Preface
Special Issue: In honor of David Jerison

Guozhen Lu*

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To celebrate the many distinguished achievements and a remarkable career of Professor David Jerison from the Massachusetts Institute of Technology, this special issue of *Advanced Nonlinear Studies* is dedicated to him on the occasion of his 70th birthday.

David Jerison is an outstanding mathematician widely known for his work on partial differential equations. He is a leading figure in the areas of harmonic analysis methods to partial differential equations. He started out studying linear elliptic boundary value problems in non-smooth domains and sub-elliptic problems, especially those related to the Heisenberg group. He proved some sharp Carleman inequalities, quantitative forms of unique continuation. He then began to consider nonlinear problems. Jerison has made many fundamental and influential contributions in all these directions and his works have been followed widely by many mathematicians in the past few decades.

The classical Minkowski problem asks whether one can find a convex polyhedron given area and normal vector of each face. The corresponding inverse problem in the plane is quite easy: find the (convex) polygon given its sidelengths and angles. Jerison noticed that the Schwarz-Christoffel formula for conformal mapping solves an analogous inverse problem for harmonic measure: find the convex polygon given its angles and the harmonic measures of its sides. He generalized this to higher dimensions, solving the problem of finding a convex polyhedron given the harmonic measure and normal vector of each face. The smooth version of this problem involves a global, fully nonlinear partial differential equation relating Gauss curvature to harmonic measure. Jerison then introduced and solved a different problem that is much closer structurally to the Minkowski problem, namely, one which substitutes electrostatic capacity for volume in Minkowski's theory. Capacity can be defined as the total flux of the equilibrium potential through the boundary, and the flux is a suitably normalized harmonic measure viewed from infinity. Equivalently, capacity is the (exterior) Dirichlet

*** Corresponding author: Guozhen Lu,** Department of Mathematics, University of Connecticut, Storrs, CT 06269, United States, e-mail: guozhen.lu@uconn.edu

energy of the equilibrium potential. There is a parallel, technically simpler, Minkowski-type problem with volume replaced by the ground state eigenvalue, which is a kind of interior Dirichlet energy of the convex body.

Inspired by seminal papers by Alt-Caffarelli, Alt-Caffarelli-Friedman, and Caffarelli, Jerison has worked for three decades on another family of nonlinear problems, free boundary problems. A major theme of this work is the parallel between free boundaries and minimal surfaces. The element that unites it with his other work is the relevance of harmonic measure. Jerison also studied an evolution equation, the Hele-Shaw equation, in which the growth of a fluid region is governed by the boundary pressure, which is none other than harmonic measure. This led him to a discrete version of this evolution known as internal diffusion-limited aggregation, a model introduced by chemists to describe corrosion and chemical polishing. Jerison's current main focus is on shapes of eigenfunctions and the curvature of their level sets. In this context, flux across level sets replaces harmonic measure.

Jerison's work on the Poincaré inequalities for Hörmander's vector fields, the CR Yamabe problem and sub-elliptic equations have influenced greatly my own research interests. My work on sharp constants and extremal functions for geometric and functional inequalities has been inspired by his pioneering work.

Jerison is a principal investigator for the Simons Collaboration on Waves in Disorder, a cross-disciplinary project based on experiments with inorganic and organic semiconductors and Bose-Einstein condensates. He serves regularly as faculty advisor to SPUR, the MIT math department's summer research program for undergraduates, as well as RSI, a high school science summer research program for students from across the world, which takes place at MIT. He is also known for his widely viewed, on-line lectures in calculus on MIT's Open Courseware and other platforms.

David Jerison was born in 1953 in Lafayette, Indiana. After college and before graduate school, he studied for a year at the University of Paris 11, at Orsay. After completing his PhD at Princeton University in 1980 under the direction of Elias Stein, he spent a year as an NSF Postdoctoral Fellow at the University of Chicago. He joined the MIT mathematics faculty in 1981, where he has remained for the rest of his career. He received a Sloan Research Fellowship and a Presidential Young Investigator Award in 1985 and delivered an invited address at the ICM in Zurich in 1994. He was selected as a Margaret MacVicar Faculty Fellow in 2004 and awarded the Bergman Prize in 2012 for his joint work with Jack Lee on the CR Yamabe problem. He received a Simons Fellowship in 2018 and a Guggenheim Fellowship in 2019. Jerison is a Fellow of the AMS and the American Academy of Arts & Sciences and served as Vice President of the AMS from 2017 to 2020.

In this special issue, we invited articles from both well-established mathematicians and young scholars. Many of them are leading researchers in the areas of harmonic analysis and partial differential equations, including some of the world's most prominent mathematicians. The wide range of topics covered in this special issue also reflect the broad scope of Jerison's research interests. I am grateful to all the contributors and reviewers for making this special issue possible.

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Guozhen Lu
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