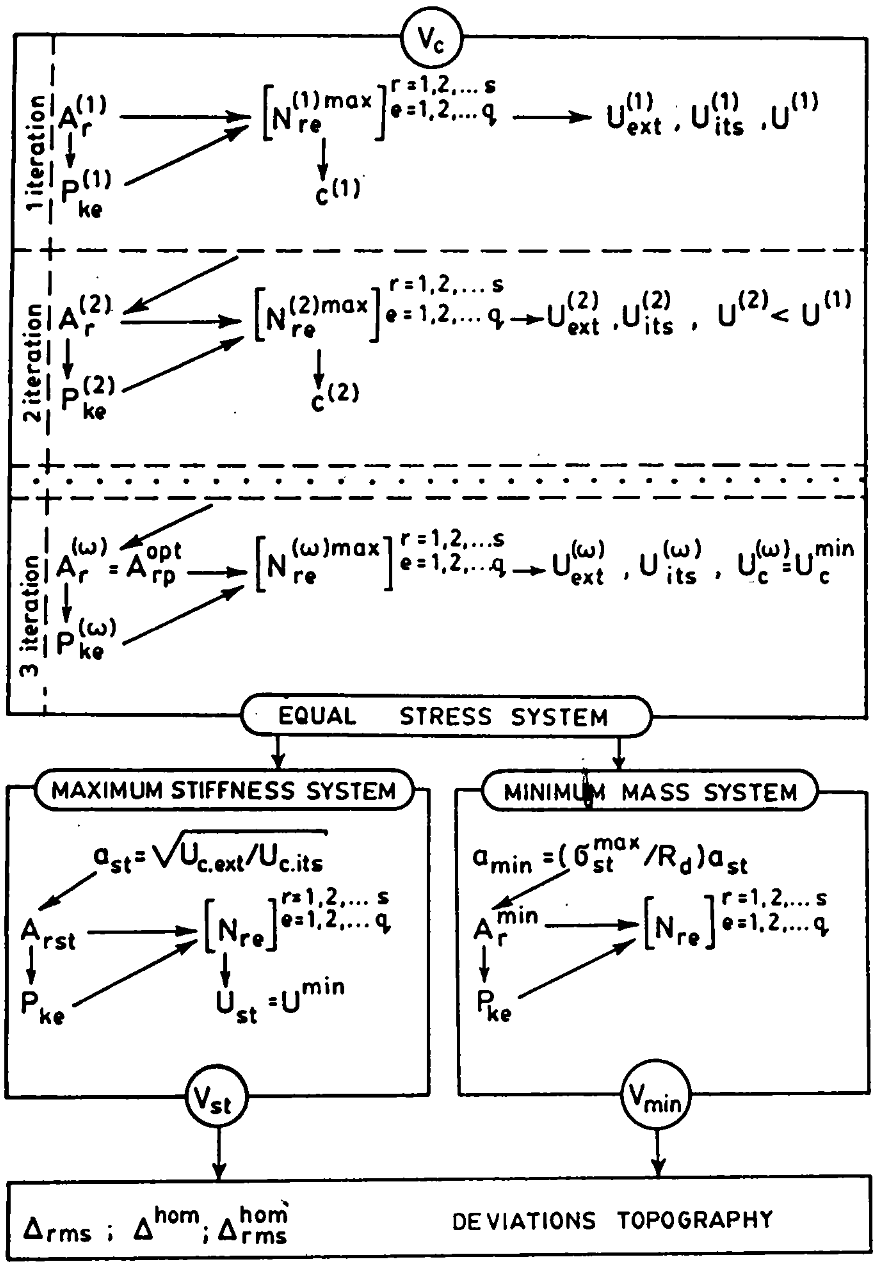

Fig. 1. An algorithm of numerical variational calculation.

Table 1. Characterictics of the radiotelescopes shown in Fig. 2

Ant.	No.	Antenna sites	D (m)	$\lambda_{min}=$ $=4\pi$rms (mm) at rms_{hom}	$\lambda_{min}=$ $=4\pi$rms (mm) at rms	λ_{min}/D (10^{-4}) at λ_{min}^{hom}	λ_{min}/D (10^{-4}) at λ_{min}
	1.	Kitt Peak (USA)	12	0.9	1.9	0.8	1.6
	2.	Onsala (Sweden)	20	2.3	X*	1.2	X
	3.	Crimea (USSR)	22	3.8	4.8	1.7	2.2
	4.	Pico de Veleta (Spain)	30	1.1	15.7	0.4	5.2
	5.	Haystack (USA)	37	9.0	12.0	2.4	3.2
	6.	Owens Valley (USA)	40	X	13.6	X	3.4
Ana-	7.	Green Bank (USA)	43	11.0	12.8	2.6	3.0
lo-	8.	Tokyo (Japan)	45	2.5	110	0.6	24.4
gue	9.	Ontario (Canada)	46	11.0	14.4	2.4	3.1
	10.	Parkes (Australia)	64	X	44.0	X	6.9
	11.	Medvezhy Ozera (USSR)	64	9.0	X	1.4	X
	12.	Goldstone (USA)	64	14.0	18.8	2.2	2.9
	13.	NRAO design (USA)	65	3.5	30.1	0.5	4.6
	14.	Crimea (USSR)	70	10.2	78.5	1.5	11.2
	15.	Jodrell Bank (England)	78	X	126	X	16.5
	16.	Effelsberg (Germany)	100	7.5	204	0.8	20.4
	17.	CNIIPSK design (USSR)	128	11.0	X	0.9	X
Max.	18.	RT-12 (calculation)	12	0.2	1.8	0.2	1.5
stiff-	19.	RT-30 (calculation)	30	0.8	8.9	0.3	3.0
ness	20.	RT-70 (calculation)	70	1.9	19.6	0.3	2.8

*X - characteristics which have no credible data.

with the help of conventional methods of version planning and designing or by making use of different methods of optimization, the maximum deviation has been lowered by 2 to 5 times. Effectiveness of the method is due to a very large set of the simultaneously optimized variables. The calculation has been carried out with simultaneous variation of cross-section areas of 2743 bars.

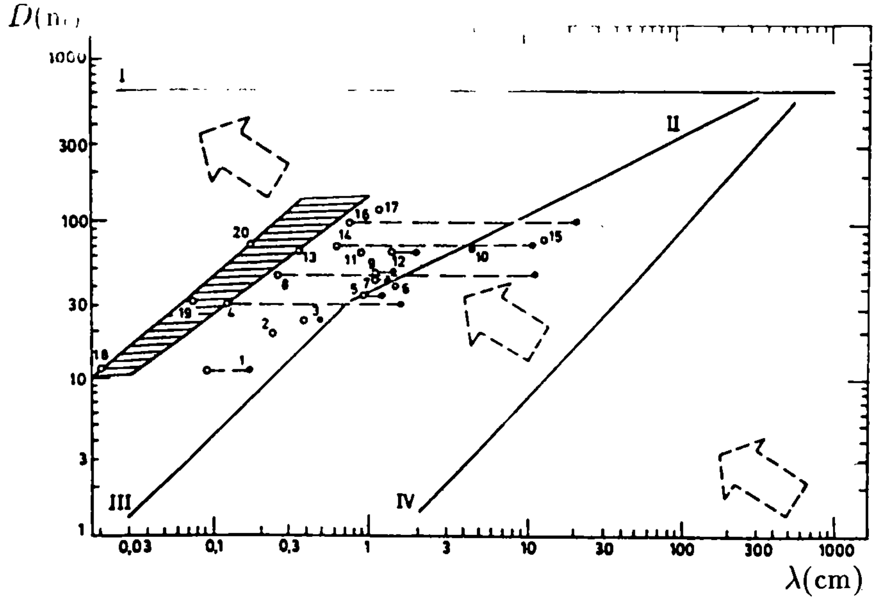

Fig. 2. Dynamics of development of steerable radiotelescopes listed in Table 1. Circles are plotted at $\lambda_{\rm min}$ corresponding to r.m.s. relative to undeformed parabola and dots are at $\lambda_{\rm min}$ corresponding to r.m.s of homologous surface. The lines I, II, III and IV are limits caused by the strength of material, deformations by own weight, thermal deformations and restrictions of building norms. The shadowed area can be assimilated only by using a double variational principle of the potential energy.

Theoretical investigation of the maximum stiffness constructions and the obtained calculation results for the existing antennae permit us to make the following conclusions:

1. The potential energy of deformation of bar systems in the ground-based conditions, having the absolute minimum value, or the integral stiffness, having the maximum value, are stipulated by a single value of total mass of the material and by a single way of its distribution.

2. The use of the double variational principle of the potential energy enables us to get over the λ_{min}/D ratio restriction for radiotelescopes.

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