Flow Regimes in Horizontal Viscous Dam-Break Flow of Clayous Mud

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Abstract:
The main characteristics of geological flows such as debris flows, avalanches and lahars is due to the relative importance of viscous versus inertial forces in the momentum balance. This paper considers the motion generated by the collapse of a dam-retaining mud, itself modeled as a power-law fluid. The equation of motion is derived in a non-dimensional form and solved analytically with the shallow-water assumption in a dry and smooth horizontal channel. Notably indicated are flow regimes and the effect of the reservoir length as well as the effect of mud rheology on flow development. Then, a parametric study of this model is produced and the effect of mud shear-thinning on flow development is pointed out.

Zusammenfassung:
Die Hauptmerkmale von geologischen Flüssen, wie beispielsweise Schuttlawinen, Lawinen und vulkanischen Schlammströmen (Lahars), hängen vom Verhältnis der Zähflüssigkeits und Trägheitskräfte in der Impulserhaltungsgleichung ab. Diese Arbeit beschäftigt sich mit den Bewegungen, die beim Bruch eines Dammes entstehen, der eine zähflüssige Substanz zurückhält, die durch eine Kraftgesetzflüssigkeit modelliert wird. Die Bewegungsgleichung wird in adimensionaler Form hergeleitet, und analytisch gelöst, unter der Seichtwasserhypothese für einem trockenen und glatten horizontalen Kanal. Insbesondere werden Wasserstandsverhältnisse und der Einfluss der Beckenlänge sowie der Einfluss der Rheologie der zähflüssigen Substanz auf die Flussentwicklung aufgezeigt. Danach wird eine Parameterstudie dieses Modelles angefertigt und der Effekt der Ausscherung der zähflüssigen Substanz auf die Flussenwicklung wird herausgearbeitet.

Résumé:
La principale caractéristique des écoulements géologiques tels que les laves torrentielles, les avalanches et les lahars est dûe à l'importance relative des forces de viscosité par rapport aux forces d'inertie dans l'équation de conservation de la quantité de mouvement. Le présent article s'intéresse au mouvement généré par la rupture d'un barrage retenant de la boue, celle-ci étant modélisée par un fluide à loi de puissance. L'équation du mouvement est formée sous forme adimensionnelle et résolue analytiquement avec l'hypothèse des eaux peu profondes dans un canal horizontal, lisse et sec à l'aval. Tout particulièrement, les régimes d'écoulement et l'effet de la longueur du réservoir ainsi que l'effet de la rhéologie de la boue sur le développement de l'écoulement sont étudiés. Par la suite, une étude paramétrique est réalisée et l'effet de la rhéofluidification de la boue sur le développement de l'écoulement est précisé.

Key words: dam failure; mudflow, debris flow, open channel, Power-law model, shallow-water

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1 INTRODUCTION

Dam-break flows generated by the sudden release of a given quantity of water contained between a fixed plate and a dam constitute a class of gravity currents [1]. They develop in two phases. When the gate is suddenly removed, the fluid rapidly collapses downstream with constant velocity following an inertial solution [2]. The main force acting on the water is due to gravity, and acts vertically downwards (positive wave), while a long wave of depression (negative wave) propagates upstream with constant velocity. This negative wave is reflected by the fixed wall and propagates away from the wall with speed slightly greater than the speed of the front. Thereafter, when the negative wave overtakes the front, the front speed which has been constant up to this point decreases [3]. Such scenario can describe the initiation of flash floods (e.g. Queensland event, Australia, 2001). After long and intense rains over a small but sloping basin, the water quantity can rapidly become catastrophic if the draining network is insufficiently sized. The wave front travels fast, destroying goods and structures and killing people and animals.

Debris flows involving water naturally ladened with sediment or debris can be initiated by dam-break event (e.g. Grand-Bornand event, France, 1987). They transport much energy and can even move rocks and boulders upon very long distances. During this trip, they are responsible of much destruction and they erode the soils. This process results in a suspension of non-colloidal particles (grains) in a dispersion of colloidal particles (clay) in water. For modeling, this mixture will be considered as a stable and homogeneous complex fluid with a rheological behavior depending on the observation scale. On a small scale, the mud is a multi-component material so, it is the seat of different interactions: between water molecules, between clay particles in water and between grain particles in the clay-water dispersion, as well as the different possible combinations. On a large scale, it is well known that as far as the concentration in coarse grains is low, the rheological behavior of this suspension is similar to that of the suspending fluid (clay-water dispersion), i.e. it exhibits a yield stress and a shear-thinning property. In this case, a third phase takes place when the viscous effects become more important than inertial effects, causing the front speed to decrease more rapidly [4, 5].

Constitutive equations are needed which relate the stresses in the fluid to the rate of deformation. In this aim, the Reiner-Rivlin form of stress proves to be useful as a monophasic model for a complex fluid mixture, as noticed in the ASCE 1997 Conference on Debris Flow Hazards Mitigation. It is known that in the field, the shearing rate is often less than 100 [6 - 8]. Furthermore, Ng and Mei [9] state that at a low shearing rate, the power-law model which only takes into account the shear-thinning property of the mud and neglects its plasticity is more appropriate [10 - 12].

In this work, the dam-break flow of clayous mud is considered. A 1D equation of creeping motion is derived by applying the mass and momentum conservation laws on a power-law constitutive equation, and made non dimensional. Then, an analytical solution is found within two limits: the short time limit and the long time limit. These solutions are combined into a single universal model. Finally, a parametric study of this model is carried out and the effect of mud shear-thinning on flow dynamic characteristics is pointed out.

2 PROBLEM STATEMENT

2.1 RHEOLOGY

Let us consider a dam obstructing a horizontal smooth channel which is dry downstream and with a given quantity of clayous mud upstream (with height h_o), contained between a fixed plate and a dam (Figure 1). At the initial time, the dam collapses and the fluid (with density ρ and apparent viscosity η) is released downstream. Let t denote the time, $\mathbf{g}$ the gravity, p the pressure and $\mathbf{V}$ the velocity, the isochoric flow of this complex fluid is described by the continuity and momentum equations $\nabla \cdot \mathbf{V} = 0$, $\rho\dot{\mathbf{V}} = \rho\mathbf{g} - \nabla p + \nabla(2\eta\mathbf{D})$, where the dot is used to indicate the total derivative and $\mathbf{D}$, the rate of deformation tensor.

Let Λ and h_o denote the two characteristic length scales of the flow in the direction of its length (x-coordinate) and of its depth (y-coordinate) respectively, with $\Lambda >> h_o$ as it generally occurs in nature, so that the motion is essentially horizontal, the vertical component of the velocity v being much smaller than u and w, the

m	α_m	β_m	γ_m	T_c	$X_f(T_c)$	H_d
1.0	0.841	18.62	1.862	0.038	0.189	0.664
1.2	0.823	17.59	1.842	0.04	0.215	0.664
1.4	0.809	16.89	1.82	0.041	0.236	0.646
1.6	0.797	16.38	1.798	0.042	0.255	0.639
1.8	0.786	16.0	1.778	0.044	0.272	0.634
2.0	0.777	15.70	1.758	0.045	0.286	0.629
2.2	0.77	15.47	1.74	0.047	0.299	0.626
2.4	0.763	15.27	1.723	0.048	0.31	0.623
2.6	0.757	15.11	1.707	0.05	0.32	0.62
2.8	0.752	14.98	1.692	0.052	0.329	0.617
3.0	0.748	14.86	1.679	0.054	0.338	0.615
3.2	0.743	14.76	1.666	0.056	0.345	0.613
3.4	0.74	14.67	1.654	0.058	0.352	0.611
3.6	0.736	14.6	1.643	0.061	0.358	0.609
3.8	0.733	14.53	1.633	0.064	0.364	0.608
4.0	0.73	14.47	1.623	0.067	0.369	0.607
4.2	0.728	14.41	1.614	0.07	0.374	0.605
4.4	0.725	14.36	1.606	0.073	0.379	0.604
4.6	0.723	14.32	1.598	0.077	0.383	0.603
4.8	0.721	14.28	1.59	0.08	0.387	0.602
5.0	0.719	14.23	1.583	0.085	0.391	0.602

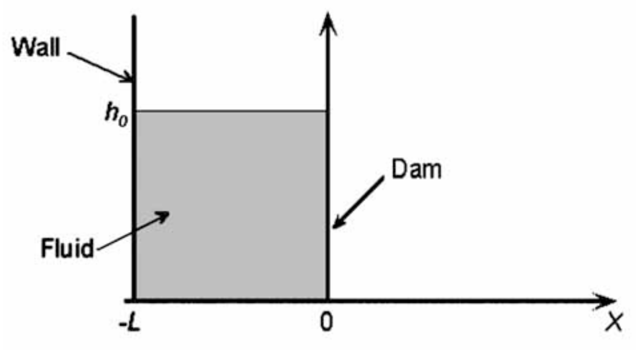


Table 1 (left): Computation of the different constants and characteristic values versus m.

Figure 1: Configuration of horizontal dam-break flow at negative time.

x - and z - components (lubrication approximation). The rheological behaviour of the mud, modelled by its shear-thinning property, obeys the so-called power-law equation, which makes it possible to represent changes in viscosity as shear increases [13]

$$\boldsymbol{\sigma} = 2\eta\left(\sqrt{-4\mathrm{D}_{\mathrm{II}}}\right)\mathbf{D} \quad and \quad \mathrm{tr}(\mathbf{D}) = 0 \tag{1}$$

$$D_{II} = -\frac{tr\left(\mathbf{D}^2\right)}{2} \quad and \quad \eta = k\left(-4D_{II}\right)^{(n-1)/2} \tag{2}$$

where $\boldsymbol{\sigma}$ denotes the extra-stress tensor, the constants k (liquid consistency) and n (power-law index) are the rheological parameters, while the two scalars D_{II} and η are the second invariant of **D** and the apparent viscosity, respectively.

The fluid is known as shear-thickening for $n > 1$, Newtonian for $n = 1$ and shear-thinning for $n < 1$. The rheological parameters are generally determined [14] using a controlled-stress controlled rate and/or controlled strain rheometer equipped with a cone-and-plate geometry [15,16] or with a concentric cylindric geometry [17, 18] as usually adopted for the formulation of suspensions. For clayous mud (this case), a value of $n = 1/3$ is generally used.

2.2 INERTIAL REGIME

Immediately after the sudden collapse of the gate (located at $x = 0$), the viscous effects (friction) are much smaller than the inertial and the physics of the non viscous problem is returned to [2]. It states that the wave front advances at a constant speed of $2\sqrt{gh_o}$, while the negative wave moves back with constant speed $\sqrt{gh_o}$. Between these two extremities, the average speed $\tilde{u}$ and the hydrograph are respectively given by

$$\tilde{u} = \frac{2}{3}\left(\frac{x}{t} + \sqrt{g h_o}\right), \quad \sqrt{g h} = \frac{1}{3}\left(2\sqrt{g h_o} - \frac{x}{t}\right) \tag{3}$$

Important characteristics of the flow can be derived from these relations. At dam site, height, average velocity and flow rate are constant and respectively given by

$$h_d = \frac{4}{9}h_o, \quad \tilde{u}_d = \frac{2}{3}\sqrt{g h_o}, \quad q_d = h_d \tilde{u}_d = \frac{8}{27}\sqrt{g h_o^3} \tag{4}$$

Assuming that the fixed plate is located at $x = -l_o$, Ritter's solution holds until the negative wave reaches it, as the non slip condition at the rear wall is violated for $t > l_o\sqrt{gh_o}$.

2.3 EQUATION OF MOTION

Introducing the power-law equation (Eq. 1) in the equations of motion approximated as the leading order equations in powers of the small parameter $\epsilon = h_o/\Lambda$, we obtain that the pressure is hydrostatic as usual in lubrication approximation and that the vertically averaged equation of motion writes:

$$\frac{\partial h}{\partial t} + \left[\frac{\rho g \cdot 2^{(n-1)/2}}{k}\right]^{1/n} \frac{\partial}{\partial x}\left\{h^{(2n+1)/n}\left|\frac{\partial h}{\partial x}\right|^{(1-n)/n}\frac{\partial h}{\partial x}\right\} = 0 \tag{5}$$

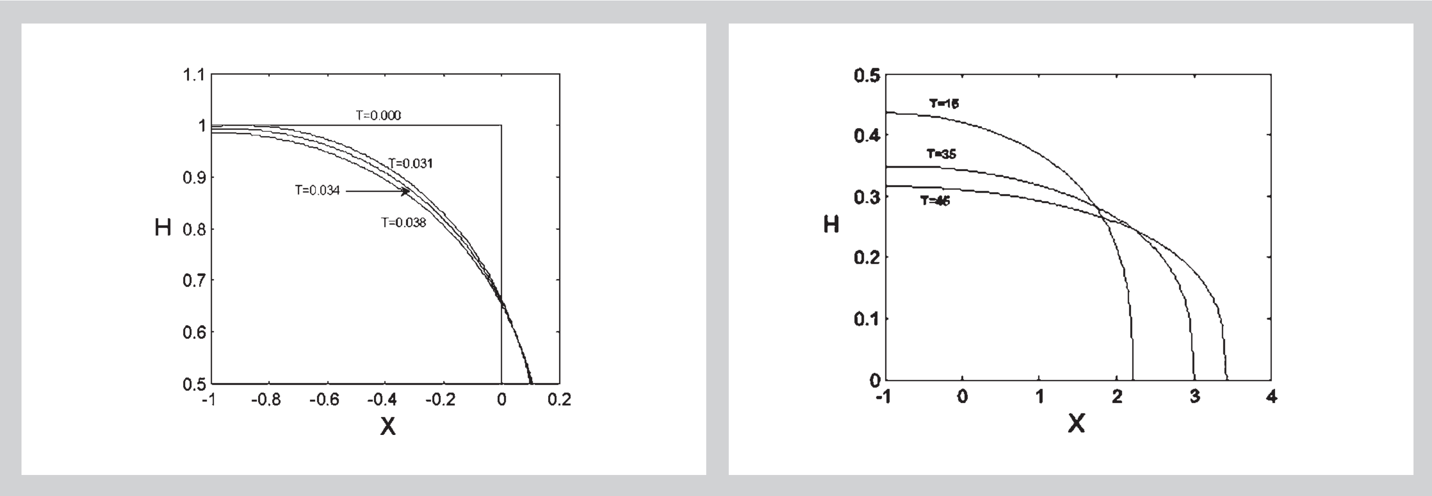


Figure 2:
Time variation of free surface profile in the short time regime (a) and in the long time regime (b).

Notice that the governing equation of Newtonian viscous dam-break flow is recovered from Eq. 5, for $n = 1$ [4]. Now, defining the following set of non dimensional variables $H = h/h_o$, $X = x/\Lambda$, and $T = t/\tau$ where

$$\tau = \frac{(m+2)\Lambda^{m+1}}{h_o^{2m+1}}\left(\frac{2k(m+1)}{\rho g m}\right)^m \tag{6}$$

and $m = 1/n$, equation of motion (Eq. 5) becomes

$$\frac{\partial H}{\partial T} + \frac{\partial}{\partial X}\left[\left[-\frac{\partial\left(H^{(2m+2)/m}\right)}{\partial X}\right]^m\right] = 0 \tag{7}$$

3 ANALYTICAL SOLUTION

3.1 FLOW REGIMES

Dam-break flow belongs to the general class of gravity currents so, the solution depends on time scale [1]. First of all, the inertial regime, characterized by a fixed height given by Eq. 4 at the dam-site, holds immediately after the dam collapse. Then, this results in a solution dominated by viscous effects. The transition occurs when the negative wave touches the rear wall. A short time regime takes place until the wave reflected by the fixed plate overtakes the front. This short time regime tends to an asymptotic form (long time solution). Such scenario was obtained for the dam-break flow of Newtonian fluid by Cavaillé [19]. In this configuration the viscous solution (short time regime for $H \geq 0.684$ and the long time regime for $H \leq 0.684$) is presented in Figure 2. Gratton and Minotti [20] describe the Boltzmann transform formalism of deriving solutions to Eq. 7, and the phase plane formalism of deriving similarity solutions as well. For consistometry purpose, Piau and Debiane [21] give a series expansion of the short time solution and focus on the shear stress and on the shear rate mean values in the long time viscous regime.

The originality of this paper is to build the analytical expression of the solution for the short time ($T << 1$) and long time ($T >> 1$) viscous solutions, and to combine them as the two viscous regimes of a single universal model. Moreover, Gratton and Minotti [20] stated that the physics of the Newtonian case can be generalized to the power-law model. So, the numerical computation of this solution will allow us to derive the effect of the mud rheology on the dynamics characteristics of the flow. Eq. 7 can be solved using the method of similarity. Assuming a solution of the form

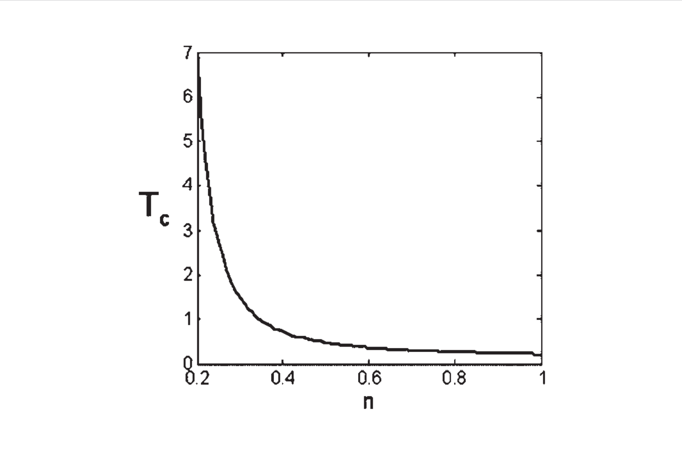
$$H(X,T) = \Omega(T)\Psi(\lambda) \quad where \quad \lambda = \frac{X-\phi(T)}{P(T)} \tag{8}$$

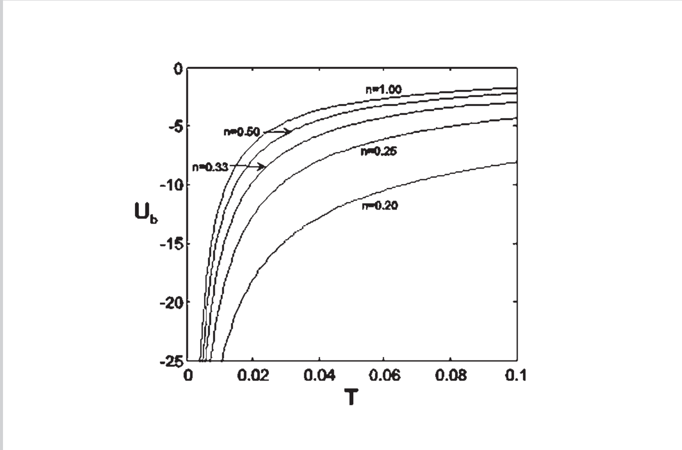
Introducing Eq. 8 in the equation of motion (Eq. 7) we get

$$\frac{d}{d\lambda}\left[\left[-\frac{d\left(\Psi^{(2m+2)/m}\right)}{d\lambda}\right]^m\right] + \frac{T'P^{m+1}}{T^{2m+2}}X - \frac{P'P^m}{T^{2m+1}}\lambda\frac{d\Psi}{d\lambda} - \frac{\phi' P^m}{T1}\frac{d\Psi}{d\lambda} = 0 \tag{9}$$

where a prime denotes the derivative of a given function with respect to its variable. To meet a scenario similar to Figure 2, let $X_b(T)$ denote the front of the back wave and $X_f(T)$ the front of the positive wave. If T_c denotes the time where the back wave front reaches the rear wall, the short time regime corresponds to a viscous solution such that $T \leq T_c$. While, for larger time, H is everywhere less than 1.

So, two regimes can be identified, which correspond to two different physical mechanisms of reservoir emptying. The short time solution is such that far downstream from the dam, the flu-





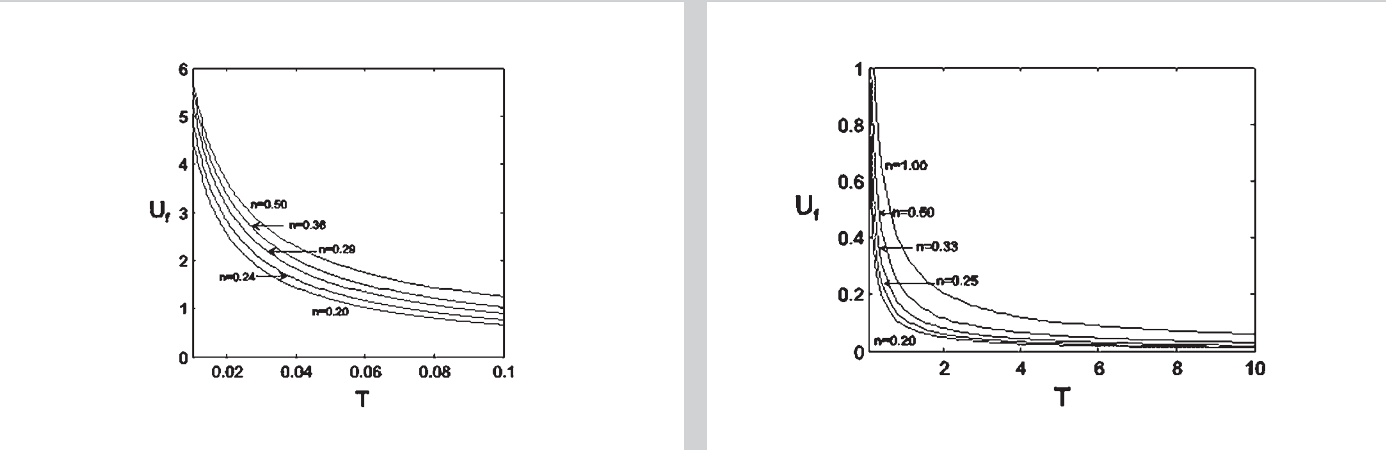


Figure 3 (left above): Variation of characteristic time with power law index.

Figure 4 (right above): Time variation of the velocity of the back wave for given power law index.

Figure 5 (below): Time variation of the velocity of the positive wave for given power law index in the short time regime (a) and in the long time regime (b).

id seems to be at rest at a depth h_o, so that the reservoir's length l_o has no effect on this flow regime, while for the long time solution, the flow only retains the initial (non dimensional) volume of the reservoir $V = l/L$ and not the details of its initial geometry.

3.2 THE SCALING LAWS

Giving the common following expression to $\Psi(\lambda)$ in both the short time and the long time viscous regime:

$$\Psi(\lambda) = \left[1-\lambda^{(1+m)/m}\right]^{m/(2m+1)} \tag{10}$$

$$T_c = \frac{1}{(m+1)\beta_m \alpha_m^{m+1}}, \qquad X_f(T_c) = \frac{1-\alpha_m}{\alpha_m} \tag{11}$$

where $\lambda = (X - X_o)/(X_f - X_b)$ in the short time regime and $\lambda = (X + 1)/(X_f + 1)$ in the long time regime while the constants α_m and β_m write

$$\alpha_m = \int_o^1 \left[1-\lambda^{(1+m)/m}\right]^{m/(2m+1)} d\lambda \tag{12}$$

$$\beta_m = \frac{\left|\frac{d}{d\lambda}\left[\left[-\frac{d}{d\lambda}\left[\left(1-\lambda^{(1+m)/m}\right)^{(2m+2)/(2m+1)}\right]\right]^m\right]\right|_{\lambda=\alpha_m}}{\int_{\alpha_m}^1 \left[1-\lambda^{(1+m)/m}\right]^{m/(2m+1)} d\lambda} \tag{13}$$

4 RESULTS AND DISCUSSION

First of all, the similar solution for the dam-break flow of a power-law fluid obtained in the previous section must be validated in the limiting case of a Newtonian fluid, by comparison with an experimental and theoretical study presented by [5], who obtained a short time viscous regime and a long time viscous regime, respectively described by the following scaling laws

$$X_{fs} \propto T^{1/2}, \quad X_{fl} \propto T^{1/5} \tag{14}$$

These results clearly agree with those of the present work for $n = 1$. Moreover, the fluid height at dam site in the short time regime found as $H_d = 0.664$ in present work agrees with the value of $H_d = 0.684$ obtained numerically in [22].

Then, the effect of the shear thinning property of the mud will be derived by computing the

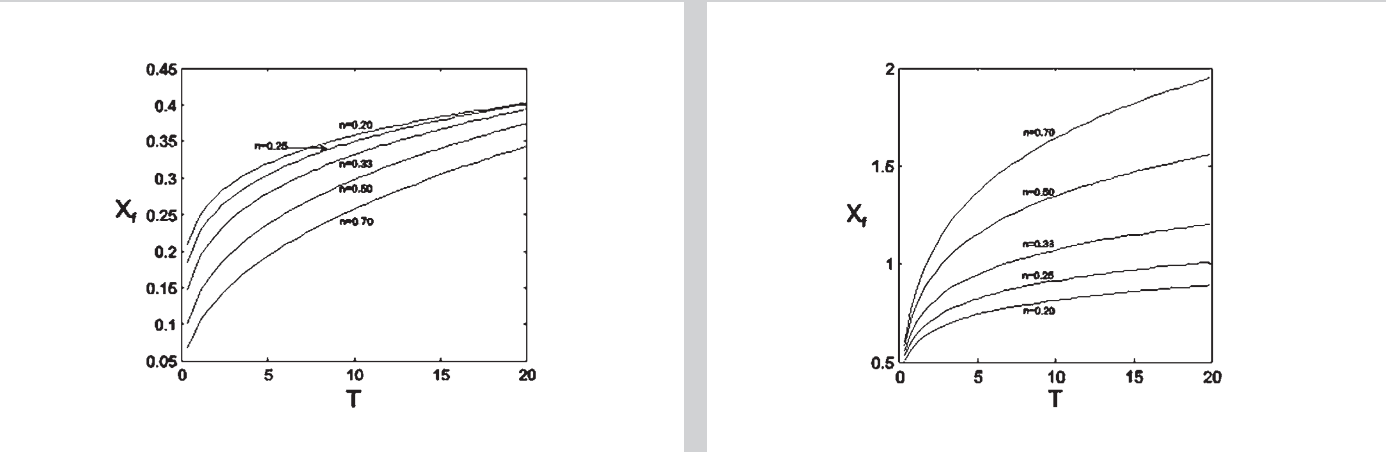


Figure 6 (above): Evolution of the front of the positive wave front for assigned power law index in short time viscous regime (a) and in the long time regime (b).

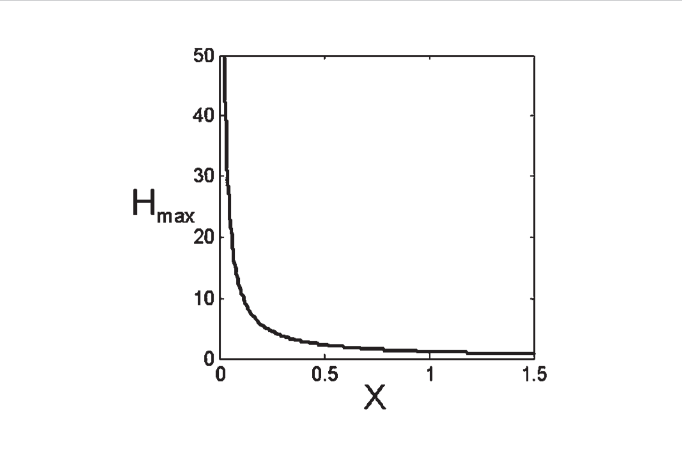


Figure 7 (left below): Variation of the maximum height at given station, vs the corresponding abscissa.

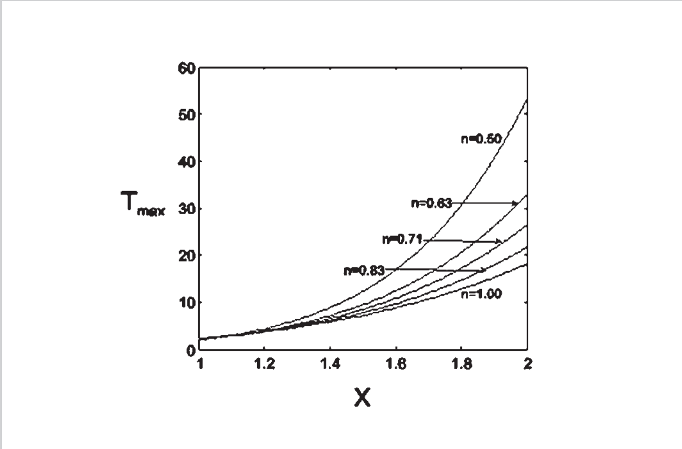


Figure 8 (right below): Variation of the time of occurrence of maximum height for given abscissa for given power law index.

solution obtained in the previous section. Indeed, the present computations showed that the general features of the physical description of the development of the dam-break flows of both the water and the mud are similar. Notably, the free surface profile obtained in the Newtonian case is similar to that of the mud, for the short time viscous regime and for the long time regime as well. Meanwhile, the effect of the power-law index on the solution will be pointed out owing to a reference to the Newtonian case.

Table 1 computes the different constants appearing in the solution together with different characteristic quantities in the range $0.2 \leq n \leq 1$. Recalling that $m = 1/n$, we see that for increasing power law index, the characteristic time decreases following the graph presented in Figure 3. This means that one effect of the shear thinning property is to slow down the back wave. This result can be verified in Figure 4 which shows that $|U_b(T)|$ increases with n. The same remark holds for the velocity of the front wave $U_f(T)$ in both the short time regime (Figure 5a) and the long time regime (Figure 5b). As a consequence, the abscissa of the positive wave $X_f(T)$ increases with the power law index, both for the short time regime (Figure 6a) and for the long time regime (Figure 6b).

Meanwhile, Figure 7 shows that for all the tested values of the power law index ($0.2 \leq n \leq 1$), the maximum heights at given station versus the corresponding abscissa were described by a single graph. Concerning the time when H_{max} occurs at given station, its graph for assigned power law index is presented in Figure 8. As expected, T_{max} is shorter for larger n which agrees with the results obtained in Figure 5b.

All these results illustrate the increasing of the velocity of the negative wave and the positive wave with power law index, due to lower friction for the water than for the mud characterized by the same consistency. Finally, Table 1 shows that for decreasing power law index, the critical height H_d decreases, which means that the quantity of fluid mobilized during the short time regime is larger for the mud than for the water. As a consequence, for a structure situated in a vulnerable zone, the impact will occur later for the mud than for the water as stated earlier in this section, but the quantity of fluid will be larger. Moreover, considering that the density of the mud is larger than that of the water, the impact of the mud is expected to be greater and potentially produce more damages. This effect was discussed in [23 - 25], among others.

5 CONCLUSION

The horizontal dam-break flow of mud modeled by a power-law fluid was considered analytically. The channel was smooth and dry at initial time. The solution of such problem depends on the time scale. The inertial regime characterized by dominant inertial forces, takes place immediately after dam collapse, and holds until the negative wave touches the rear wall. In such an inviscid solution (Ritter solution), the rheological behaviour of the mud has no influence. Then, the viscous forces become the dominant forces and a viscous solution is obtained. Applying the conservation of mass and momentum with the shallow water approximation, an equation of motion was derived and made non-dimensional. The analytical solution of this viscous flow was worked out in terms of wave front dynamics and spatio-temporal variations in the fluid height, with a self-similar form. The short time solution holds until the wave reflected by the (fixed) rear wall overtakes the front and then the long time one governs the asymptotical flow dynamics and shape. The limiting case of a Newtonian fluid derived from this solution was successfully compared with previous experimental, analytical and numerical solution. Then the shear thinning property of the mud was pointed out. Firstly, it was shown that the general features of the physical description of the development of the dam-break flows of both the water and the mud are similar. Moreover, all these results illustrate the increasing of the velocity of the negative wave and the positive wave with power law index, due to lower friction for the water than for the mud characterized by the same consistency. For all the tested values of the power law index ($0.2 \leq n \leq 1$), the maximum heights at given station versus the corresponding abscissa were described by a single graph. Finally, it was shown that for a structure situated in a vulnerable zone, the impact will occur later for the mud than for the water but the quantity of fluid will be larger, and considering that the density of the mud is larger than that of the water, the impact of the mud is expected to be greater and potentially produce more damages.

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