Research Article

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Comments on the Signaling Theory of IPO Underpricing and Investor Protection Laws

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Abstract: It has been widely known that good firms use lower IPO prices to signal their superior prospects to investors. The underlying intuition is that good-type firms signal their type by underpricing their initial issue of shares, because investors can rationally infer that only the best can recoup the signaling cost from subsequent issues. In this paper, we argue that the intuition is not complete. We show that a good firm always has an incentive to deviate to raise the IPO price slightly from its equilibrium price if the price is the only signaling device, implying that signaling by underpricing is not an equilibrium phenomenon in the case of one-dimensional signal. Then, we show that if the firm can choose the equity fraction to be sold as well, a good-type firm can signal its high profitability by choosing a low equity fraction. In this case, a good-type firm engages in underpricing, but it cannot be a signal because both types choose same prices in equilibrium. We also discuss the effect of investor protection laws on IPO underpricing.

Keywords: investor protection laws; IPO; IPO puzzle; separating equilibrium; signaling; underpricing

JEL Classification: G10; G12; G30

1 Introduction

A company often announces initial public offering (IPO) when it decides to raise funds through sale of securities or shares for the first time to the public. The main purpose of IPO is to raise capital for the future growth of the company. The offering price is usually determined not only by many quantitative factors including future profitability and cash flow etc. but also by a strategic motive.

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It is well known that IPO prices are often underpriced.[1] Underpricing is the practice of listing IPO at a price below its real value in the stock market, which is called the IPO puzzle. Many explanations for this anomaly have been offered. Among others, Allen and Faulhaber (1989) provided a signaling explanation.[2] Their main insight is that firms with good prospects find it optimal to signal their type by underpricing their initial issue of shares, and investors know that only the best can recoup the cost of this signal from subsequent issues.

In this paper, we argue that the intuition behind their main result is not complete. We show that a good firm always has an incentive to deviate to raise the IPO price slightly from its equilibrium price if the price is the only signaling device. This implies that there is no separating equilibrium, that is, signaling by underpricing does not occur in equilibrium in the case of one-dimensional signal. If the firm can choose the equity fraction to be sold as well as the price, however, a high-type (good-type) firm can signal its high profitability by choosing a low fraction of equity. In this case, a high-type firm still engages in underpricing in the sense that it sets a lower IPO price than the real value of the firm, but underpricing cannot be a signal because both types choose the same price in equilibrium.

Recently, the effect of investor protection laws on IPO underpricing has received attention and several authors has studied the issue. Since long ago, governments addressed certain policy issues around firms raising money from the public with respect to information asymmetries, misleading information, and conflicts of interest. Investor protection laws are to enhance investor protection and disclosure primarily by mandating disclosure of certain information such as profitability and ownership structure so that investors can value companies more easily.

Most of the literature on this line examine the issue empirically. For instance, La Porta, Lopez-De-Silanes, and Vishny (1997) show that higher levels of investor protection are associated with a larger number of IPOs. However, they do not investigate the relation between the level of underpricing of IPOs and the level of investor protection. Engelen and Van Essen (2008) empirically demonstrate a negative relationship between a higher level of investor protection and the level of underpricing. Chen et al. (2022) also show that IPO underpricing is negatively related to the level of investor protection. In this paper, we use a proxy for disclosure requirement to measure an investor protection law and show that it has no effect on IPO underpricing, contrary to the empirical literature.

The paper is organized as follows. In Section 2, we consider a simple benchmark model. In Section 3, we analyze the benchmark model to show that there is no

1 Evidence of underpricing is well documented. See Ibbotson (1975), Loughran, Ritter, and Rydqvist (1994). For recent evidence, see Ritter (2020).

2 Allen and Faulhaber (1989) borrow the insight from Ibbotson (1975).

separating equilibrium in which IPO underpricing signals a firm's profitability. In Section 4, we analyze a model with two-dimensional signals, and briefly discuss the effect of investor protection laws on IPO underpricing. In Section 5, we discuss the robustness of our result by considering extended models, in particular, investor protection laws. Concluding remarks follow in Section 6.

2 Simple Benchmark Model

We closely follow the central assumptions of Allen and Faulhaber (1989) – which will be abbreviated as AF. Basically, our model is a reduced version of their model.

There is a firm and investors in the IPO market. The total number of shares outstanding of the firm is normalized to one. The firm is going to offer a certain fraction α of its equity to the public (homogeneous investors) in an IPO at some IPO price p to acquire the capital needed for its new project which is denoted by $K(>0)$. We assume that $\alpha p \geq K$ because the IPO sales revenue must finance K. We will call this financing constraint (FC).

Let π be the firm's future profit (its future value).[3] The future profit is either high (H) or low (L) with $H > L > 0$. We assume that the true value of π is known to the firm but not known to the investors. The investors only know the prior probability that $\pi = H$ which will be denoted by $\theta \in (0, 1)$.[4] We assume that the investors are risk-neutral in the face of uncertainty about the firm's type.

The interaction between the firm and the investors goes as follows. In the first period, the firm offers the IPO price p to sell the fraction α of its equity, and then the investors decide whether to buy the shares (invest) or not. After that, π is realized in the second period. We can interpret π as the stock price equivalently, since the total number of shares is one. We assume no dividend.[5]

The payoff of the firm's owner is $\alpha p + (1 - \alpha)\pi$ if α of the equity is sold, while it is L if it is not sold. We are assuming that the high-type firm's innovation by the new project succeeds with probability one ($\lambda = 1$) in the notation of Allen and Faulhaber (1989). Note that both types of the firm get L because even a high type cannot

3 Since our model is a reduced version of Allen and Faulhaber (1989)'s multi-period model, our π corresponds to their V. That is, all the relevant information about future earnings flow in V is compressed in π.

4 Our model follows a lottery interpretation of an incomplete information game. Alternatively, we can interpret our model as having infinitely many firms of which the proportion θ is H type. This is called a random-vector interpretation. It is well known that the two interpretations are analytically equivalent. See the classical article of Harsanyi (1967).

5 In Allen and Faulhaber (1989), dividends play a crucial role in computing the firm's value. In our model, we assume that the firm's value is realized in the second period without assuming dividends.

implement the profitable project if it cannot ensure the required capital by IPO.[6] The payoff of the investors is $\alpha(\pi - p)$ if they invest by buying the shares at the price p and is zero if they do not invest.

3 No Separating Equilibrium in the Benchmark Model

In this section, we analyze the benchmark model. Our analysis will focus on the possibility that a separating equilibrium in which signaling by underpricing occurs exists. We will use the (weak) Perfect Bayesian Equilibrium (wPBE) as our main equilibrium concept. Roughly speaking, wPBE is defined by a strategy profile and a belief satisfying that (i) the strategy profile is sequentially rational given the belief in the sense that at each information set, each player chooses the optimal strategy given the other player's strategy and the belief, and (ii) the belief must be weakly consistent with the equilibrium strategy profile in the sense that the belief must be updated according to Bayes' law whenever it is possible.[7]

To figure out the configuration of the possible separating equilibrium, we will resort to the first best outcome under complete information.

3.1 Complete Information Case

The equilibrium can be found by backward induction. Since this is a complete information game, it suffices to find the subgame perfect equilibrium in this dynamic game.

It is clear that given any IPO price p, the investors accept the price offer (buy the shares at the price) if $p \le \pi$.

Now, consider the firm's pricing decision. Let $p^*(\pi)$ be the equilibrium price of type π. Taking the investors' decisions into account, the firm will choose the IPO price p which is the maximal price that the investors will accept. Therefore, the equilibrium prices under full information must be $p^*(H) = H$ and $p^*(L) = L$. It is also clear that the H type prefers this outcome to no investment outcome which is obtained when it

6 This assumption follows the spirit of Allen and Faulhaber (1989) who assumes that a bad firm can never be a good firm by innovations. The only difference is that a good firm remains a good firm certainly if it succeeds in financing in our model, whereas it can become a bad firm with some probability even if it succeeds in financing in their model.

7 For the formal definition of wPBE, see Mas-Colell, Whinston, and Green (1995).

offers $p > H$, because $\alpha H + (1 - \alpha)H = H > L$, while the L type is indifferent between investing and not investing because $\alpha L + (1 - \alpha)L = L$.

3.2 Incomplete Information Game

Our main interest in this section is whether a separating equilibrium in which an IPO price signals the value of the firm is possible. We denote the separating equilibrium price of the high type and the low type by p_H and p_L respectively, and the investors' posterior belief updated after observing p by $\theta\ (p)$. Instead of θ , we may denote by π_1^e the perceptions of investors about π in the first period. That is, $\pi_1^e \equiv \theta\, H + (1 - \theta\,)L$, so $\pi_1^e = H$ if $\theta\ = 1$ and $\pi^e = L$ if $\theta\ = 0$.

If the true value of the firm is not known to the investors, a low type wants to pretend to be a high type because he could sell his shares at a higher price by doing so. However, if the true value of the firm is revealed in the second period and is fully reflected in π, i.e., $\pi_2^e = \pi$ where π_2^e is the second-period perception of incestors about π, regardless of the IPO price p, a low type gains nothing in the second period by pretending a high type. If $p_H > p_L$, a low type can successfully imitate the high type by offering p_H which will be always accepted. So, it cannot be an equilibrium. If $p_H < p_L$, a low type loses by pretending to be a high type. In this case, he has no incentive to deviate from p_L. So, a necessary condition for a separating equilibrium is that $p_H < p_L$.

Suppose $p_H < p_L$, i.e., a high type underprices in equilibrium. It is easy to see that the low-type firm's separating equilibrium price is not distorted, i.e., $p_L = p^*(L)$, because the low type would deviate to $p^*(L)$ if $p_L \neq p^*(L)$, under the most pessimistic belief, because $p^*(L)$ is the best price for the low-type firm among the prices such that $\theta\ (p) = 0$. The equilibrium price of the high-type firm must satisfy the following incentive compatibility condition of the type:

$$ap_H + (1 - \alpha)H \geq \max\{L, ap + (1 - \alpha)H\}. \tag{1}$$

Inequality (1) is the incentive compatibility condition of the high type which will be called [ICH1]. It requires that a high-type firm has no incentive to deviate to any other price than its equilibrium price p_H. The right hand side of (1) is the high type's payoff when it deviates from p_H. If it deviates to $p > L$, investors do not buy the shares, so its payoff is just L. If it deviates to $p \leq L$, investors invest and thus the firm's payoff is $ap + (1 - \alpha)H$.[8]

8 If $p_H < p_L = L$, the incentive compatibility condition of a low type, which will be abbreviated as [ICL1], is trivially satisfied: $L \geq ap_H + (1 - \alpha)L$.

It is not difficult to see that this incentive compatibility condition cannot be compatible with the optimal decision of the investors. Since $p_H < L$, the high-type firm would deviate to $p \in (p_H, L)$ because such a price would always be accepted by the investors. The investors would infer from this price that the firm must be a low type, but finds the price still attractive enough to buy the shares because the price is too low ($p < L$);[9] hence, no IPO underpricing in equilibrium. To summarize, we have

Proposition 1. *There exists no separating equilibrium in this model with the one-dimensional signal (IPO price).*

Proof. The proof is immediate from the argument that any deviation to $p \in (p_H, L)$ is profitable for a high-type firm. □

Some may suspect that this result is an artifact of the assumption that only pure strategies are available. What if we allow mixed strategies of the investors? Unfortunately, it turns out that separation (semi-separating equilibrium) is not possible even with mixed strategies.

Let $r(p) \in [0, 1]$ be the probability that investors accept the IPO price p. It is true that the incentive of a high-type firm to deviate to $p \in (p_H, L)$ could be deterred if we allow mixed strategies of the investors so as to make the probability that the investors accept p strictly less than one. However, the mixed strategy $r(p) < 1$ for any $p \in (p_H, L)$ cannot be optimal for the investors because they always strictly prefer investing at the price p to not investing, because $\alpha(\pi^e - p) > 0$ for any $p \in (p_H, L)$ and for any belief π^e. As long as the investors always buy the shares at $p \in (p_H, L)$ with probability one, the high type always deviates to p from the equilibrium price $p_H (< L)$.

4 Model of Two-Dimensional Signals

So far, we assumed that the fraction of shares that is sold at the market for IPO is exogenously fixed. In this section, we consider an extended model in which the firm can choose the fraction to be sold as well as the IPO price.

9 In Allen and Faulhaber (1989), the strategy of investors and their posterior belief off the equilibrium path are not clearly defined. We presume that their definition of investors' strategy is the same as ours from their phrase "investors will not pay more for the firm than its value to them." Then, the investors' decisions depend on the assumption on the off-the-equilibrium belief because it determines the expectation of the future value of the firm. What we have shown is that investors are willing to pay the price $p < L$ under the most pessimistic belief or even under any belief. Taking this optimal decision of the investors into account, a high-type firm will deviate to such a price $p \in (p_H, L)$; hence, no separating equilibrium.

This model is motivated by the following observation. The main reason why no separating equilibrium exists in the previous model is that a high-type firm can always profitably deviate by increasing the IPO price slightly ($p > p_H$) which will be still accepted by the investors. However, if the firm loses something by increasing the price, it may not profitably deviate to $p \in (p_H, L)$. For example, if the firm must sell more shares at a higher price and keep less shares, a high type may not deviate from a low IPO price p_H, and accordingly, the underpriced IPO price $p_H(<L)$ may be a separating equilibrium price. Below, we will investigate this possibility.

Let (α_H, p_H) and (α_L, p_L) be the equilibrium pair of choices of the high-type firm and the low-type firm respectively where $p_H < p_L = p^*(L)$. Note that the decision of the investors remain unaffected for any α_H and α_L, i.e., the investors buy the shares α_H at price p_H if $p_H \leq H$ and if $p \neq p_H$, they buy the fraction α if $p < L$ for any α. This implies that $p_H(<L)$ can never be an equilibrium price of a high-type firm even in this model, because a high type can always profitably deviate to (α_H, p) with $p \in (p_H, L)$, insofar as the investors' decision is unaffected for any α_H. This implies that it must be that $p_H = p_L = L$ in a separating equilibrium. That is, a high-type firm can signal its type only by the fraction of shares to be sold in the IPO market, i.e., $\alpha_H \neq \alpha_L$. Accordingly, $\theta\ (\alpha_H, L) = 1$ and $\theta\ (\alpha_L, L) = 0$ in equilibrium. Again, we impose the most pessimistic off-the-equilibrium belief, i.e., $\theta\ (\alpha, p) = 0$ for any $(\alpha, p) \neq (\alpha_H, L)$.

The equilibrium fractions of equity α_H and α_L must satisfy two incentive compatibility conditions. Let $V(\alpha, p; \pi)$ be the payoff of π-type firm when it chooses α and p. Then, the incentive compatibility conditions require (i) $V(\alpha_H, p_H; H) \geq V(\alpha, p, ; H)$ for any (α, p), and (ii) $V(\alpha_L, p_L; L) \geq V(\alpha_H, p_H; L)$. To elaborate, we have

$$(i)\alpha_H p_H + (1 - \alpha_H)H \geq \max\{L, \alpha p + (1 - \alpha)H\}, \forall (\alpha, p) \neq (\alpha_H, L), \qquad \text{[ICH2]}$$

$$(ii)L \geq \alpha_H p_H + (1 - \alpha_H)L, \qquad \text{[ICL2]}$$

where $p_H = L$.

Equation [ICH2] is the incentive compatibility condition for the high-type firm and [ICL2] is the incentive compatibility condition for the low-type firm. The right hand side of [ICL2] is the low type's payoff when it imitates the high type by choosing α_H. It is easy to see that [ICL2] is trivially satisfied for any $\alpha_H \neq \alpha_L$ if $p_H = L$. In inequality [ICH2], $\alpha p + (1 - \alpha)H$ is the high-type firm's payoff when $p < L$ so that its deviant offer (α, p) is accepted by the investors, and L is his payoff when $p > L$ so that (α, p) is rejected. Since it is clear that $\alpha_H L + (1 - \alpha_H)H > L$, it suffices to consider the case that $p < L$.

We know that (FC) imposes s lower bound for α_H because $\alpha_H p_H = \alpha_H L \geq K$ implies that $\alpha_H \geq \frac{K}{L} \equiv \bar{\alpha}_H$. Since $V(\alpha_H, L; H)$ is decreasing in α_H, setting α_H as low as possible, i.e., $\alpha_H = \bar{\alpha}_H$ is the optimal equity fraction of the high type. Now, any deviation (α, p)

also has to satisfy (FC) condition requiring that $\alpha p \geq K$. Since $p < L$, it implies that $\alpha > \alpha_H$. Then, it is easy to see that inequality [ICH2] is satisfied for any (α, p) such that $\alpha > \alpha_H$ and $p < L$, i.e.,

$$\alpha_H L + (1 - \alpha_H)H \geq \alpha p + (1 - \alpha)H.$$

This implies that a high type has no incentive to deviate from p_H, either.

Proposition 2. *If the IPO firm can chooses the IPO price and the fraction of equity to be sold in the IPO market, there exists a separating equilibrium in which the high-type firm chooses* $(\bar{\alpha}_H, L)$ *and the low-type firm chooses* $(\bar{\alpha}_L, L)$ *with* $\alpha_L > \alpha_H$.

Proof. The proof is immediate from the above argument. □

This proposition implies that a high-type firm underprices in equilibrium. Underpricing in this paper does not mean that the high-type firm's price is lower than the low-type firm's price. It means that the high-type firm's price is lower than its first-best price, i.e., $p_H < p^*(H)$. However, this underpricing in equilibrium is possible only by the accompanying choice in the equity fraction to be sold in the IPO market. If a firm sells a smaller fraction of equity in the IPO market, it signals a high profitability of the firm. Note that a high-type firm cannot signal by underpricing, because both types choose the same low IPO price so that the public cannot tell by the IPO price. Although a high-type firm engages in underpricing in equilibrium, it signals by choosing a low equity fraction to be sold, not by underpricing. Then, how can a high-type firm signal its type with the same price as a low-type firm? Since the cost of selling the shares is higher for a high-type firm who knows that its future value will be higher, it will sell a lower fraction of equity which cannot imitated by a low-type firm who prefers increasing the monetary revenue by selling a higher fraction of equity. This is consistent with the insight of Myers and Majluf (1984) that equity financing may be a bad signal of the firm's profitability.

At this point, it is important to compare this with the result of Allen and Faulhaber (1989). Their incentive compatibility conditions of a good firm and a bad firm are shown in their (9a) and (9b) as follows;

$$R_G(p_0, \lambda) \geq R_G(V_0(0), 0), \tag{2}$$

$$R_B(p_0, \lambda) \geq R_B(V_0(0), 0). \tag{3}$$

In these inequalities, λ is the probability that a good firm remains to be good after innovation. As we put in Footnote 3, we assume that $\lambda = 1$. Also, p_0 is the IPO price and $V_0(0)$ is the value of the firm when $\theta = 0$. Inequality (2) compares the firm's payoff when it chooses the good type's equilibrium price (p_0) and the bad type's equilibrium price ($V_0(0)$). However, they did not check whether the left hand side of (2) is not less than the good firm's payoff when it deviates to another price $p' \neq V_0(0)$, as we checked in [ICH1] and [ICH2]. In fact, we showed in this paper that a good type (a high type) always has an incentive to slightly increase the IPO price to $p' = p_0 + \epsilon$ for $\epsilon > 0$. Such a deviation $p' \in (p_0, V_0(0))$ is always accepted and so it is profitable for a good firm, insofar as $p_0 < V_0(0)$. Some may think that the profitability of the deviation depends on the off-the-equilibrium belief of the deviation when the investors observe p'. However, as we argued in Proposition 1 of this paper, it is profitable even under the most pessimistic belief, implying that it is profitable regardless of the belief.

To close this section, it is worthwhile to note Allen and Faulhaber (1989) remark that a necessary condition for separation to occur is $0 < \lambda < 1$. They argue that if "a good firm could somehow signal its type" and $\lambda = 1$ as we assumed in this paper, the investors do not need further observations of dividend outcomes whatever to tell whether it is a good firm. It may be correct that they do not need information of dividend outcomes, but even if $\lambda = 1$, the benefit of the signal differs across types insofar as the additional capital increases the future profitability only for the high type, i.e., the probability of successful innovations for a high type and a low type ($\lambda = 1$ vs. $\lambda = 0$) differs. Contrary to the argument of Allen and Faulhaber (1989), a separating equilibrium is possible in our model, although we assume that $\lambda = 1$.

5 Discussions

In this section, we briefly discuss the possibility that the separating outcome of signaling by underpricing can be recovered by extending the model into various directions.

5.1 Investor Protection Laws

We examine how investor protection laws can affect IPO underpricing. Since many rules and amendments to enhance investor protection in IPO consider information asymmetries as one of the main causes in IPO underpricing, we may identify a stronger investor protection law by stronger a disclosure requirement.

Suppose that the firm releases some required report to the public before IPO pricing. Then, the investors receive some signal $s = G$ or B from the report. We assume that investors receive G with probability $q\,(>\frac{1}{2})$ or B with the remaining probability $1 - q$ if $\pi = H$ and they receive G with probability $1 - q$ or B with q if $\pi = L$. It means that a report from a high-type firm is not always interpreted as a good news (G). It may be interpreted as a bad news (B) by some investors. If the disclosure gets more reliable, q gets higher, that is, $q = 1$ if disclosure is perfect, and $q = \frac{1}{2}$ if the report is completely unreliable. We assume that q is higher as the investor protection law is stronger.

In the first period, the firm sells a fraction α of its equity by offering the IPO price p. Then, investors decide whether to purchase the shares, based on the observations of s and p.

In this modified game with investor protection laws, the investors' purchasing decision is slightly modified. Their posterior belief is updated based on p and s. So, we will denote the resulting posterior belief by $\theta\ (p, s)$. Then, their perceptions about π in the first period is $\pi^e = \theta\ (p, s)H + (1 - \theta\ (p, s))L$. It is clear that the investors buy the shares if $p \le \pi_1(p, s)$.

Since IPO underpricing can occur only in a separating equilibrium, we will restrict our attention to separating equilibria. Then, it is not difficult to see that the analysis for a separating equilibrium is not affected very much. This is mainly because in a separating equilibrium, the true value of the firm is revealed by its strategy (price or the equity shares), regardless of the signal s. This implies that there is no separating equilibrium in the model with one-dimensional signal. The same intuition applies. In a separating equilibrium, if $p_H < p_L$, the high-type firm would profitably deviate to any $p \in (p_H, p_L)$ because investors will always buy the shares under the most pessimistic belief. If $\theta\ (p, s) = 0$, investors' net benefit from buying the shares is $\pi_1^e - p = L - p > 0$ for any $p \in (p_H, p_L)$. This is true even for the most optimistic belief, because the investors' benefit is then $\pi_1^e - p = H - p > L - p > 0$ for any $p \in (p_H, p_L)$. The result that we obtained in the model with two-dimensional signals is not affected, either. It is clear that $p_H = p_L = L$ for the same reason. Now, suppose that $\alpha_H \neq \alpha_L$. The definition of a separating equilibrium requires us to infer that $\theta\ (\alpha_H, L\,; s) = 1$ and $\theta\ (\alpha_L, L\,; s) = 0$ for any $s = G, B$. Then, under the most pessimistic belief, the incentive compatibility conditions given by [ICH2] and [ICL2] are not affected. Since [FC] condition also remains unaffected, Proposition 2 saying that a high-type firm can signal its type only by selling a lower fraction of equity is still valid, regardless of q. This implies that investor protection laws have no effect on IPO underpricing at all.

5.2 Partial Revelation of Information

In this paper, we assumed that all uncertainty regarding the profitability of the firm is resolved at the end of the first period, but readers may be interested in whether this assumption is crucial to our result. So, in this section, we will examine whether the result would be affected if the true value of the firm is not completely realized in the final (second) period so the second-period value depends partially on the first-period belief of the investors, as in Allen and Faulhaber (1989).

Let us assume that some investors become fully informed of π in the second period and others are not. Let $\mu \in (0, 1)$ be the proportion of investors who are uninformed of π in the second period. Then, we can assume that the firm's perceived valuation in the second period is $\pi_2^e = \mu\pi_1^e + (1 - \mu)\pi$.

Under this alternative assumption of partial realization, let us check the incentive compatibility conditions. Now, [ICH1] and [ICL1] are modified as the following [ICH3] and [ICL3]:

$$\alpha p_H + (1-\alpha)H \geq \max\{L, \alpha p + (1-\alpha)(\mu L + (1-\mu)H)\}, \forall p < L \qquad \text{[ICH3]}$$

$$L \geq \alpha p_H + (1-\alpha)(\mu H + (1-\mu)L). \qquad \text{[ICL3]}$$

Note that the most profitable deviation of an H-type firm is to $p = L$ which will be accepted by investors. The firm's profit in this case, which is the right hand side of [ICH3], is $\alpha L + (1-\alpha)(\mu L + (1-\mu)H) = L + (1-\alpha)(1-\mu)(H-L)$. So, [ICH3] is satisfied if $\mu \geq \frac{\alpha}{1-\alpha}\frac{L-p_H}{H-L}$, or equivalently,

$$p_H \geq L - \frac{1-\alpha}{\alpha}\mu(H-L). \qquad (4)$$

The inequality [ICH3] can be more easily satisfied if the punishment for a deviation from p_H by the worst belief is more severe (μ is larger) or the price distortion is less severe (p_H is higher).

On the other hand, a low-type firm will have no incentive to imitate a high type by choosing p_H if p_H satisfies [ICL3] which boils down to $\mu \leq \frac{\alpha}{1-\alpha}\frac{L-p_H}{H-L}$, or equivalently,

$$p_H \leq L - \frac{1-\alpha}{\alpha}\mu(H-L). \qquad (5)$$

Again, note that it is easier to satisfy [ICL3] if the punishment for a deviation to p_H is more severe (i.e., μ is smaller) or the price that he will imitate is lower (p_H is lower). Since (4) and (5) are contradictory to each other except for the knife-edge case that both types are just indifferent between choosing its own equilibrium price and the other type's equilibrium price, we can say that there is essentially no meaningful separating equilibrium in which a high type signals its type by underpricing.

The intuitive reason for the nonexistence of a separating equilibrium is quite clear. Since the low-type firm's IPO price cannot be distorted from $p_L = L$ in any equilibrium, it remains to check whether it is possible that $p_H > L$ or $p_H < L$ in equilibrium. If $p_H > L$, a low type always has an incentive to deviate to p_H to imitate a high type. If $p_H < L$, a high type always has an incentive to deviate to $p' \in (p_H, L)$, as we argued, because such a price p' is always accepted insofar as the price is lower than the worst perception on the firm value; hence, no separating equilibrium. Note that those two intuitions are both so robust that this result cannot be affected by any assumption on the second-period perceived value of the firm.[10]

5.3 Model with Issuing Costs

To examine the possibility that the robust intuition for nonexistence of a separating equilibrium (in the case of a single-dimensional signal) may break down, we may consider a more enriched model by adding more realistic features. As suggested by a commentator, we will incorporate the issuing cost into the model, although the paper by Allen and Faulhaber did not. The motivation for considering the issuing cost is that a low type may not want to imitate p_H which is higher than L if imitation incurs additional costs.

Let $c > 0$ be the issuing cost. Then, the firm first chooses whether to go public (enter the IPO market) with incurring additional costs or stay private and forgo the investment opportunity. After the decision, the game assumed in Section 2 proceeds.

It is easy to see that there is no separating equilibrium in which the high type enters the IPO market and the low type does not enter. The reason is obvious. If there is such an equilibrium, the type of the firm is fully revealed by the entering decision, implying that the high type must choose $p = H$ after it enters. Unfortunately, however, this cannot be an equilibrium. If c is small enough, i.e., $c < H - L$, a low type will imitate the strategy by choosing $p = H$ after entering the IPO market. If $c > H - L$, a high type will not enter because $H - c < L$. Anyway, there will be no separating equilibrium involving underpricing in this game.

If the firm chooses the issuing decision and the pricing decision simultaneously,[11] underpricing may occur in a separating equilibrium. Consider an outcome in which a high type enters the IPO market and chooses p_H while a low type does not enter. The incentive compatibility conditions in this case require the following inequalities;

$$\alpha p_H + (1-\alpha)H - c \geq \max\{L, \alpha p + (1-\alpha)H - c\}, \qquad \text{[ICH4]}$$

$$L \geq \alpha p_H + (1-\alpha)L - c. \qquad \text{[ICL4]}$$

10 Some commentator argued that the main difference between this model and AF model is that the payoff to the firm in the second period does not depend on the beliefs of investors in this model, whereas the payoff depends in AF model so the result of AF model will be recovered if the second-period payoff also depends on the posterior belief of the investors that is formed in the first period. However, the analysis in this section shows that the conjecture is false.

11 We suspect that this is a very unlikely situation in reality, though.

First, to prevent a low type from imitating p_H, p_H must be very low. More specifically, the gain from imitation $\alpha(p_H - L)$ must not be higher than the entry cost c, i.e.,

$$p_H \le L + \frac{c}{\alpha}. \tag{6}$$

Second, a high type firm's incentive to deviate from choosing p_H should be prevented. The most profitable deviation of a high type may be to enter or not to enter, depending on c. If $c < (1 - \alpha)(H - L)$, the high type prefers entering and deviating to L to not entering, so $\max\{L, \alpha p + (1 - \alpha)H - c\} = \alpha L + (1 - \alpha)H - c$. Then, [ICH4] simply implies that $p_H \ge L$ because the equilibrium price p_H must be better (higher) than or equal to the highest price (except for p_H) that can be accepted which is L.

If $c > (1 - \alpha)(H - L)$, not entering is better than entering to the high type. So, max $\{L, \alpha p + (1 - \alpha)H - c\} = L$. In this case, [ICH4] implies that $\alpha p_H + (1 - \alpha)H - c \ge L$, i.e.,

$$p_H \ge \frac{L}{\alpha} - \frac{1-\alpha}{\alpha}H + \frac{c}{\alpha}. \tag{7}$$

Note that $\frac{L}{\alpha} - \frac{1-\alpha}{\alpha}H + \frac{c}{\alpha} > L$, since $c > (1 - \alpha)(H - L)$. This implies that $p_H > L$. Therefore, in both cases, i.e., whether the entering cost c is greater or less than the gain from deception, $(1 - \alpha)(H - L)$, a high type can signal its type by $p_H \in (L, L + \frac{c}{\alpha}]$. This is consistent with our discussion in Section 3 that $p_H < L$ cannot be a separating IPO price of a high type.

Intuition goes as follows. With a positive issuing cost, a low type cannot imitate the price of a high type even if $p_H > L$ so that imitation gives him some gain by $p_H - L$. Also, with a positive issuing cost, a high type would always deviate if $p_H < L$. To prevent this incentive, either c must be large or $p_H > L$. In fact, even if c is large, p_H must be greater than L. Otherwise, the high-type firm's payoff when he enters the market with incurring a significant entering cost would be lower than his payoff when he does not enter. To see this, suppose that $p_H < L$ when $c > (1 - \alpha)(H - L)$. Then, the high-type firm's equilibrium payoff is lower than his payoff when he does not enter the IPO market, since $\alpha p_H + (1 - \alpha)H - c < \alpha L + (1 - \alpha)H - c < \alpha L + (1 - \alpha)H - (1 - \alpha)(H - L) = L$. Thus, a high type can separate himself by underpricing. Note that there is a discontinuity at $c = 0$. Although there is no separating equilibrium when $c = 0$, there does exist a separating equilibrium for any $c > 0$. This is mainly because the low-type firm cannot imitate a high-type's equilibrium price p_H which is higher than L if it must bear a positive cost c.

The upshot of this section is that the AF result of signaling by underpricing can be recovered with considering the issuing cost only if the firm can choose its IPO price at the same time as it enters the IPO market.

6 Conclusion and Caveats

In a simple model, we showed that a good firm signals its type not by underpricing of IPO price but by its choice of the amount of equity to be sold in the market, although it engages in underpricing.

The purpose of this paper is two-fold. Since the primary purpose is to correct the proof of the well-known IPO underpricing result, we admit that it does not address other related issues in the finance literature, for example, the puzzle of long-term IPO underperformance. This issue could be addressed by relaxing our assumption that the firm's value is fully realized in the second period. By relaxing the assumption, we could also obtain some implications of the bullish IPO market or bearish IPO market on the IPO underpricing result. The second purpose is to derive some implications on the relationship between investor protection laws and IPO underpricing based on our theoretical analysis. We also admit the analysis is just preliminary. However, we believe that our result and the general analytic methodology provided in this paper will help to resolve those issues and enrich our understanding for IPO underpricing in the near future.

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