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What Is It to Be Real? Numbers as Real Species of a Category in the Late Medieval Debate about the Ontological Status of Numbers

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Abstract: The article deals with one of the facets of a hitherto almost completely unexplored subject: the late medieval debate about the ontological status of numbers. It focuses on one of the most popular arguments in favor of a realist view of number. The realist view holds that a number adds a new form over and above the things that it numbers. The argument for the view is based on the thesis that since the categories are a classification of extramental reality, and number belongs to the category of Quantity, it, together with its form, must have extramental existence. The second part of the paper is devoted to an exposition and analysis of various rebuttals of this argument advanced by critics of realism about numbers. Throughout the whole discussion, the paper also gives an evaluation of the dialectical moves made by the participants in the debate.

Keywords: ontology of numbers, categories, hylomorphism, quantity, medieval ontology

1 Introduction

Still unbeknownst even to most historians of the period, in the late Middle Ages (from around 1270s onwards) there was an interesting, sophisticated, and heated debate concerning the ontological status of numbers. Its main *locus* were commentaries on Peter Lombard's *Sentences*, Book I, Distinction 24, which concerns the number three of the Persons of the Holy Trinity, and the question of whether positing a number in the Trinity does not undermine the unity of God.[1] In the final

1 Besides the *Sentences* commentaries, the main sources for reconstructing the history of the debate include commentaries on Aristotelian works such as the *Categories*, *Physics*, and *Metaphysics*, as well as various quodlibetal disputations.

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decades of the thirteenth century, more and more authors became interested in the issue of the ontological status of numbers. They did so first and foremost in order to solve the theological problem of the number of Divine Persons versus the unity of God. But as time passed, more and more commentators gained interest in the ontology of numbers as a philosophical problem in its own right. The key question throughout the whole debate – one facet of which I shall look at in this paper – is whether a number is really distinct from what it numbers, and if so, what kind of existence (extramental or mental) it possesses. For example, is the number three of the three stones something distinct from these stones, and if so, does it exist in reality or only in the mind of, say, the person currently counting these stones?

The late medieval debate about this issue was shaped by two key metaphysical presuppositions. The first of them was Aristotle's classification of number as belonging to the category of Quantity, to the species of Discrete Quantity, whose characteristic feature (unlike that of the other species of Quantity, i. e., Continuous Quantity) is having discontinuous parts, that is, parts that are separate (even if contiguous) entities.[2] The second key presupposition was conceiving of numbers in hylomorphic terms. Authors who posited that a number adds something more over and above the numbered things held that the numbered things, as numbered units, are the *matter* of a number; for example, three stones, as three numbered units, are the matter of a number. The reality of these three stones, if they really exist, is uncontroversial. The late medieval debate was instead focused on the *form* of number: whether for the stones to be three in number, one needs to posit a form of threeness, which is an accident belonging to the species of Discrete Quantity and which makes the stones be three in number (not in the sense of efficient but in the sense of formal causality). The question that immediately arises for anyone wanting to posit such a form is whether it exists in the extramental reality or only in the mind.[3]

Three main positions, each with many different variants, emerged in the debate: *realists* held that the form of number exists in the extramental reality, *conceptualists* held that it exists in the mind only, whereas *reductionists* argued that it cannot be posited at all and that all that there is is only the *material* aspect of number: three stones, which are three in number by themselves and not in virtue of some superadded form.[4]

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2 See Aristotle, *Categories* 6, 4b20–21.

3 See Sorokina 2022.

4 Realists include thinkers such as Hervaeus Natalis and most of the Thomists, Thomas Wylton, Walter Burley, and Paul of Venice. Conceptualists include figures such as John Duns Scotus and most later Scotists, Peter Auriol, Francis of Marchia, and Walter Chatton; lastly, reductionists include thinkers such as William of Ockham, Gregory of Rimini, William of Rubio, and Blasius of Parma.

Realists about numbers had several standard arguments in favor of their position, which, especially given the metaphysical framework within which they were formulated, can initially seem compelling. But both the conceptualists and the reductionists, to whom I will sometimes, for the sake of brevity, refer together as 'anti-realists,' found none of these arguments to be conclusive and instead proposed alternative accounts of the intuitive or authoritative premises on which the realist arguments were based. This paper is meant as a case study: I shall focus on one of the most commonly invoked realist arguments, having to do with the claim that number is a true species of a category, and only what has extramental reality can be such a species. I will first reconstruct the realist argument, noting several of its slightly different variants. After that, I will turn to the anti-realist critique of the argument. Throughout my exposition and analysis, I will also try to give an assessment of various dialectical moves made by the authors engaged in the debate.

2 Realist Argument: Number as a True Species of a Category

I begin by sketching out the realist argument. It relies on a key premise, which some authors articulate explicitly while others leave it implicit. The premise, in a nutshell, is that the categories are a classification of extramental being.[5] One of the thinkers making this premise explicit is the Franciscan Francis of Meyronnes (ca. 1288–1328). Here is how he articulates it in his final *Sentences* commentary, the *Conflatus*:

1. Being is first divided by the metaphysician into real being and being of reason, before it is divided into ten genera.[6]

5 This premise is, of course, itself controversial and was a subject of a heated debate throughout the whole of the Middle Ages. However, I cannot get into its details here. But see, e. g., Gracia and Newton 2016.

6 Francis of Meyronnes, *In quatuor libros Sententiarum (Conflatus)* I.24.1, ed. Venice 1520, 79rb: "Ens primo dividitur a metaphysico per ens reale et rationis antequam dividatur in decem genera." Unless indicated otherwise, all translations shall be mine. Francis's formulation of the above premise is itself controversial, since it relies on the assumption that being is the highest genus that is divided into ten categories; a thesis that was famously championed by Scotus and Scotists but rejected by others, for example, Aquinas and Thomists. While there is no room here for a further discussion of this controversy either (the reader may want to consult, among many others, numerous mentions of this debate in Aertsen 2012), whether one adheres to this assumption or not is of peripheral importance for the whole of the present realist argument. For even the

It is a commonplace in our period to divide all that exists into *real beings*, that is, beings existing in extramental reality, and *beings of reason*, which exist only in the human mind, as conceived by the human mind. The realists about numbers argue that Aristotle's tenfold categorial scheme laid out in the *Categories* is a classification of real beings only. Even if there were a classification of beings of reason, it would not be identical with, or a part of, the categories that Aristotle lays out in the *Categories*. What our authors think is characteristic of real beings and of their tenfold Aristotelian classification is their mind-independent character; that is, that they exist regardless of whether there is anyone conceiving of them; and so that they exist outside the mind. As the Augustinian Michael of Massa (d. ca. 1337), for example, puts it:

2. The species of a real category does not have its complete being from the soul, because otherwise if the soul stopped thinking about it this category would cease to exist in actuality, which is absurd.[7]

Focusing on the real categories, that is, categories of real beings only, one can say, for the reasons I have just explained, that, to use the Augustinian Thomas of Strasbourg's (d. 1357) wording:

3. A true species of a real category is something real.[8]

Or, as Francis of Meyronnes puts it:

3*. What is *per se* in some category is a real being.[9]

Something is *per se* in a category if it belongs to it directly, that is, if it is unqualifiedly its member, or one of its members. By contrast, a given thing is in a cate-

realists who deny that the ten categories are all species of the genus *being* will still hold that the categories (either all of them, or at least enough of them to include the category of Quantity) are a classification of extramental entities, not just beings of reason; which is all that is needed for the first premise to hold.

7 Michael of Massa, *Commentarius in I Sententiarum* 24.2, ms. Bu, 201va: "Species praedicamenti realis non habet suum esse completum ab anima; alias anima non considerante desineret esse habere actualiter, quod est absurdum." Or, as Peter Nigri puts it, albeit without adducing any proof, "Nulla species praedicamenti realis est ens rationis vel ens factum ab anima" (*Clypeus*, pars Praed. 29, ms. M, 104va).

8 Thomas of Strasbourg, *Commentarius in I Sententiarum* 24.3, ed. Venice 1564, 83vb: "Species vera praedicamenti realis est quid reale."

9 Francis of Meyronnes, *In quatuor libros Sententiarum (Conflatus)* I.24.1, ed. Venice 1520, 79rb: "Quod per se est in aliquo praedicamento est ens reale."

gory *per reductionem* if it belongs to a category, or categories, in virtue of its parts. For example, some thinkers in our period hold that a substance taken as a whole, that is, as a hylomorphic composite, belongs to the category of Substance indirectly (*per reductionem*), namely, because it is composed of prime matter and substantial form, which belong to the category of Substance directly.[10] Now, it is clear that Aristotle classifies number as belonging to one of the two basic species of Quantity: Discrete Quantity (the other species being Continuous Quantity). The most famous locus of this statement are the *Categories*.[11] To quote Thomas of Strasbourg again:

> 4. Number is a true species of quantity, as is clear in the *Categories* and in Book V of the *Metaphysics*.[12]

Even if the thesis that number belongs to the category of Quantity was contested by some, especially by some Scotists,[13] who argued instead that, properly speaking, it belongs to the category of Relation, all that the realists need for the argument to work is the concession that number belongs to *some* real category – and this is something that their opponents (regardless of which category they have in mind) seem to be willing to concede. Thus, a perhaps better (because broader) formulation of Premise 4 is one put forward by the Dominican Paul Soncinas (d. 1494/5):

> 4*. <A number such as> number two is in a category, as is conceded by all.[14]

From this, given (3) and (4), it follows that:

> 5. A number is a real being.

10 This matter is much more complex since various thinkers held different views on which kinds of hylomorphic composites fell under a given category *per reductionem*. On this issue, see, e. g., Conti 2022, Section 2.

11 See Aristotle, *Categories* 6, 4b20–21.

12 Thomas of Strasbourg, *Commentarius in I Sententiarum* 24.3, ed. Venice 1564, 83vb: "Sed numerus est vera species quantitatis, ut patet in *Praedicamentis* et quinto *Metaphysicae*." The reference to Book V of the *Metaphysics* is problematic insofar as the distinction between Continuous and Discrete Quantity, despite the impression given by Thomas's passage, is never explicitly mentioned there.

13 Despite the fact that one of the few Scotist realists, Francis of Meyronnes, asserts in *In quatuor libros Sententiarum (Conflatus)* I.24.1, ed. Venice 1520: 79rb, that this premise is "concessa ab omnibus."

14 Paul Soncinas, *Quaestiones metaphysicales acutissimae* X.8, ed. Lyon 1579, 239b: "Binarius est in praedicamento, ut omnes concedunt."

One could perhaps assume that this should be the end of the argument. However, an obvious problem that is still lingering is that an anti-realist could insist that a number owes its *reality* to its material element alone, that is, its unities; and that this suffices to make it be *real* enough to belong to a (real) category, despite the fact that its form comes from the mind. It seems to be because of this worry that some of the realists add further steps to their arguments to show that in order for number to belong to a real category, its form, too, must be real, that is, it must have extramental existence.

I will here briefly consider two of such additional reasonings, advanced by two Dominicans: Bernard of Auvergne (d. after 1307) and Paul Soncinas. They both reason along very similar lines, since the arguments of both share the same implicit premise, which is that:

6. Everything that belongs to a category must be put into that category by something.

In other words, there needs to be some factor responsible for the categorial identity, so to speak, of each categorial item. To begin with Bernard of Auvergne's expansion of the argument, he adds to the above another premise, namely that:

7. Everything is put into its category first and foremost by its form.[15]

Note that Premise 7 is formulated in ontologically neutral terms, in that it does not specify whether the form in question has extramental existence; and for this reason it would be acceptable to anti-realist conceptualists, too, since they too hold that it is the form of number that makes it belong to its category; it is only that, unlike the realists, they think that the form in question exists in the human mind only.

From the preceding steps Bernard derives the conclusion that:

8. If <the form of number> were not something outside the intellect, then Discrete Quantity that is a number would not be in the category of Quantity.[16]

But Premises 6 and 7 do not seem to me to warrant this inference, even if we include the other assumptions that Bernard, just like other realists, is also committed to, namely that the categories are a classification of extramental being and that what

15 Bernard of Auvergne, *Reprobationes Henrici de Gandavo Quodlibet* IV.6, ms. Ba, 21ra: "Unumquodque maxime reponitur in praedicamento per suam formam."

16 Bernard of Auvergne, *Reprobationes Henrici de Gandavo Quodlibet* IV.6, ms. Ba, 21ra: "Nisi enim <forma numeri> esset aliquid extra intellectum, tunc quantitas discreta quae est numerus non esset in praedicamento quantitatis."

belongs *per se* to a given category is a real being. Bernard clearly assumes not only that if something belongs *per se* to a category, then it is a real being, but also that in such a case what puts that thing into its category is itself a real being. But why? We need some extra steps to justify this key inference, and Bernard does not give us any.

The same problem arises in the case of a later argument by Paul Soncinas. As I have said, Paul's argument shares the same assumption with Bernard's argument, namely Premise 6: everything that belongs to a category must be put into that category by something. Now, we already know that the realists also hold that only real beings can belong to real categories. Based on these two theses, Paul reasons that:

> 7*. If nothing can be in a category unless it be a real being, then that by which this thing is put in the category will even more so be a real <being>.[17]

I find 7* to be problematic. An anti-realist could agree that indeed, (a) it is necessary that, if X belongs to a (real) category, then X must be a real being. But he could then demand additional explanation for why from (a) it would have to follow that (b) it is thus also necessary for that which puts X into its category to be "even more so" a real being. The problem with Soncinas's Premise 7* is exactly the same as the problem with Bernard of Auvergne's reasoning: the sole fact that X is a real being in a real category makes it by no means obvious that what puts X into its category is a real being, too.

One could perhaps initially suspect that these premises are implicitly based upon the scholastic model of causality, whereby a cause must previously possess what it is to pass onto what it causes, so that nothing can give what it does not first itself possess.[18] Is it not the case that, because X is a real being belonging to a real category, what puts X into its category must be real, too? This, however, would be to mistake the causal role of the thing, call it Y, that puts X into its category. Y does not make X real; Y only puts X into its category. Of course, the realists are convinced that Y is *also* the thing that makes X be a real thing, since they believe that Y is the extramental, real form of number; but to invoke this belief here would be to beg the question. In principle, there seems to me to be nothing in the structure and content of the realist reasoning that would preclude the possibility that Y may be a being of

17 Paul Soncinas, *Quaestiones metaphysicales acutissimae* X.8, ed. Lyon 1579, 239b: "Si nihil est in praedicamento nisi ens reale, multo magis illud erit reale per quod ponitur in praedicamento."

18 There is surprising scarcity of general introductions to medieval theories of causality, but see, e. g., Marenbon 2009. The great late scholastic Francisco Suarez also gives a decent summary of the dominant medieval theories of causality that survived until his period in his monumental *Disputationes metaphysicae*, beginning with Disp. 12 and then going over the four Aristotelian causes in Disp. 13–24.

reason. In fact, I am not even sure if, and if so, how, this kind of transmission theory of causality could be applied to the case at hand.

In any case, Paul's next step is to state that we know that a number belongs to a (real) category:

8*. <A number such as> number two is in a category, as is conceded by all.[19]

Because of Premise 6, Paul then considers two possible candidates for what could put number into its category:

9*. Therefore, a number is in a category either
a. thanks to its formal nature, or
b. thanks to its material element, that is, thanks to its unities.[20]

Paul makes it clear that this this an exhaustive alternative because metaphysically speaking, a number, as a hylomorphic composite, consists of only two elements: its material element, which are its units, and its formal element, which is the form of number. He then goes on to reject 9*b: the possibility that number could belong to its category thanks to its material element:

10*. [9*b] Not thanks to its unities [...] <because> unities are in a category only reductively as principles, just as parts of substances. Hence, that by which they are reduced to a category, namely the whole, is primarily in a category.[21]

By way of elimination, then, this only leaves the possibility that number is placed in its category by its form:

11*. [9*a] A number, such as number two, is placed in its category by its formal and quidditative nature (*ratio formalis et quidditativa*).[22]

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19 Paul Soncinas, *Quaestiones metaphysicales acutissimae* X.8, ed. Lyon 1579, 239b: "Minor probatur. Binarius est in praedicamento, ut omnes concedunt."

20 Paul Soncinas, *Quaestiones metaphysicales acutissimae* X.8, ed. Lyon 1579, 239b: "Ergo vel <binarius> est in eo per suam rationem formalem, aut per id quod est materiale in ipso, scilicet per unitates."

21 Paul Soncinas, *Quaestiones metaphysicales acutissimae* X.8, ed. Lyon 1579, 239b: "Sed non per unitates; ergo per seipsum. Maior patet, quia in binario formaliter accepto nihil est aliud nisi suae unitates et eius forma. Minor probatur. Unitates sunt in praedicamento tantum reductive tamquam principia, sicut etiam partes substantiarum. Ergo id per quod reducuntur ad praedicamentum, scilicet ipsum totum, per prius est in praedicamento."

22 Paul Soncinas, *Quaestiones metaphysicales acutissimae* X.8, ed. Lyon 1579, 239b: "Numerus, puta binarius, ponitur in praedicamento per suam rationem formalem et quidditativam."

Thus, from 7* and 11* it follows that number, also in respect of its form, must be a real being, that is, being existing in extramental reality:

> 12*. Therefore, number two taken in respect of its form (*formaliter acceptus*) is a real being.[23]

By using these and similar arguments based on belonging to Aristotelian categories, the realists purport to demonstrate that number as a whole, that is, both in respect of its matter and of its form, must have extramental existence.[24]

3 Anti-Realist Critique of the Realist Argument

3.1 Introduction

Among the conceptualist anti-realists, the most common reaction to the realist argument sketched out above, that number belongs to a (real) category, and only what is a real being can belong to a (real) category, is to simply deny its major premise. As the Franciscan Peter Auriol (ca. 1280–1322) puts it succinctly:

> A category is not called real because everything that is contained within it has being outside the soul.[25]

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23 Paul Soncinas, *Quaestiones metaphysicales acutissimae* X.8, ed. Lyon 1579, 239b: "Ergo binarius formaliter acceptus est ens reale."

24 An interesting but completely different argument based on the categorial belonging was proposed by Peter Nigri in *Clypeus*, pars Praed. 29, ms. M, 102vb. Since the argument had, to the best of my knowledge, no later reception (in fact, Paul's text is full of original arguments for realism, which had no reception either), I decided to relegate it to a footnote. The argument goes as follows. (1) If we take items which are in fact one and the same thing, each of these items (given that they are a single thing) must belong to the same category. (2) But according to the anti-realists, if we take, say, a substance and its quality, they are both the same thing: a single number, i. e., number two. (3) Number belongs to the category of Discrete Quantity. (4) Since substance and its quality are a number, the absurd conclusion follows that they belong to the category of Discrete Quantity. Peter's argument is interesting but it would have little success against the anti-realists. The main problem I see with it has to do with the formulation of Premise 2, which misunderstands or misrepresents the view held by the anti-realists: they never claim that, to take our example, a substance and its quality somehow merge into a single thing that is a number. Rather, the anti-realists claim the exact opposite: that a number, as far as extramental reality is concerned, is a mere multitude and no single thing.

25 Auriol, *Reportatio in I Sententiarum* 24.1, mss. Be, 66ra; P, 97ra; Vb, 108va: "Praedicamentum non dicitur reale quia omnia quae sunt in praedicamento habeant esse reale extra animam."

3.2 John Duns Scotus

As Maria Sorokina has observed, the foundations for this reply were laid by the Franciscan John Duns Scotus (1265/6–1308), who states that, both according to Aristotle and according to the truth, one and the same category can contain both real beings and beings of reason. Thus, in addressing the issue of how it is possible that a number is not a real being if it is a member of the category of Quantity, Scotus explains that Quantity comprises both quantity with extramental existence, the Continuous Quantity, and quantity which, at least as far as its form is concerned, has mental being only: the Discrete Quantity. The soul, by counting, measures a given discrete quantity and gives it its name; this name signifies primarily the measure in the soul, which is the form of number, and which can then be denominatively predicated of the multitude of counted things existent in reality.[26]

3.3 Henry of Harclay

Henry of Harclay (1270–1317), who was Scotus's student and follower, expands on the ideas originally found in Scotus. He rejects the major premise of the realist argument, namely, that the form of an item belonging to a real category must be itself real. He gives counterinstances to this premise: for example, not only real relations but also relations of reason (ones where one or both terms do not have real existence) belong to the real category of Relation.[27] Similarly, not all species of Quality are real: for instance, shape is not, and is in fact reducible to continuous quantity. Shape is classified as a quality only *secundum dici* (presumably because

26 See Scotus, *The Examined Report of the Paris Lecture. Reportatio I-A* 24, eds. Wolter and Bychkov, 40: "Dicendum quod sicut secundum intentionem Philosophi ens dividitur in ens reale et ens secundum rationem, et similiter relatio in relationem rei et rationis, sic quantitas prima sui divisione dividitur in quantitatem continuam ut in speciem realem, et quantitatem discretam ut speciem et ens secundum rationem, quae tantum est in anima, quam anima mensurat per aliquod sui; et illi potest anima nomen imponere quod significabit primo illam quantitatem quae est in anima, et denominative praedicare potest de multitudine extra animam, quae tantum est hoc, hoc, hoc, quia sine anima non habet aliquam formam unitatis numeralis." For further discussion, see Sorokina 2022, 47–49.

27 See Henry of Harclay, *In I Sententiarum, q. 48 (d. 24)*, ed. Sorokina, 90: "Ad primum dico quod sicut in relatione est quaedam relatio secundum esse et quaedam secundum dici, et similiter quaedam relatio realis et quaedam rationis, eodem modo est in aliis generibus." Henry's view can then be found in, e. g., Auriol, *Scriptum* I.24.1, ed. Sorokina, 851–855; ed. Rome 1596, 554a; Francis of Marchia, *Scriptum in I Sententiarum* 24.3, ed. Etzkorn, 48; and Lombardi, *In I Sententiarum* 24.1, mss. E, 99ra; L, 46rb.

it responds to the question of what a thing is like, which is standardly a question seeking quality – this is a reply given by many later Scotists[28]), but *in re* it is identical with continuous quantity because it is nothing more than the "closure of lines containing continuous quantity."[29]

Henry holds that just as both mere beings of reason (relations of reason) and real beings (real relations) belong to the category of Relation, so the category of Quantity consists of the genus of Continuous Quantity, which has under itself real things, that is, things existing in the extramental reality, and the genus of Discrete Quantity, which has under itself, among others, time and number, which have no extramental existence but only mental existence, since they exist objectively in the mind.[30]

3.4 Francis of Marchia

The Franciscan Francis of Marchia (ca. 1285/90–after 1344) expands the point (taken either from Henry of Harclay or, what is more likely, from Auriol, who himself borrowed it from Henry) about relations. The challenge that Auriol, Francis of Marchia, and others, raise against the realists is this. All the thinkers that I am familiar with do recognize that, despite the difference in their ontological status, both types of relation fall under the same category, namely, *Relation* itself. Thus, Francis points

28 See Majcherek, forthcoming.

29 See See Henry of Harclay, *In I Sententiarum, q. 48 (d. 24)*, ed. Sorokina, 92: "Praeterea, est similiter reperire aliquam quantitatem quae tamen secundum dici est qualitas. Verbi gratia, figura non est realiter alia res a quantitate continua. Probatio. Nam impossibile est considerare quantitatem praecise nisi sub aliqua figura; et tamen figura secundum dici ponitur in quarta specie qualitatis; nam ut est in genere qualitatis non est aliud nisi clausio linearum continentium quantitatem continuam, et dicit sic modum qualitatis; sed ut est in genere quantitatis dicit continuum lineis terminatum et clausum; unde solum differunt secundum modum et secundum dici." For further discussion of shape as *terminus quantitatis*, see Majcherek, forthcoming, Ch.1.

30 Henry of Harclay, *In I Sententiarum, q. 48 (d. 24)*, ed. Sorokina, 91: "Eodem modo, dico ego, est in quantitate, quod alia est quantitas secundum rem, ut continuum, linea, superficies et corpus, alia autem secundum rationem et secundum dici, ut numerus. Nec aliud intelligit Aristoteles, si volumus eum glossare." Followed by Auriol, *Scriptum* I.24.1, ed. Sorokina, 851–857; ed. Rome 1596, 554a–b: "Non valet etiam sextum: Quoniam tempus et numerus sunt in praedicamento quantitatis; nec ex hoc sequitur quod habeant esse reale. Ad idem enim praedicamentum reducuntur relationes reales et rationis, et qualitates reales, cuiusmodi est figura, et rationis, qualis est pulchritudo, quae non addit ad figuram et colorem ex quibus consurgit aliquam formam absolutam realem. Similiter etiam ad praedicamentum quantitatis pertinent quantitates continuae permanentes, quae sunt reales, et tempus et numerus, quae non habent esse nisi in anima obiective, sicut Philosophus dicit."

out that if the realists wanted to stick with their original major premise, that only what is a real being falls under a category, they would need to deny the existence of relations of reason or insist that all relations are real relations:

> <To the argument> that number is a species of Quantity, which is a real genus, I reply that if this were a good argument, it would follow that no relation in the world would be a relation of reason, because every relation of reason, or at the very least some, is a species of Relation, which is a real genus.[31]

Thus, Francis emphasizes, real relation and relation of reason are two species belonging to the same genus, that is, to the same category of Relation. This shows that a certain class of entities can belong to a category without possessing extramental existence. Thus, going back to Quantity: even if one concedes that the other basic species of Quantity, Continuous Quantity, is real, this will have no import for the ontological status of Discrete Quantity, just like in the above case the status of real relations has no import for the status of relations of reason. Thus, the anti-realists add, on this understanding, analogous to the case of real relations and relations of reason, Continuous Quantity, a genus of real being, and Discrete Quantity, a genus of being of reason, will be two genera which together constitute one category, despite the fact that number has no real, extramental being.[32] This presents the realists with a serious dilemma, to which, I think, they have no satisfying reply: if they admit that relations of reason, despite their lack of extramental existence, are species of the genus of Relation, why does number, based on the fact that it is a species of the genus of Quantity, have to be a real being?

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31 Francis of Marchia, *Scriptum in I Sententiarum* 24.3, ed. Etzkorn, 48: "Tunc ad rationes alterius partis. Ad primam, quod numerus est species quantitatis quae est genus reale, dico quod si ista ratio esset bona, sequeretur quod nulla relatio de mundo esset relatio rationis, quia quaelibet relatio rationis, vel saltem aliqua, est species relationis, quae est genus reale."

32 See Francis of Marchia, *Scriptum in I Sententiarum* 24.3, ed. Etzkorn, 48: "Ideo dico quod absolutum et respectivum formaliter diversificant genus, sed reale et non-reale non diversificant genus sed bene diversificant speciem. Et ita numerus, licet sit tantum in anima, et quantitas continua, licet sit extra, non differunt genere, licet differant specie, sicut relatio realis et relatio rationis non differunt genere, licet differant specie." See also Francis of Marchia, *Francisci de Marchia sive de Esculo Commentarius in IV libros Sententiarum Petri Lombardi. Distinctiones primi libri a undecima ad vigesimam octavam (Reportatio)* 24.2a, ed. Mariani, 439a: "Dico quod ens in anima et ens extra animam, licet necessario distinguantur specie, non tamen genere; relatio enim realis et relatio rationis eiusdem generis sunt, licet specie distinguantur. Nisi enim ens rationis esset in eodem genere cum ente reali, essent plura praedicamenta quam decem, et ita sicut relatio non facta per animam et relatio facta per animam sunt in eodem genere, ita etiam quntitas non facta per animam, sicut est quantitas continua et quantitas facta per animam ut discreta, cuiusmodi est numerus."

3.5 Evaluation

One of the most basic problems with these objections is, of course, their heavy reliance on analogies with other cases which the anti-realist authors take to involve categories containing both real beings and beings of reason. For the realist authors could simply deny that, for example, a shape is not a real being. Indeed, there is even disagreement about this issue among the authors using the argument: as I have said above, Henry of Harclay lists shape as an example of an item in the category of Quality that is not a real being. By contrast, Peter Auriol states explicitly that shape is a real being.[33]

But, setting these quibbles about shape and so on aside, there remains a much more serious problem for the realists, which was pointed out by their opponents: that of real relations and relations of reason. Most late medieval thinkers, regardless of their ontological sympathies, do recognize the distinction between real relations and relations of reason.[34] On the standard medieval way of conceiving of a relation, each relation has two terms: its foundation and its terminus. For example, if Socrates is similar to Plato because both Socrates and Plato are wise, then Socrates's wisdom is the foundation, and Plato's wisdom is the terminus, of the relation of similarity existing between Socrates and Plato.[35] Moreover, on the standard account, a relation is real if both its foundation and its terminus are real. By contrast, a relation of reason does not require the reality of both of its terms. If, for example, I am thinking of a golden mountain, there is a relation of *thinking of* existing in me and the golden mountain, but only one of its terms, namely, me, is real, whereas the golden mountain is a mere being of reason. Thus, the case of real relations and relations of reason seems to undermine the premise that the categories encompass real being only, and thus to undercut the whole realist rationale for realism about numbers based on categorial belonging.

33 Nevertheless, Auriol does claim that Quality comprises real beings and beings of reason; for him, shape is a real being, but another item listed under the category of Quality, beauty (*pulchritudo*), is not, so the main point still stands. See the quote in the note above.

34 On medieval theories of relations, see, e. g., Henninger 1989; and Brower 2018.

35 On the standard medieval theory of relations, endorsed by most medievals, relations are *monadic* accidents: Socrates's similarity to Plato is a relation distinct from Plato's similarity to Socrates. On this peculiar (compared to our contemporary understanding of such properties, i. e., as polyadic properties) feature of medieval theories of relations, see, besides the two works referenced above, Penner 2016.

3.6 Aftermath

As a result of the above considerations, some thinkers, such as Auriol and Francis of Marchia, are led to the rejection of the major premise of the original realist argument that they criticized, namely, that only real being can belong to a real category, and thus allow for all kinds of categorial items that are mere beings of reason. They are happy to concede that, notwithstanding that number is merely a being of reason, it is nevertheless a species of the category of Quantity, just as a relation of reason is a species of the category of Relation.

But other thinkers take a different, somewhat more radical route, because they deny that number is properly speaking a species of Quantity at all. For example, the Franciscan Scotist Aufredus Gonteri Brito (fl. ca. 1320) explicitly states that in the *Categories*, Aristotle enumerates kinds of Quantity, including number, "which are not *per se* quantities," that is, which do not belong *per se* to the category of Quantity and thus cannot constitute one of its species. Many medieval Scotists are rather skeptical about the Aristotelian classification of the ten categories. For example, many of them hold, against Aristotle, that properly speaking, shape, which Aristotle classifies in the *Categories* as the fourth species of Quality, is in fact a relational accident.[36]

I have made this digression for a purpose, since Aufredus uses his description of the twofold classification of shape to illustrate the status of number. He says that "other <items>, such as number, time, and place, are called quantities <merely> in respect of their concept and manner of speaking (*secundum rationem et dici*); nor did Aristotle mean otherwise."[37] This of course still leaves the question open as to what, if anything, numbers are *in re*, and unfortunately Aufredus never fills this gap in his account. One can only suspect that he would perhaps go on to say that in reality they are reducible to the numbered things.

Why, though, given that, on Aufredus's account, number is not a quantity *in re*, did Aristotle, rather misleadingly, describe it as if it were? Here Aufredus presents

36 Auferus follows into Henry of Harclay's footsteps when he defends a view (which, incidentally, would then be endorsed by many famous thinkers defending a reductive account of shape, such as John Buridan) on which shape is Quantity *de re* and Quality *de dici*; that is to say, shape is in reality a Quantity "because it is that which is enclosed by a terminus or termini, and in this way it is in the genus of Quantity, that is, it is quantity *per se*"; why shape is a Quality *de dici*, that is, according to our common way of speaking, Aufredus does not explain, but the usual explanation given by other thinkers endorsing this view was that this is so because pointing to shape responds to the question of 'what something is like' (*quale est*), which is a question concerning the *quality* of a given thing. See Gonteri Brito, *Commentarius in I et II Sententiarum* I.24.2 (ms. V: 127vb). See also Majcherek forthcoming, Ch. 1.

37 Gonteri Brito, *Commentarius in I et II Sententiarum* I.24.2 (ms. V: 127vb): "Aliae dicuntur quantitates secundum rationem et dici, ut numerus, tempus et locus; nec aliud intelligit Aristoteles."

us with two explanations, one more specific and one more general. Aufredus's specific explanation relies heavily on his detailed account of what number is, which I can only here sketch very briefly. Following Henry of Harclay, Aufredus thinks that a number consists in a replication of unities performed by the intellect and in the intellect: I start with one and replicate it as many times as needed in order to obtain a given number. Now, Aristotle says in Book X of the *Metaphysics* that a characteristic feature of quantity is that it is a measure.[38] Weaving these threads together, Aufredus tries to show how his concept of number as a replication of unities and the concept of number as a measure could (mis)lead some into believing that number belongs to the category of Quantity:

> If we want to explain [...] why number is posited by Aristotle to be a quantity, the reason is as follows: Because the nature of measure belongs first of all to quantity, as is said in Book X of the *Metaphysics*, for this reason, because unity considered by the intellect or imagination by its replication can measure the continuum that is divisible in diverse parts, for this reason such a replication and comparison of distinct part or parts performed by the comparing intellect is called Quantity, and such a replication of unities is nothing else than a number, which is a measured multitude, as is said in Book X of the *Metaphysics*, that is, in reality a number is a multitude or continuity divided into parts measured by one and by the replication of one.[39]

There is an easy way to translate this rather convoluted passage into simpler English, which is to go back to the case of shape. As I have mentioned, one of the usual reasons given by thinkers who reclassified shape from Quality (where it had originally been put by Aristotle) to one of the relational categories (usually Position) was that shape is quality *de dici* because it responds to the question of what something is like, which is the question normally seeking quality. In a somewhat analogous way, Aufredus explains that because the proper function, as it were, that Aristotle attributes to Quantity is being a measure, and a number seems to be a measure (and thus, albeit this remains implicit in the passage, a number also pro-

38 In fact, in *Metaphysics* X, Aristotle seems to attribute the property of being a measure to unity first and to number only derivatively, insofar as it is composed of units. See, e. g., *Metaphysics* X.1, 1052b19–24; X.2, 1054a1–13.

39 Gonteri Brito, *Commentarius in I et II Sententiarum* I.24.2, ms. V, 127vb: "Si voluimus enim glossare de numero aut de quo est quaestio praesens, quare numerus ponitur ab Aristotele esse quantitas, ratio est ista: Quia quantitati primo debetur ratio mensurae, decimo *Metaphysicae*. Quia ergo unitas considerata secundum intellectum vel imaginationem per sui replicationem potest mensurare continuum quod est divisibile in diversas partes, ideo talis replicatio et comparatio partis distinctae a parte vel partibus per intellectum comparantem dicitur quantitas, et talis replicatio unitatum non est aliud quam numerus qui est multitudo mensurata, decimo *Metaphysicae*, id est in rei veritate numerus est multitudo vel continuitas distincta in partes mensurata uno et per replicationem unius."

vides a reply to the question of 'how many,' which seeks Quantity), it looks as if number is, properly speaking, a species of the category of Quantity.

This was the more specific reason why, according to Aufredus, Aristotle described number in a way that might suggest that it is a quantity. What is the more general reason? Here Aufredus borrows his idea from Simplicius, who, in struggling with explaining and reconciling various theses expressed in Aristotle's *Categories*, said about the theses that he found impossible to explain that, because the *Categories* are an introductory work to Aristotle's logic, and thus the whole of his philosophy, Aristotle sometimes includes in them the views commonly held by others that he himself does not share but does not (yet) attempt to disprove. Aufredus claims that this may be the case with numbers, namely, that Aristotle borrowed the classification of number as quantity from others, as a popular view, without himself espousing it. (Needless to say, there is absolutely no textual evidence in Aristotle for this being so.)[40]

3.7 Simpler Solution? Francis Marbres

A solution that is at least partially different, and perhaps also somewhat simpler compared to the ones sketched out above, was proposed by Francis Marbres (d. ca. 1330). Francis's reply focuses on what he takes to be different senses of being *real*: a stronger sense, which he thinks does not apply to numbers, and a weaker sense, in which he thinks numbers can be called real, but which does not require one to posit that their form has extramental existence. After laying out and endorsing a conceptualist view of numbers, Francis is confronted with the by now familiar objection: that the species of a real category must have real being, that is, being in extramental reality, and since number is a species of the category of quantity, it must have extramental existence and hence cannot be made by the mind.[41] In response to this objection, Francis introduces a distinction:

40 See Gonteri Brito, *Commentarius in I et II Sententiarum* I.24.2, ms. V, 127vb: "Ratio autem una generalis quare Aristoteles in *Praedicamentis* assignat species quantitatis et qualitatis quae non sunt per se quantitates vel qualitates est secundum Simplicium, quia in libro *Praedicamentorum*, qui est quasi prologus logicae, et per consequens totius philosophiae, loquitur secundum famositatem aliorum, et non semper secundum opinionem propriam."

41 Francis Marbres, *Quaestiones in octo libros Physicorum* IV.7, ed. Venice 1520, 47vb: "Sed contra istam conclusionem arguitur primo sic. Quia species praedicamenti realis est in re extra. Sed numerus est species praedicamenti realis, quia est de genere quantitatis. Ergo etc. Maior patet, quoniam nullum dependens ab intellectu est intrinsecum alicui facto per intellectum; aliter aliquid unum fieret ex ente in anima et ex ente extra animam."

> I respond by saying that *that something is real* [*aliquid esse reale*] can be understood in two ways.
>
> (1) In one way, <something can be real> because it has being regardless of any act of the intellect and because it does not depend upon the soul either objectively or effectively; for example, a stone and others of this kind <have this kind of reality>.[42]

This is the strong sense of being real, or having real being, which is completely independent of the existence or act of the intellect. A thing possessing this kind of real being is independent of the intellect both *objectively* – that is, it does not need to be the object of thinking of the intellect – and *effectively* – that is, it does not need to be made up by the intellect. A stone is a good example, since, while I might think of it (in this way it will have objective existence in my mind), it will still be out there regardless of whether I or anyone else think of it at any time. This, as one can expect, is not the way in which the reality of number should be understood according to Marbres. For, he says, there is another way of conceiving of it:

> (2a) In another way, <something can be real> because it depends on the soul objectively but not effectively; for example, a number.
>
> (2b) Or, if it depends on the soul effectively, it nevertheless informs the intellect and is its proper accident; for example, an act of understanding and of knowing.[43]

Marbres makes it clear that for a species of a real category to be real, or for that species to even qualify as a suitable candidate for a species of a real category, it is not necessary that it be real in the first sense but it suffices that it be real in the second sense.[44]

In the above passage, Marbres introduces a further sub-distinction within categorial items with mental existence: (2a) some of them depend on the soul objectively only but not effectively (see above for what this jargon means), whereas (2b) others depend on the soul effectively. Both the structure of the passage, which is clearly split into two distinct parts, and the fact that each part concludes with a different example illustrating a given type of being real, where numbers only serve as an example for (2a), makes clear that according to Marbres numbers fall under (2a) only.

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42 Francis Marbres, *Quaestiones in octo libros Physicorum* IV.7, ed. Venice 1520, 48ra: "Respondeo ad primum; et dico quod aliquid esse reale potest esse dupliciter. Uno modo quoniam habet esse circumscripto omni actu intellectus et quoniam ab anima non dependens obiective nec effective, ut lapis et talis."

43 Marbres, *Quaestiones in octo libros Physicorum* IV.7, ed. Venice 1520, 48ra: "Alio modo quoniam est dependens ab anima obiective et non effective, ut numerus; vel si effective, tamen informativum ipsius intellectus et eius proprium accidens sicut actus intelligendi et sciendi."

44 Marbres, *Quaestiones in octo libros Physicorum* IV.7, ed. Venice 1520, 48ra: "Primo modo non oportet speciem praedicamenti realis esse realem, sed sufficit secundo modo."

My interpretation is also supported by how Marbres responds to a different realist argument. In a nutshell, the realists argue that a real science must be concerned with what is real; and given that arithmetic is a real science (rather than science occupied merely with our thinking), its object must be real; so numbers must have their own reality. In his reply to this argument, Marbres not only emphasizes that what suffices for a given item to be an object of a real science is that it have objective being in the soul, but he is equally emphatic that this is *the only* kind of being in the soul which is sufficient for number to be real and so be an object of a real science: if number had soul as its efficient cause, it could not be an object of a real science, presumably because then it would be made up by the mind. Number can be a species of a real category because it "depends on the soul objectively but not effectively."[45] This means that the soul does not make up numbers but only contains forms of numbers.

This is interesting because it shows that Marbres is insistent on preserving some degree of mind-independence of numbers: he emphasizes that if the mind were their efficient cause, they would not be real and so could not be a species of a real category. Worth adding is that the *objective* being that numbers have in the mind, while it of course does involve the mind, is nevertheless tied to reality, to grasping the multitude of objects presenting themselves via our senses to our mind.

3.8 A Nominalist Take: William of Ockham

The debate summarized above was based on various versions of the broader assumption that the categories are a classification of all that exists; in other words, that they carve up exhaustively the whole of reality. Of course, this means that thinkers who do not share this assumption consider the above to be a pseudo-problem: if the categories are an exhaustive list but merely of the ways we speak or think about reality, then, while of course reality taken as a certain whole remains

45 See Marbres, *Quaestiones in octo libros Physicorum* IV.7, ed. Venice 1520, 47vb and 48ra: "[Objection:] Item, subiectum scientiae realis est extra animam et reale. Sed numerus est subiectum scientiae realis, quoniam arithmeticae, quae est scientia realis. Ergo etc. Maior patet, quoniam secant scientiae quodammodum et res de quibus sunt, secundo *De anima*; et scientia similiter habet denominationem a subiecto. Tum etiam quoniam subiectum est mensura scientiae; et mensura perfectior est mensurato, et sunt unigenea, decimo *Metaphysicae*. Sed omnia ista deficiunt hic. Ens enim rationis non mensurat ens reale, nec est perfectius, nec unigeneum. […] [Reply:] Ad secundam dico quod sufficit quod subiectum scientiae realis non dependeat ab intellectu effective, licet bene obiective; et quod sit reale, hoc est, quia non dependet effective ab intellectu."

the basis for the categorial scheme (after all, as I have said, these would be ways of speaking or thinking *about reality*), still the categorial items would not need to satisfy any criterion of *reality* in order to be counted as such.

One of the most famous representatives of this position is, of course, the Franciscan William of Ockham (ca. 1287–1347). When, in his discussion of number in his *Sentences* commentary, Ockham attacks the alternative, realist views of numbers, and, what is now of most relevance, the realist argument based on categorial identity of number, he begins by giving a brief summary of the relevant aspects of his view of the categories:

> Some [...] would say[46] that some of the categories do not signify things distinct from other things belonging to other categories, but that instead <these categories> signify the same things by different modes of signification. For example, according to some, the category of relation does not signify a thing distinct from the absolute thing of another category but rather signifies that <absolute> thing of another category together with connoting one other thing, either of the same category or of a different category.[47]

This is admittedly a rather complicated passage, but in essence what Ockham means to say is that, with the exception of some categories (according to him, Substance, some of the species of Quality, and relations in the Trinity),[48] (a) the other categorial names do not stand for a thing that would be different from other things in the world or from the other categories and that (b) the categories are distinct from each other by having different modes of signification. In the case of relations, mentioned in the passage above, in order to provide an exhaustive account of the ontology of relations, say, of how it is that X is similar to Y, one does not need to posit some third thing over and above X and Y. If a realist about, say, the relation of similitude wanted to force me into admitting that if Plato is similar to Socrates, this can only happen in virtue of similitude, which is an extra thing over and above Plato, Socrates, and, say, the qualities that are compared (say, wisdom), I could reply that the statement that Plato is similar to Socrates signifies Plato (and the relevant

46 The editors of Ockham's *Opera omnia* assume at this point that Ockham is referencing Peter John Olivi's view; Ockham simply uses the term 'aliqui,' which he often uses to introduce his own view, in cases where it is a controversial one. See, e. g., Pasnau, 2011, Ch. 12.4.

47 Ockham, *Scriptum in librum primum Sententiarum. Distinctiones XIX–XLVIII* 24.2, OTh IV, 112: "Aliqui [...] dicerent quod praedicamenta aliqua non significant alias res a rebus aliquorum aliorum praedicamentorum, sed easdem res significant alio et alio modo significandi; sicut praedicamentum relationis non significat secundum aliquos aliam rem ab omni re absoluta et alterius praedicamenti, sed significat rem alterius praedicamenti connotando unam aliam rem vel eiusdem praedicamenti, vel alterius."

48 See, e. g., Adams, 1987, 267–276; and Henninger, 1989, 140–145.

quality being the source of similitude, such as wisdom) while connoting, that is, signifying secondarily, Socrates (and his quality of wisdom); this is all that is needed for Plato to be similar to Socrates, without positing any intermediary thing, as it were, that would somehow point from Plato to Socrates. Thus, the peculiar mode of signifying of similitude is signifying one quality while connoting another of the same kind; say signifying Plato's quality of wisdom while connoting Socrates's wisdom.[49]

So far, so good, but how does this apply to number, which is a species of Discrete Quantity? According to Ockham, the explanation of the characteristic mode of signification of number, and of Discrete Quantity in general, on the account that he endorses is that "discrete quantity would be a concept signifying things themselves and connoting that they do not make up something one *per se*." In other words, a numerical term signifies the numbered things while connoting their discretion, that is, the fact that they are not united into a single *per se* whole. The species of number, then, can never signify an individuum but rather always signifies a plurality of things or of terms that stand for a plurality of things.[50]

This is not a question that Ockham ever dealt explicitly in his texts, but one can easily see that a realist opponent could raise the following objection to his account: If a number is just a concept or a term that we apply to a multitude of things while thinking of them as discrete from one another, what basis do we have for assigning a given concept or term to a given multitude? For example, if I apply the concept or term 'four' to the chairs standing in my room, what is the truthmaker of the mental or verbal proposition 'There are four chairs in my room'? Either this imposition is completely arbitrary, and I could assign any other numerical concept or term just as I please, or there must be something in reality that would make my proposition

49 See Ockham, *Scriptum in librum primum Sententiarum. Distinctiones XIX–XLVIII* 24.2, OTh IV, 112: "Sicut 'simile' significat aliquam qualitatem, aliam qualitatem connotando cum illa exsistere. Sicut Sortes non dicitur similis nisi habeat albedinem, et unus alius habeat unam aliam albedinem, vel propter aliquam aliam qualitatem. Et posito quod Sortes habeat albedinem, et unus alius similiter, omni alio circumscripto Sortes est similis illi."

50 See Ockham, *Scriptum in librum primum Sententiarum. Distinctiones XIX–XLVIII* 24.2, OTh IV, 112: "Similiter, quantitas discreta esset unus conceptus significans ipsas res connotando eas non facere unum per se. Et secundum istum modum ponendi, sicut de ratione aliquarum specierum vel generum est significare res quarum quaelibet sit una per se, ita quod non potest vere praedicari nisi tantum de termino supponente praecise pro talibus rebus, [...] ita sunt aliquae species et genera quae praecise significant multas res; nec possunt verificari nisi de multis vel de terminis supponentibus pro multis. Et talis species est numerus, et similiter ternarius et quaternarius, quia concreta illorum nullo modo possunt de uno per se sumpto verificari sed praecise de multis, quia quocumque uno demonstrato haec est falsa simpliciter: 'Hoc est quatuor vel quinque', sed demonstratis pluribus haec est vera 'Ista sunt quatuor vel quinque', et sic de aliis."

true (or false). Ockham, I am sure, would agree with the latter statement, just as he does in so many other contexts.[51] But he would refuse to make the key step that the realists deem inevitable, namely, to say that there is a further thing in reality that makes the chairs in my room be four in number and thus serves as the truth-maker of my proposition about their number. Rather, Ockham would insist that there being a specific multitude of chairs in my room is a brute fact which cannot be explained any further.

To summarize, Ockham is convinced that in order to explain the categorial identity of numbers, it suffices to employ the notion of the mode of signification that is peculiar to each category, in this case to Discrete Quantity; and nothing more is needed in order to make number belong to such a species, for reasons (at least partly) laid out above. Of course, as I have said, this is not so much a solution of the original problem as it is a dismissal of it, which stems from a radically different approach to the status and distinction of the categories. But this broader topic lies far beyond the scope of the present study, so I shall leave it at this point.[52] Ultimately, it seems to me, because the original source of disagreement between Ockham and the realists about numbers lies so deep, namely, it concerns the status of the Aristotelian categories, it is difficult to assess how well Ockham's view of number fares compared to the realist account. Rather, I suggest, it is best to think of these two as two very different, and incompatible, paradigms.

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51 For another instance, see Majcherek 2022.

52 For the interpretation of Ockham closest to mine, see Brower-Toland 2023.

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