Materials and Structural Design for Lightweight Primary Mirrors

C.J. Hamelin, and J. Beddoes,
Department of Mechanical & Aerospace Engineering,
Carleton University, Ottawa, Canada, K1S 5B6
jbeddoes@mae.carleton.ca

J. Lo
CANMET, Natural Resources Canada,
568 Booth St., Ottawa, Canada, K1A 0G1

ABSTRACT

A deflection analysis for seven potential primary mirror materials is presented for loading due to gravitational or external forces. Based on this analysis a finite element model of a plano-plano mirror is developed for three mirror materials including aluminum, Zerodur and an Mg particulate-reinforced composite. Comparison of mirror deflection according to the finite element model to interferometer-measured deflections of a Zerodur mirror indicate good accuracy is obtained from the model. Using the calibrated finite element model various geometrical light weighting options are evaluated. These include the fabrication of between three and six circular pockets on the back face of the mirror. It can be concluded that while both pocket depth and diameter contribute to mass reduction of the mirror, mass reduction through increases in pocket depth are more advantageous than increases in pocket diameter, as this results in greater mirror stiffness for the same mass reduction. Also, an increase in the number of light weighting pockets for the same mass reduction has less impact on the mirror optical performance.

INTRODUCTION

There are two methods used to reduce primary mirror weight. The first involves geometrical optimization – the shape of the mirror is chosen to reduce weight while enhancing overall stiffness. Design examples include contoured-back, radially ribbed, open-back, sandwich, and foam core structures /1/. The second method of light weighting is via materials selection. During the past 40 years a continuous advance in the design of new lightweight optical materials has occurred. While many mirrors have been constructed of aluminum, other materials including low-expansion glass, beryllium, and silicon carbide are available. Recently, several state-of-the-art optical mirrors have been fabricated from beryllium, Zerodur, Ultra-Low Expansion (ULE) glass, silicon carbide, and metal-matrix composites (MMCs).

The mechanical, physical, optical, metallurgical, and fabrication properties of candidate materials can influence their probability of selection /2/, along with a significant impact of cost and availability. Recently, in Canada, aluminum and Zerodur have been selected for reflective telescope mirror fabrication /3/, with beryllium mirrors selected for military applications. By careful selection of the individual components of a composite material system, the physical properties of composites can to some extent be tailored for particular application requirements. As such, there is value to examining composites with good stiffness-to-weight ratios.

By optimizing the geometry and materials selection, the primary mirror best suited for a particular application can be specified. However, if this optimization and selection is performed through prototyping, the time and cost of producing multiple prototype mirrors of various geometry and material is excessive. Therefore, a simple, rapid, and accurate method is needed for the optimization of primary mirror design and materials selection. Given the foregoing, the purpose of the work described in this paper is to develop a finite element model to successfully predict the behaviour of plano-plano mirrors with no light weighting for various candidate mirror materials. The accuracy of the model is established through calibration with interferometric analysis, and subsequently used to examine the benefits of structural light weighting.

MATERIAL SELECTION

The relevant physical properties of six materials frequently used for primary mirrors are listed in Table I, with a relative comparison of cost, manufacturing ease and optical performance given in Table II. Included in these tables is preliminary data for a proprietary MMC consisting of a particulate-reinforced magnesium matrix. From these tables it is immediately apparent why aluminum dominates primary mirror fabrication. Nevertheless, the excellent performance of beryllium has been utilized for several critical applications, such as the primary mirror of the Mars Orbiter Laser Altimeter /4/.

By applying thin plate bending theory /5/ and using aluminum as a baseline, the primary mirror surface deflection is compared in Figure 1, when subjected to either external or gravitational loading, with larger deflections having a greater impact on the optical performance. Typically, external loads are most important during manufacturing, with gravitational loads encountered in service (for space-based applications, particularly upon launch). Beryllium and silicon carbide offer the best optical performance with aluminum and ULE glass performing less well. For the purposes of the current investigation a detailed finite element model was applied to mirrors fabricated from aluminum, Zerodur and the magnesium MMC.

ANALYSES & RESULTS

To validate the analysis, the results of a finite element analysis (FEA) applied to mechanically loaded plano-plano mirrors are compared to experimental interferometric data obtained for the same plano-plano mirror. This FEA is described in the next section and compared to the interferometric data in the subsequent section.

Finite Element Analysis

The simplified I-DEAS™ finite element model of the plano-plano mirror is shown in Figure 2. The model

Table I

Mechanical and thermal properties of lightweight mirror materials /3/

Material	ρ kg/m^3 10^3	*CTE* ppm/K	*Specific Heat* J/kg-K	*Thermal Conductivity* W/m-K	*E* GPa	*Thermal Distortion Coefficient* μm/W	*Specific Stiffness* MH/kg 10^6
Al	2.7	25	899	237	76	0.105	28.1
Be	1.85	11.4	1880	216	303	0.053	164
ULE	2.20	0.03	708	1.3	67	0.023	30.4
Zerodur	2.55	0.15	820	6.0	90	0.025	35.3
Al-SiC MMC	2.96	12.2	-	120	130	0.102	40
SiC	3.21	2.4	700	250	466	0.010	145
Mg matrix composite	2.2				125		56.8

Table II
Comparison of mirror materials for candidate selection.

Material	*Cost/Availability*	*Ease of Manufacture*	*Optical Performance*
Aluminum	Very good	Very good	Poor
Beryllium	Poor	Very poor	Very good
ULE glass	Good	Poor	Very poor
Zerodur	Good	Good	Good
SXA	Very poor	Poor	Good
Silicon carbide	Poor	Very poor	Very good
Mg matrix composite	Good	Poor	Good

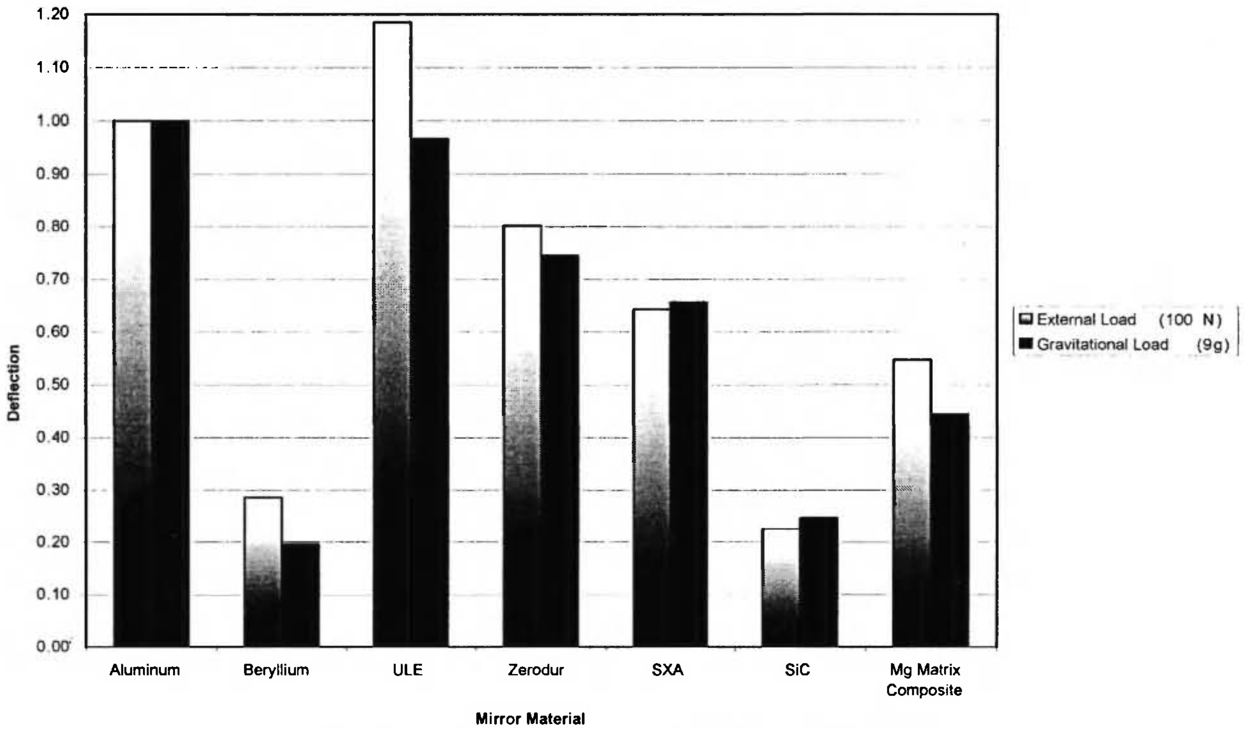


Fig. 1: Central deflection of solid plano-plano loaded mirrors.

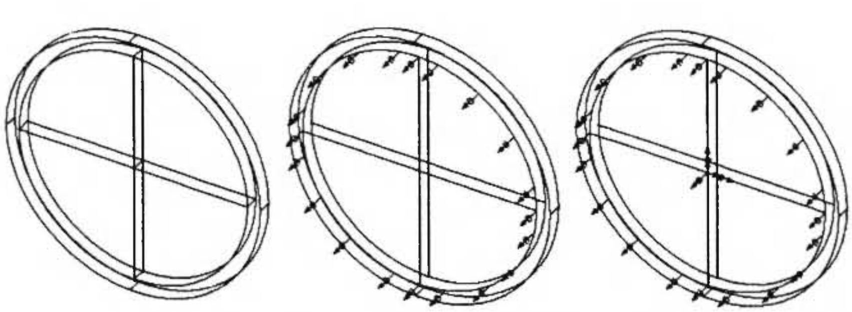


Fig. 2: Placement of geometric boundary conditions. *Left:* the partitioned model, *Centre:* the model with a ring mount restraint, *Right:* the fully restrained model.

mirror diameter is 108.2 mm with a thickness of 6 mm, giving a diameter-to-thickness ratio of approximately 18. Typically, smaller mirrors such as these have a ratio of 6:1, but for the current work the maximum load that could be applied during interferometric analysis limited the mirror thickness. A concentric ring of 97.28-mm diameter supports the mirror, such that the model is constrained in the thickness direction at this location (shown in the centre of Figure 2). Consistent with the loading during interferometric analysis, a 36.95-N load is applied to the bottom centre of the mirror (shown at right, Figure 2). A finite element mesh consisting of 10-node tetrahedrons is used throughout the modelled mirror. To improve the capability of the model to simulate various more complex lightweight configurations, solid elements are used instead of plate elements and axisymmetric simplifications were not applied. Varying the thickness normalized element length between 0.4 and 1 resulted in virtually no change to the predicted deflection of the top surface centre.

Table III
FEA predicted top surface central-point deflection for three different mirror materials.

Material	*Deflection (μm)*
Aluminum	3.125
Zerodur	2.492
Mg matrix composite	1.763

Furthermore, this deflection compared well with that calculated based on thin plate bending theory /5/. Therefore, an element length of 0.5 is applied for all analyses, corresponding to the mesh of 11,305 elements shown in Figure 3. Three models were constructed for aluminum, Zerodur, and the Mg matrix composite. The resulting top surface central deflection is listed in Table III.

Interferometric Analysis

Interferometric analyses utilized a Zygo GPI XPS interferometer and a Zerodur mirror. The central load was applied to the centre of the back surface. Figure 4 illustrates the interferometric deflection data for the Zerodur mirror subjected to a 36.95-N load, with a corresponding peak to valley (P-V) deflection of 3.345 μm. The interferometer software filters the results to minimize errors that occur as a direct result of off-axis mounting or initial surface sag of the mirror. Application of the software filters results in a central peak to valley deflection of 1.948 μm. Table IV compares the FEA-predicted deflection with this interferometric analysis for the Zerodur mirror. The FEA-predicted deflection is between the interferometric values obtained with and without application of the software filters. It is believed that filtering eliminates the effect of concavity that appears as an initial condition or state of the mirror: however, the load application at the centre of the mirror will produce

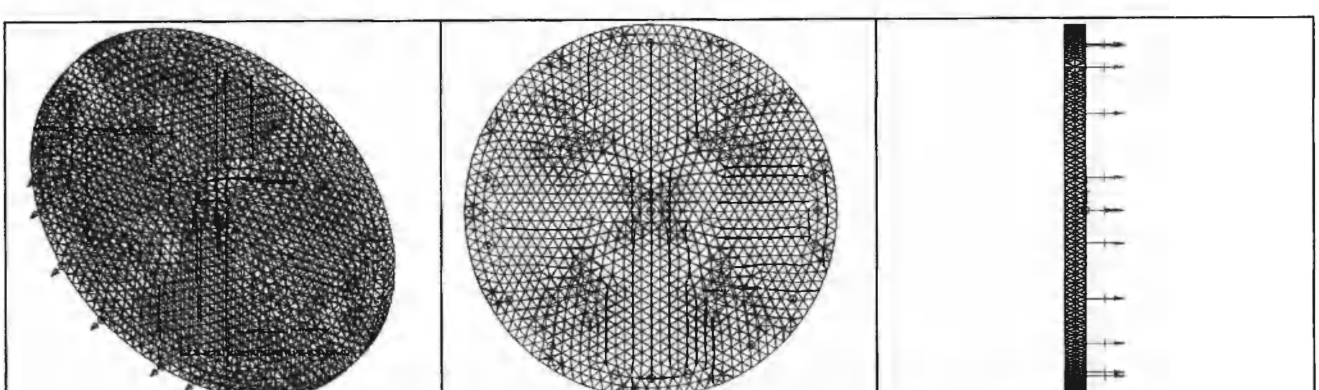


Fig. 3: Finite element mesh of just over 11,000 tetrahedral elements used for all analyses.

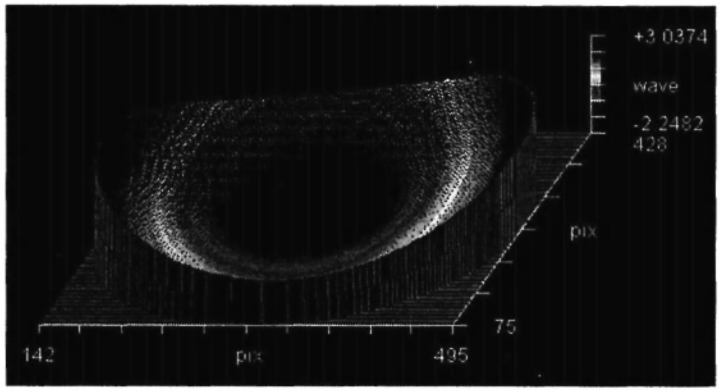


Fig. 4: Interferometer measured profile of Zerodur mirror reflecting surface at 36.95 N load. Peak to valley defection of 3.037 + 2.248 = 5.285 × wavelength of laser or 3.345 μm.

Table IV

Comparison of deflection data between FEA and Interferometer results

Software Filter Status	*Interferometer P-V Deflection (μm)*	*FEA Deflection (μm)*	*Deviation (μm)*	*% Error*
Filtering Not Applied	3.345	2.492	+ 0.853	+ 34
Filtering Applied	1.948	2.492	- 0.544	- 22

deflection effects similar to natural concavity without loading. Thus, that the FEA-predicted deflection is between the two interferometric values is reasonable, as this would be the approximate deflection of a completely defect-free Zerodur mirror. Regardless, the error between either interferometer result and the FEA values is satisfactory given the very small displacements modelled.

STRUCTURAL LIGHT WEIGHTING

The first structural light weighting step was to create circular pockets in the back of the mirror – an easy to remove material during manufacture. To evaluate this option a detailed FEA parametric study to examine the effect of multiple circular pockets with variations in pocket depth and diameter is presented. For this analysis, more than 100 lightweight configurations were completed with three to six circular light weighting pockets in a Zerodur mirror.

Three Circular Lightweight Pockets

Figure 5 shows an example of the structure imposed on the back of the mirror for a three pocket design. Variations to the dimensions of this model take two forms: the diameter of each pocket is varied, and the depth of the pocket ($t0$) is varied for each pocket radius ($r1$). In each case – given a specified pocket depth and diameter – the mass of the mirror and the mirror deflection are calculated for a load of 36.95 N applied to the back surface. This load allows comparison of the deflections between plano-plano and lightweight designs. Figure 6 shows mass reduction and deflection data for the three pocket design, as both pocket depth ($t0$) and diameter ($r1$) are varied. The term $r2$ refers to the radius from the mirror centre to the centre of the light weighting pocket: for all models, $r2$ is equal to one quarter the mirror diameter (27.05 mm) and the mirror thickness (t) is 6 mm. For the three pocket lightweight design, combining a large pocket diameter with sufficient depth (i.e. for large percentages of mass

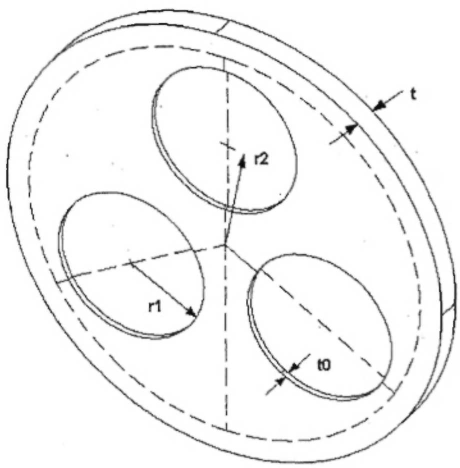


Fig. 5: Lightweight mirror, consisting of three light weighting pockets on the back of the mirror. Dashed lines represent partitions used for FEA operations.

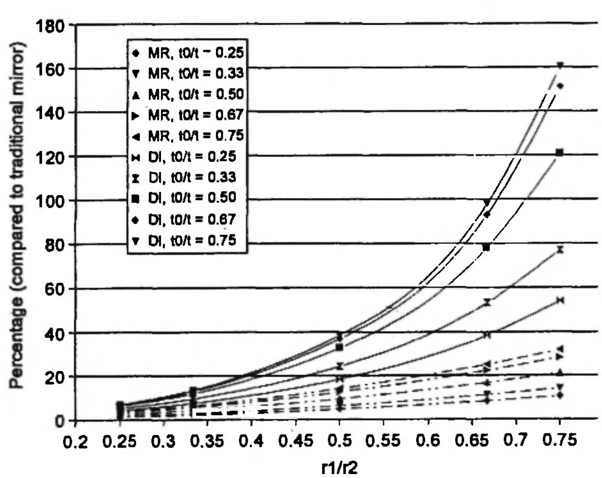


Fig. 6: Sensitivity of the three pocket design to pocket depth and diameter (DI = increase in deflection, MR = mass reduction).

removal) results in deflection values greater along the radial direction of the pocket centres. This illustrates the need to better distribute the mass reduction, leading to the analyses of lightweight designs with a larger number of pockets.

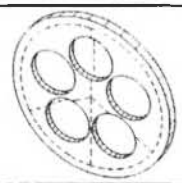
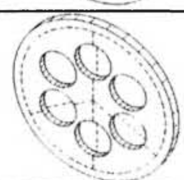
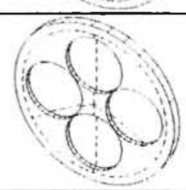
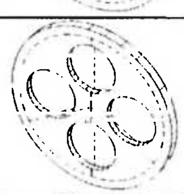
Four To Six Pocket Designs

To allow comparison of the various designs, sets of 25 analyses were carried out for each number of pockets, with variation of both the pocket depth and diameter. For all analyses, the finite element models contained approximately 10,000 to 12,000 elements. From Table V it is evident that as the number of pockets is increased, the deflection at a given mass is noticeably reduced. Also included in Table V is a comparison of two four-pocket designs for which the mass reduction is nearly identical, but the pocket depth and diameter are varied, and as a result the deflection data is significantly different.

Comparison of light weighting through an increase of pocket depth and diameter reveals that for the same degree of mass reduction, the increase in deflection that arises due to pocket diameter increase is greater than that when pocket depth is increased. However, there is a limit to the depth of a light weighting pocket, and hence a suitable pocket diameter must be chosen. Table V illustrates this result – comparing two lightweight, four-pocket designs with nearly identical values in mass reduction illustrates that the design with a smaller pocket diameter and a larger pocket depth resulted in less mirror deflection under the same loading conditions. This characteristic is invaluable when attempting to reduce the mass of a mirror by a given amount – to maximize the extent of light weighting requires use of the largest pocket depth and diameter permissible. However, maintaining optical performance of the mirror dictates how aggressively geometrical light weighting can be applied. Clearly, a trade-off exists between the degree of light weighting possible while meeting optical performance criteria.

The advantages of using a smaller pocket diameter are further exemplified by comparison of three-, five-, and six-pocket designs, illustrated in Table V. As the number of pockets is increased, the pocket depths and diameters are adjusted such that the mirrors of Table V experience approximately the same degree of mass reduction, by reducing pocket diameter and increasing pocket depth as the pocket number increases. It can be seen however that while the mass reduction values are similar, the six-pocket design has the smallest deflection at the same loading conditions – the deflection is almost

Table V
Comparison of selected deflection data for multiple pocket designs.

Number of pockets	*t0/t*	*r1/r2*	*% Mass Reduction Compared to Plano-Plano Mirror*	*% Deflection Increase Compared to Plano-Plano Mirror*
	0.50	0.75	21.1	120.7
	0.67	0.55	25.2	101.5
	0.75	0.45	22.8	67.4
	0.50	0.70	24.5	155.4
	0.67	0.60	24.0	95.4

half that of the three-pocket design. Clearly therefore, it is beneficial to increase the number of light weighting pockets in the mirror design.

SUMMARY AND CONCLUSIONS

Finite element analysis and interferometric data indicate that the finite element method can be used to successfully model mirror deflections. The deflection of the mirror centre predicted by FEA deviated <1 μm from the deflection measured by interferometry.

Using the validated FEA model, parametric mirror light weighting studies have been undertaken. It can be concluded that while both pocket depth and diameter contribute to mass reduction of the mirror, mass reduction through increases in pocket depth are more advantageous than increases in pocket diameter, as this results in greater mirror stiffness for the same mass reduction. Also, an increase in the number of light weighting pockets for the same mass reduction has less impact on the optical mirror performance.

ACKNOWLEDGEMENTS

The authors thank the late Martin High for his cooperation and guidance of this research, without which it would not have been possible. One of the authors (CJH) is grateful for financial support through the Ontario Graduate Scholarship for Science and Technology.

REFERENCES

1. D. Vukobratovich. "Lightweight laser communications mirrors made with metal foam cores," *Proc. SPIE,* **1044**, 216-226 (1989).
2. R.A. Paquin. "Selection of materials and processes for metal optics," *Proc. SPIE*, **65**, 12-19 (1975).
3. CANMET/APS, "Advanced Manufacturing Technology for Space Telescopes", 1999.
4. P. Generie and B. Hayden. "Estimation of the on-orbit distortion of the Mars Orbiter Laser Altimeter (MOLA II) Primary Mirror," *Proc. SPIE*, **2857**, 45-56 (1996).
5. S. Timoshenko and S. Woinowsky-Krieger, *Theory of Plates and Shells*, McGraw-Hill, 1959.