Nonlinear Stress Analysis of Unidirectionally Reinforced Symmetric Aluminum Metal-Matrix Laminated Beams under a Bending Moment

Onur Sayman[a], Hasan Çallıoglu[a], Cesim Ataş[a] and N. Sinan Köksal[b]

[a]*Department of Mechanical Engineering, Dokuz Eylül University, 35100, Bornova, İzmir, Turkey*
[b]*Department of Mechanical Engineering, Celal Bayar University, Muradiye, Manisa, Turkey*

ABSTRACT

This study deals with elastic-plastic behavior of aluminum metal-matrix laminated cantilever beam subjected to a bending moment at the free end. The Bernoulli-Euler theory is utilized in the solution and small plastic deformations are considered. The beam consists of four layers and its material is assumed to be linearly hardening. A few ply arrangements such as $[90°/0°]_S$, $[30°/-30°]_S$, $[45°/-45°]_S$ and $[60°/-60°]_S$ are taken into consideration for such an analysis. The moment values that initiate plastic flow at any point of the beam are carried out for diverse stacking sequences. The variation of the elastic, elastic-plastic and residual stress components versus increasing plastic zone spread are given in tables and figures.

INTRODUCTION

Due to their specific stiffness, high temperature performance and low density, metal-matrix composites have been used in structures and commercial applications for a long time. Recently, many production techniques of MMCs such as casting and powder metallurgy methods have been utilized in numerous investigations /1-3/.

Bahai-El-Din and Dvorak /4/ investigated the elastic-plastic behaviour of symmetric metal-matrix composite laminates under in-plane mechanical loading. Dadras /5/ presented an elastic-plastic stress analysis of plane strain pure bending of a strain-hardening curved beam. In that study only a linear hardening case has been analyzed. Fares /6/ presented a modified version of Reissner's mixed variational formula for investigating generalized non-linear thermoelasticity problem in composite laminated beam. Liu and Soldatos /7/ assessed the accuracy of the distribution of the interlaminar (transverse shear and transverse normal) stresses through the entire beam thickness. Khdeir and Reddy /8/ presented an exact solution for the bending of thin and thick cross-ply laminated beams by using the classical first-order, second-order and third-order theories in the analysis. Chattopadhyay and Guo /9/ developed non-linear structural design sensitivity analysis for structures undergoing elastoplastic deformation. Sayman and Zor /10/ investigated elastic-plastic stresses in a thermoplastic cantilever beam loaded uniformly. Karakuzu and Özcan /11/ carried out an elasto-plastic stress analysis on aluminum composite cantilever beam loaded by a single force at the free end and a uniformly distributed force at the upper surface by using an analytical solution. Sayman and Çallıoglu /12/ carried out an elastic-plastic stress analysis in composite beams under a bending moment by using the Bernouilli-Navier hypotheses.

On the other hand, in this study an elastic-plastic stress analysis is carried out in steel fiber reinforced symmetric aluminum metal-matrix laminated composite beams consisting of four orthotropic layers subjected to a bending moment. They are produced by a squeeze casting method. The Tsai-Hill theory is used as a yield criterion during the solution.

ELASTIC SOLUTION

Analysis of laminated beams subjected to pure bending can be developed from the Bernoulli-Euler theory /13/. According to this theory, the longitudinal normal strain at a distance from the neutral surface is given as

$$\varepsilon_x = \frac{z}{\rho} \tag{1}$$

where ρ is the radius of curvature of the neutral surface during flexure, z is the distance from neutral surface by the xz plane, as shown in Figure 1. The longitudinal stress in the *j*th ply is written by

$$(\sigma_x)_j = (E_x)_j\,(\varepsilon_x)_j \tag{2}$$

where $(E_x)_j$ is the Young's modulus of *j*th ply along the x axis and $(\varepsilon_x)_j$ is the longitudinal strain in the *j*th ply along the x axis. From the Eqs. (1) and (2) the longitudinal stress can be evaluated as,

$$(\sigma_x)_j = (E_x)_j \frac{z}{\rho} \tag{3}$$

The stress component σ_x must be related to the bending moment as,

$$M = 2\int_0^c \sigma_x\, z\, dz \tag{4}$$

or

$$M = \frac{2t}{3\rho}\sum_{j=1}^{N/2} (E_x)_j\,(z_j^3 - z_{j-1}^3) \tag{5}$$

where 2c and t are the height and thickness of the beam, and N is the total number of plies and z_j is the distance from the neutral surface to the outside of the *j*th ply. For an even number of plies of uniform thickness $z_1 = j\,2c/N$ and Eq. (5) becomes

$$M = \frac{16tc^3}{3\rho N^3}\sum_{j=1}^{N/2} (E_x)_j\,(3j^2 - 3j + 1) \tag{6}$$

or, it is written as

$$M = \frac{E_f\, I_{yy}}{\rho} \tag{7}$$

where I_{yy} is the inertia moment of the cross-section of the beam, E_f is the effective flexural modulus of the beam which is

$$E_f = \frac{1}{c^3}\sum_{j=1}^{N/2} (E_x)_j\,(z_j^3 - z_{j-1}^3) \tag{8}$$

The stress component can also be written by eliminating the radius of curvature;

$$(\sigma_x)_j = \frac{M}{E_f\, I_{yy}}(E_x)_j\, z = \frac{M z}{I_{yy}}\left[\frac{(E_x)_j}{E_f}\right] \tag{9}$$

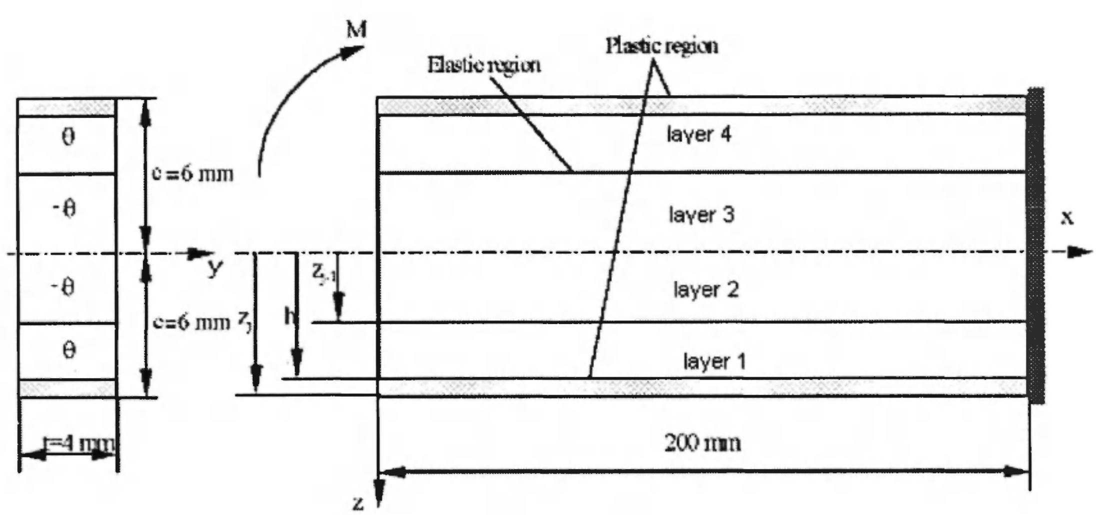

Fig. 1: A laminated composite cantilever beam.

The strain-stress relation in the composite laminated beam is written as,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \varepsilon_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{16} \\ \bar{a}_{12} & \bar{a}_{22} & \bar{a}_{26} \\ \frac{\bar{a}_{16}}{2} & \frac{\bar{a}_{26}}{2} & \frac{\bar{a}_{66}}{2} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

where $\bar{a}_{ij}$ are the components of the compliance matrix /14/:

$$\begin{aligned}
\bar{a}_{11} &= a_{11}m^4 + (2a_{12} + a_{66})m^2n^2 + a_{22}n^4 \\
\bar{a}_{12} &= a_{12}(m^4 + n^4) + (a_{11} + a_{22} - a_{66})m^2n^2 \\
\bar{a}_{22} &= a_{11}n^4 + (2a_{12} + a_{66})m^2n^2 + a_{22}m^4 \\
\bar{a}_{16} &= (2a_{11} - 2a_{12} - a_{66})nm^3 - (2a_{22} - 2a_{12} - a_{66})n^3m \\
\bar{a}_{26} &= (2a_{11} - 2a_{12} - a_{66})n^3m - (2a_{22} - 2a_{12} - a_{66})nm^3 \\
\bar{a}_{66} &= 2(2a_{11} + 2a_{22} - 4a_{12} - a_{66})m^2n^2 + a_{66}(m^4 + n^4)
\end{aligned} \tag{11}$$

where m=cosθ, n=sinθ, $a_{11}=1/E_1$, $a_{22}=1/E_2$, $a_{12}=-\nu_{12}/E_1$, $a_{66}=1/G_{12}$. Eq. 3 can also be written as,

$$(\sigma_x)_j = \frac{z}{\rho(\bar{a}_{11})_j} \tag{12}$$

ELASTIC-PLASTIC SOLUTION

During the elastic and elastic-plastic solution, it is assumed that the Bernoulli-Euler hypotheses are protected. According to these assumptions plane sections, which are normal to the longitudinal axis, remain plane and normal during flexure. Thus the unit strain for both the elastic and elastic-plastic cases is written as,

$$\varepsilon_x = \frac{z}{\rho} \tag{13}$$

where ρ is the radius of the curvature of the beam.

The Tsai-Hill theory is used as a yield criterion due to the same yield points of the plies of the metal-matrix composite beams in the tension and compression. X and Y are the yield strength in the 1st and 2nd principal material directions, respectively. The yield strength in the 3rd direction, (Z) is assumed to be equal to the transverse yield point (Y). Also it is assumed that the shear strength in 2-3 and 1-3 planes is equal to S which is the shear strength in the 1-2 planes.

The yield condition according to this criterion can be written as,

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 \tag{14}$$

multiplying it by X gives the equivalent stress in the first principal material direction as,

$$\sigma_{eq} = \bar{\sigma} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\frac{X^2}{Y^2} + \tau_{12}^2\frac{X^2}{S^2}} \tag{15}$$

For a linear strain hardening material, the yield stress is given by the Ludwik equation as,

$$\sigma_Y = \sigma_0 + K\varepsilon_p \tag{16}$$

where σ_0 is equal to X which is the yield strength in the first principal material direction, K and ε_p are the plasticity constant and equivalent plastic strain, respectively. In the plastic region, the equations of equilibrium are written as,

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xz}}{\partial z} = 0$$
$$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\sigma_z}{\partial z} = 0 \tag{17}$$

After integration of the first equation σ_x is determined as *C(z)*. Therefore, at any section, in the plastic region σ_x is only a function of z.

The stress components in the principal material directions for the orientation angle θ are written as,

$$\sigma_1 = \sigma_x m^2,\ \sigma_2 = \sigma_x n^2,\ \tau_{12} = \sigma_x mn \tag{18}$$

where m=cosθ, n=sinθ. Putting them in Eq. (15) gives the yield strength for the orientation angle θ,

$$X_1 = \frac{X}{N} \tag{19}$$

where,

$$N = \sqrt{m^4 - n^2m^2 + \frac{X^2n^4}{Y^2} + \frac{X^2n^2m^2}{S^2}} \tag{20}$$

The plastic strain increments in the materials directions can be found by using the potential function $f = \bar{\sigma} - \sigma_Y(\varepsilon_p)$ /15/ as,

$$\begin{Bmatrix} d\varepsilon_1^p \\ d\varepsilon_2^p \\ d\varepsilon_{12}^p \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial f}{\partial \sigma_1} d\lambda \\ \dfrac{\partial f}{\partial \sigma_2} d\lambda \\ \dfrac{\partial f}{\partial \tau_{12}} d\lambda \end{Bmatrix} \tag{21}$$

The total strain increments in the principal material directions are written as /16/,

$$\begin{aligned} d\varepsilon_1 &= d\varepsilon_1^e + d\varepsilon_1^p = a_{11} d\sigma_1 + a_{12} d\sigma_2 + \frac{2\sigma_1 - \sigma_2}{2\sigma_Y} d\lambda \\ d\varepsilon_2 &= d\varepsilon_2^e + d\varepsilon_2^p = a_{12} d\sigma_1 + a_{22} d\sigma_2 + \frac{-\sigma_1 + \dfrac{2\sigma_2 X^2}{Y^2}}{2\sigma_Y} d\lambda \\ d\varepsilon_{12} &= d\varepsilon_{12}^e + d\varepsilon_{12}^p = \frac{a_{66}\, d\tau_{12}}{2} + \frac{2\tau_{12}\dfrac{X^2}{S^2}}{2\sigma_Y} d\lambda \end{aligned} \tag{22}$$

The stress component σ_x for the orientation angle θ can be written as $\sigma_x = \frac{\sigma_Y}{N}$ and $d\lambda$ is equal to the equivalent plastic increment $d\varepsilon_p$. Putting σ_1, σ_2 and τ_{12} into Eq. (22) and integrating them produces

$$\begin{aligned} \varepsilon_1 &= a_{11}\sigma_1 + a_{12}\sigma_2 + \frac{2m^2 - n^2}{2N}\varepsilon_p + C_1 \\ \varepsilon_2 &= a_{12}\sigma_1 + a_{22}\sigma_2 + \frac{-m^2 + 2n^2 \dfrac{X^2}{Y^2}}{2N}\varepsilon_p + C_2 \\ \varepsilon_{12} &= \frac{a_{66}\tau_{12}}{2} - \frac{2mn\dfrac{X^2}{S^2}}{2N}\varepsilon_p + C_3 \end{aligned} \tag{23}$$

Integration constants C_1, C_2 and C_3 are determined at the boundary of the elastic and plastic regions by using the boundary conditions since at the boundary ε_p is zero and elastic and plastic strains are equal each other. Using Eq. (10) and the transformation formula, the strain components in the principal material directions for the elastic region are written as,

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \varepsilon_{xz} \end{Bmatrix} \tag{24}$$

$$\begin{aligned} \varepsilon_1 &= \sigma_x \left(\bar{a}_{11} m^2 + \bar{a}_{12} n^2 + \bar{a}_{16} mn\right) \\ \varepsilon_2 &= \sigma_x \left(\bar{a}_{11} n^2 + \bar{a}_{12} m^2 - \bar{a}_{16} mn\right) \\ \varepsilon_{12} &= \sigma_x \left(-\bar{a}_{11} mn + \bar{a}_{12} mn + \frac{\bar{a}_{16}}{2}(m^2 - n^2)\right) \end{aligned} \tag{25}$$

where $\sigma_x = X_1$ which is the yield strength of a ply for the orientation angle θ. Equating the strain components at the boundary of the elastic and plastic regions gives the integration constants. Then the strain components in the plastic region are written as,

$$\begin{aligned} \varepsilon_1 &= \sigma_x \left(a_{11} m^2 + a_{12} n^2\right) + X_1\left[(\bar{a}_{11} - a_{11}) m^2 + (\bar{a}_{12} - a_{12}) n^2 + \bar{a}_{16} mn\right] + \frac{2m^2 - n^2}{2N}\varepsilon_p \\ \varepsilon_2 &= \sigma_x \left(a_{12} m^2 + a_{22} n^2\right) + X_1\left[(\bar{a}_{11} - a_{22}) n^2 + (\bar{a}_{12} - a_{12}) m^2 - \bar{a}_{16} mn\right] + \frac{-m^2 + 2n^2\dfrac{X^2}{Y^2}}{2N}\varepsilon_p \\ \varepsilon_{12} &= -\frac{\sigma_x}{2} a_{66} mn + X_1\left(-\bar{a}_{11} mn + \bar{a}_{12} mn + \frac{\bar{a}_{16}}{2}(m^2 - n^2) + \frac{a_{66}}{2} mn\right) - \frac{2mn\dfrac{X^2}{S^2}}{2N}\varepsilon_p \end{aligned} \tag{26}$$

The strain components in the x and z directions are obtained from the principal material directions by using the transformation formula,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \varepsilon_{xz} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix} \tag{27}$$

They are determined as

$$\varepsilon_x = \bar{a}_{11}\sigma_x + B_1\varepsilon_p, \quad \varepsilon_z = \bar{a}_{12}\sigma_x + B_2\varepsilon_p, \quad \varepsilon_{xz} = \frac{\bar{a}_{16}}{2}\sigma_x + B_3\varepsilon_p \tag{28}$$

where

$$B_1 = \frac{2m^4 - 2m^2n^2 + 2n^4 \dfrac{X^2}{Y^2} + 4m^2n^2 \dfrac{X^2}{S^2}}{2N}$$

$$B_2 = \frac{2m^2n^2 - n^4 - m^4 + 2m^2n^2 \dfrac{X^2}{Y^2} - 4m^2n^2 \dfrac{X^2}{S^2}}{2N} \quad (29)$$

$$B_3 = \frac{3m^3n - mn^3 - 2mn^3 \dfrac{X^2}{Y^2} + (-2m^3n + 2mn^3)\dfrac{X^2}{S^2}}{2N}$$

The stress component σ_x in the elastic region varies linearly in terms of the curvature of the radius as,

$$\sigma_x = \frac{\varepsilon_x}{\bar{a}_{11}} = \frac{z}{\rho\ \bar{a}_{11}} \quad (30)$$

Therefore, the distances between the plastic regions and the x axis are equal to +h and –h, as seen from Fig. 1.

At the yield point, σ_x is equal to X_1 and $\sigma_x = X_1 = \dfrac{h}{\rho\ \bar{a}_{11}}$, hence, the curvature of the radius is determined as,

$$\rho = \frac{h}{X_1\ \bar{a}_{11}} \quad (31)$$

The total strain in the plastic region is written as,

$$\frac{z}{\rho} = \left|\frac{\bar{a}_{11}}{N}\cdot\sigma_Y\right|_j + \left|B_1\right|_j \varepsilon_p \quad (32)$$

Putting σ_y in the equation gives ε_p as $|\varepsilon_p|_j = |a + b\,z|_j$ where,

$$a = \left|\frac{-\sigma_0\ \bar{a}_{11}}{K\ \bar{a}_{11} + B_1\ N}\right|_j \quad b = \left|\frac{N}{\rho\left(K\ \bar{a}_{11} + B_1\ N\right)}\right|_j \quad (33)$$

DETERMINATION OF BENDING MOMENT

The moment at any section can be evaluated according to the boundary of the plastic region. The moment of the stress component of σ_x has to be equal to the bending moment M. If the plastic region is spread in the first layer (bottom layer), the bending moment

$$M = 2\left[\int_{z=0}^{c/2} \frac{z}{\rho|\bar{a}_{11}|_2} z\,t\,dz + \int_{z=\frac{c}{2}}^{h} \frac{z}{\rho|\bar{a}_{11}|_1} z\,t\,dz + \int_{z=h}^{c} \left|\frac{X + K\varepsilon_p}{N}\right|_1 z\,t\,dz\right] \quad (34)$$

where t is the thickness of the beam and $\varepsilon_p = a + bz$ found in the previous section; X is the yield strength of the first layer. The curvature is evaluated from the yield strength of the first layer as

$$\sigma_x = X_1 = \frac{h}{|\bar{a}_{11}|_1 \rho}, \quad \rho = \frac{h}{|\bar{a}_{11}|_1 X_1} \quad (35)$$

where X_1 is the yield strength of the first layer and is equal to *X/N*.

If the plastic region is spread from the bottom layer (first layer) into the second layer, the bending moment

$$M = 2\left[\int_{z=0}^{h} \frac{z}{\rho|\bar{a}_{11}|_2} z\,t\,dz + \int_{z=h}^{c/2} \left|\frac{X + K\varepsilon_p}{N}\right|_2 z\,t\,dz + \int_{z=\frac{c}{2}}^{c} \left|\frac{X + K\varepsilon_p}{N}\right|_1 z\,t\,dz\right] \quad (36)$$

where indices show the mechanical properties of the layers, $\varepsilon_p = |a + bz|_j$, a and b are different in each layer.

RESIDUAL STRESSES

If h is known, the bending moment M in Eqs. (34) and (36) can be calculated. Afterwards, elastic and elastic-plastic stress components of σ_x for a each ply can be calculated from the following equations, respectively:

$$(\sigma_x)_e = \frac{M\,z}{I}\left[\frac{(E_x)_j}{E_f}\right] \quad (37)$$

$$(\sigma_x)_p = \frac{\sigma_Y}{N} = \left|\frac{\sigma_0 + K\varepsilon_p}{N}\right|_j = \left|\frac{\sigma_0 + K(a+bz)}{N}\right|_j \quad (38)$$

The superposition of the elastic and elastic-plastic stresses gives the residual stress components as,

$$(\sigma_x)_r = (\sigma_x)_p - (\sigma_x)_e \quad (39)$$

where subscripts r, p and e indicate residual stress, elastic-plastic stress and elastic stress, respectively.

A SAMPLE AND DISCUSSION

The analytical method is carried out on the symmetric aluminum metal-matrix composite laminated cantilever beam reinforced steel fibers unidirectionally. The beam is manufactured by the squeeze casting method. The mechanical properties of the ply are given in Table 1.

The bending moment starting plastic yielding at the upper and lower surfaces for the orientations of plies is given at Table 2. As seen from this table, the bending moment value is found to be the highest for $[30°/-30°]_s$, that is [30°/-30°/-30°/30°], orientation. When the orientation angle is chosen as $[45°/-45°]_s$, $[60°/-60°]_s$ and $[90°/0°]_s$ the bending moment value which starts plastic yielding decreases gradually. It is 11360 Nmm for the laminated beam of $[30°/-30°]_s$. It is the lowest for $[90°/0°]_s$ orientation as 9553 Nmm.

Elastic, plastic and residual stress components of σ_x and equivalent plastic strain for h=5, 4, 3, 2, 1 and 0.5 mm are given in Table 3. The equivalent plastic strain is found to be the highest for $[90°/0°]_s$ orientation for the h=0.5 mm which is the boundary of elastic and plastic regions. It is 0.0133 at the upper and lower surfaces for the $[90°/0°]_s$ orientation and h=0.5 mm. The magnitude of the residual stress component of σ_x is found to be maximum at the upper and lower surfaces. However, if the plastic zone is expanded further the highest residual stress component of σ_x occurs at the boundary of the elastic and plastic regions. It is the highest (124.15 MPa) at the boundary of the elastic and plastic regions for h=0.5 mm and $[90°/0°]_s$ orientation. The equivalent plastic stress in the principal material directions is the highest for $[90°/0°]_s$ orientation and then gradually decreases for $[30°/-30°]_s$, $[60°/-60°]_s$ and $[45°/-45°]_s$ orientations. However, the equivalent plastic stress component of σ_x approaches nearly the same value at the principal material axes.

The distribution of the plastic, elastic and residual stresses for h=2.5 mm is shown in Figs. 2, 3, 4 and 5 for $[90°/0°]_s$, $[30°/-30°]_s$, $[45°/-45°]_s$ and $[60°/-60°]_s$ orientations, respectively. It is seen from Figure 2 that plastic and residual stress components for cross-ply laminated beam change dramatically at the region where 90° and 0° plies are bonded to each other due to the different elastic moduli. At this region, plastic and residual stress components of σ_x gain higher values,

Table 1

Mechanical properties and yield strengths of a layer.

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	Axial strength X (MPa)	Transverse strength Y (MPa)	Shear strength S (MPa)	Plasticity constant K (MPa)
82	73	28	0.29	140	98	64	980

Table 2

Bending moment values starting plastic yielding in the beam.

Orientation angles	$[90°/0°]_s$	$[30°/-30°]_s$	$[45°/-45°]_s$	$[60°/-60°]_s$
Bending moment (Nmm)	9553	11360	10288	9700

Table 3
Elastic, elastic-plastic, residual stress component and plastic strain at the lower surface, and the residual stress at the elastic and plastic boundary.

Orientation angles	M (Nmm)	h (mm)	$\varepsilon_p.10^{-4}$	At the lower surface $(\sigma_x)_p$ (MPa)	$(\sigma_x)_e$ (MPa)	$(\sigma_x)_r$ (MPa)	In principal material axes $(\sigma_{eq})_p$ (MPa)	At the elastic-plastic boundary $(\sigma_x)_r$ (MPa)
$[90°/0°]_s$	11022	5	2	98.13	113.00	-14.94	140.18	3.77
	12252	4	5	98.32	125.69	-27.37	140.46	14.20
	13264	3	9	98.64	136.07	-37.43	140.91	33.66
	15022	2	26	99.80	154.11	-54.30	142.58	82.30
	15840	1	62	102.25	162.50	-60.25	146.07	109.58
	16508	0.5	133	107.13	169.35	-62.22	153.05	124.15
$[30°/-30°]_s$	13098	5	2	118.46	136.44	-17.97	140.16	4.63
	14529	4	4	118.66	151.35	-32.69	140.39	17.43
	15659	3	8	118.99	163.12	-44.13	140.78	36.77
	16507	2	16	119.65	171.95	-52.30	141.55	61.02
	17167	1	40	121.62	178.82	-57.20	143.89	88.53
	17663	0.5	87	125.56	183.99	-58.43	148.55	103.00
$[45°/-45°]_s$	11862	5	1	107.27	123.57	-16.30	140.13	4.20
	13157	4	3	107.42	137.05	-29.63	140.32	15.80
	14176	3	7	107.66	147.67	-40.01	140.64	33.33
	14934	2	13	108.15	155.57	-47.42	141.28	55.32
	15502	1	33	109.63	161.48	-51.86	143.21	80.26
	15892	0.5	72	112.57	165.54	-52.97	147.06	93.38
$[60°/-60°]_s$	11183	5	1	101.13	116.49	-15.36	140.13	3.96
	12404	4	3	101.27	129.20	-27.93	140.34	14.90
	13365	3	7	101.52	139.22	-37.70	140.67	31.42
	14082	2	14	102.00	146.69	-44.69	141.34	52.16
	14624	1	34	103.45	152.33	-48.88	143.35	75.64
	15003	0.5	75	106.36	156.28	-49.92	147.38	88.01

whereas the elastic stress component of σ_x reaches its highest value at the upper and lower surfaces. However, the plastic and residual stress components of σ_x are the highest at the elastic and plastic boundary, as shown in Fig. 2 for cross-ply laminated beam. The magnitude of the plastic and elastic stress components of σ_x is found to be maximum at the upper and lower surfaces for angle-ply laminated beams. But the residual stress component of σ_x is the highest at the elastic and plastic boundary, as shown in Figure 3 for $[30°/-30°]_s$ orientation. The distribution of the stress component of σ_x for $[45°/-45°]_s$ and $[60°/-60°]_s$ orientations is similar to $[30°/-30°]_s$ orientation, as shown in Figs. 4 and 5. Similarly plastic and elastic stresses are found to be maximum at the upper and lower surfaces, whereas the residual stress component is found to be the highest at the boundary of the elastic and plastic regions.

The distribution of the residual stress component of σ_x for h=1, 2, 3, 4 and 5 mm is shown in Figure 6 for $[90°/0°]_s$ orientation. As shown in this figure, the magnitude of the residual stress component is the highest at the upper and lower surfaces. When the

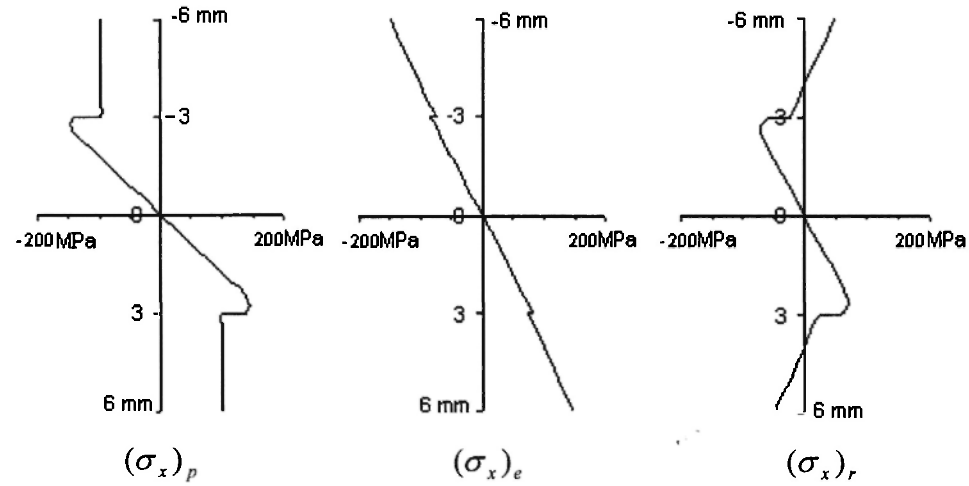

Fig. 2: The distribution of the elastic-plastic, elastic and residual stress component of σ_x for the $[90^\circ/0^\circ]_s$ orientation, h=2.5 mm.

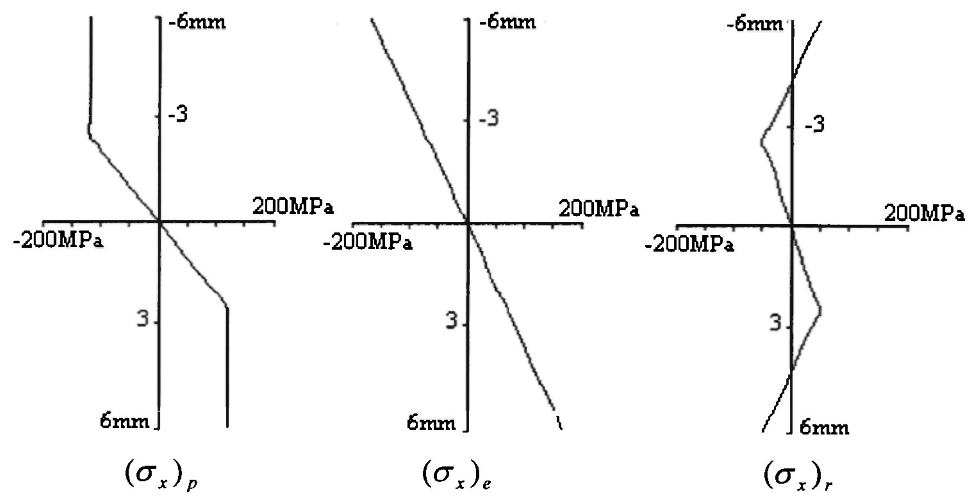

Fig. 3: The distribution of the elastic-plastic, elastic and residual stress component of σ_x for the $[30^\circ/-30^\circ]_s$ orientation, h=2.5 mm.

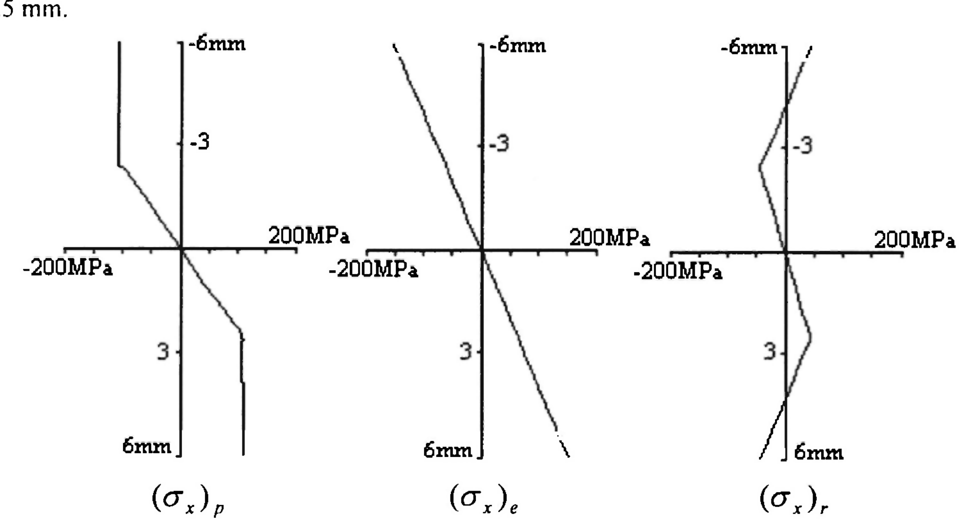

Fig. 4: The distribution of the elastic-plastic, elastic and residual stress component of σ_x for the $[45^\circ/-45^\circ]_s$ orientation, h=2.5 mm.

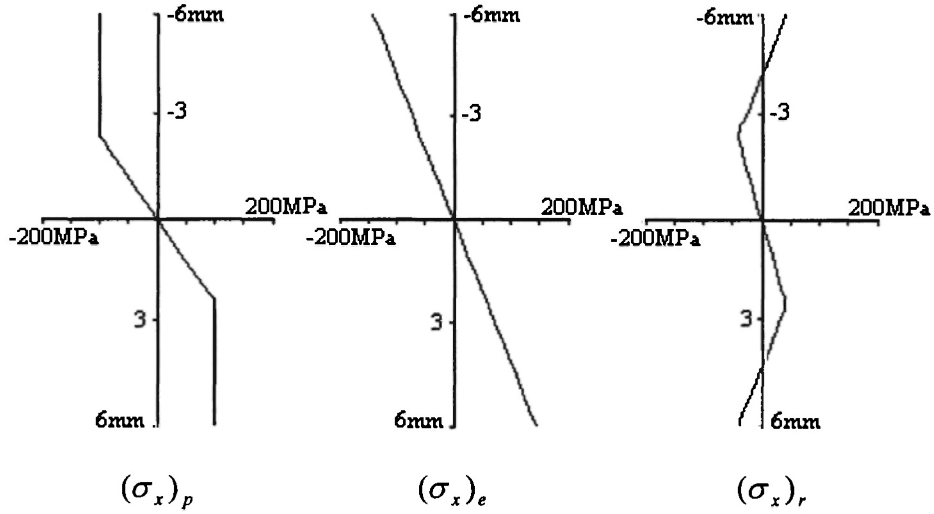

Fig. 5: The distribution of the elastic-plastic, elastic and residual stress component of σ_x for the [60°/-60°]$_s$ orientation, h=2.5 mm.

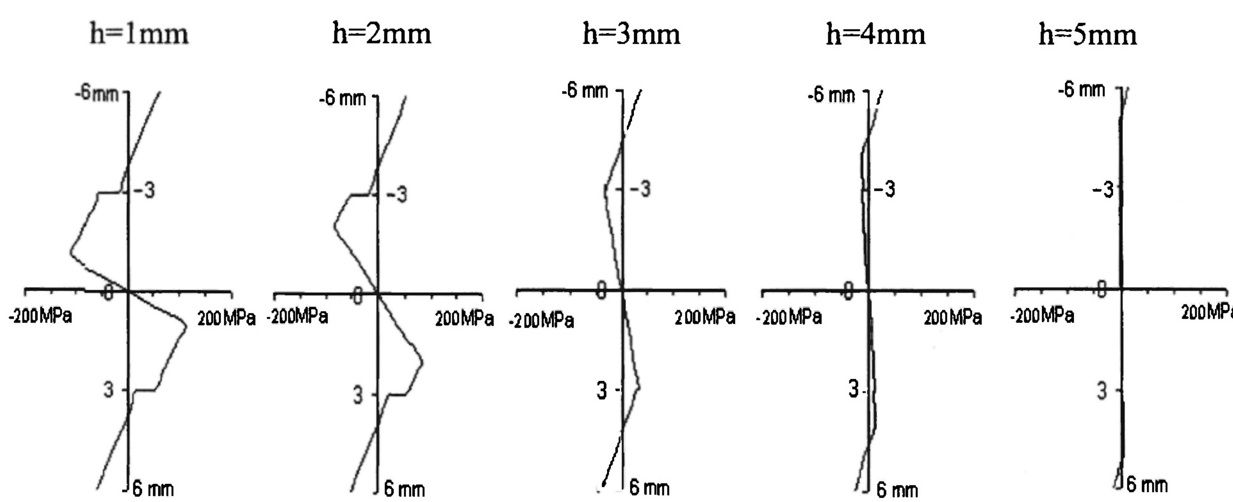

Fig. 6: The distribution of the residual stress component of σ_x for the [90°/0°]$_s$ orientation.

plastic region is further expanded it becomes the highest at the boundary of the elastic and plastic regions. The distribution of the residual stress component of σ_x for [30°/-30°]$_s$, [45°/-45°]$_s$ and [60°/-60°]$_s$ orientations is shown in Figs. 7, 8 and 9. As shown in all the figures, it is the highest at the upper and lower surfaces; however when the plastic zone is further expanded it is the greatest at the boundary of the elastic and plastic zones.

CONCLUSION

In this study, an analytical elastic-plastic stress distribution is carried out on a steel fiber reinforced symmetric aluminum metal-matrix laminated composite beam under the action of bending moment applied at the free end. The composite beam is linearly strain hardening and Bernoulli–Euler hypotheses are valid in the solution; the following are concluded:

- The plastic region starts first at the upper and lower surfaces.
- The elastic-plastic stress solution gives the maximum stress of σ_x at the boundary of the layers for cross-ply laminated beam.
- The elastic-plastic stress solution gives the highest stress of σ_x at the upper and lower surfaces for

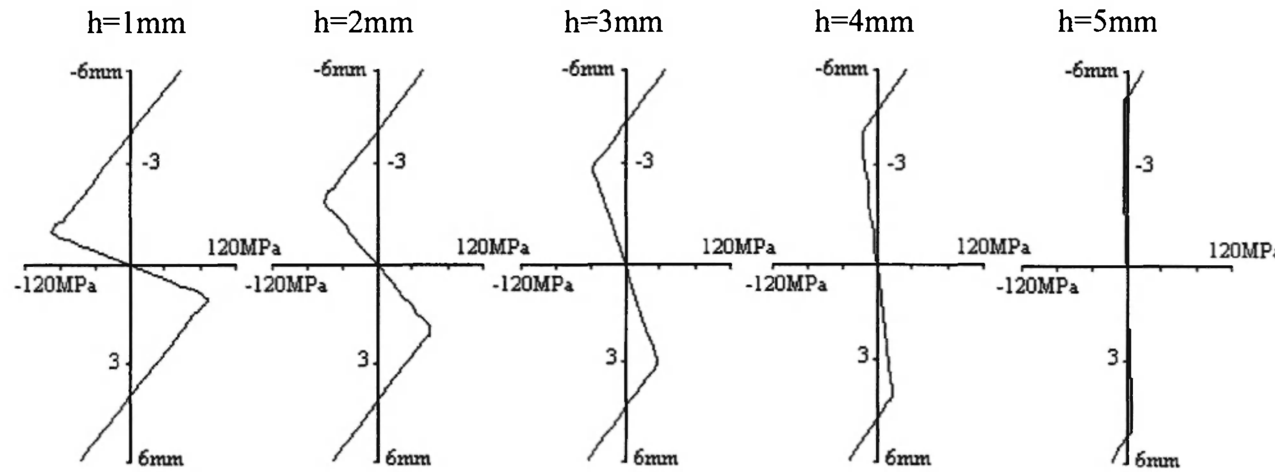

Fig. 7: The distribution of the residual stress component of σ_x for the $[30°/-30°]_s$ orientation.

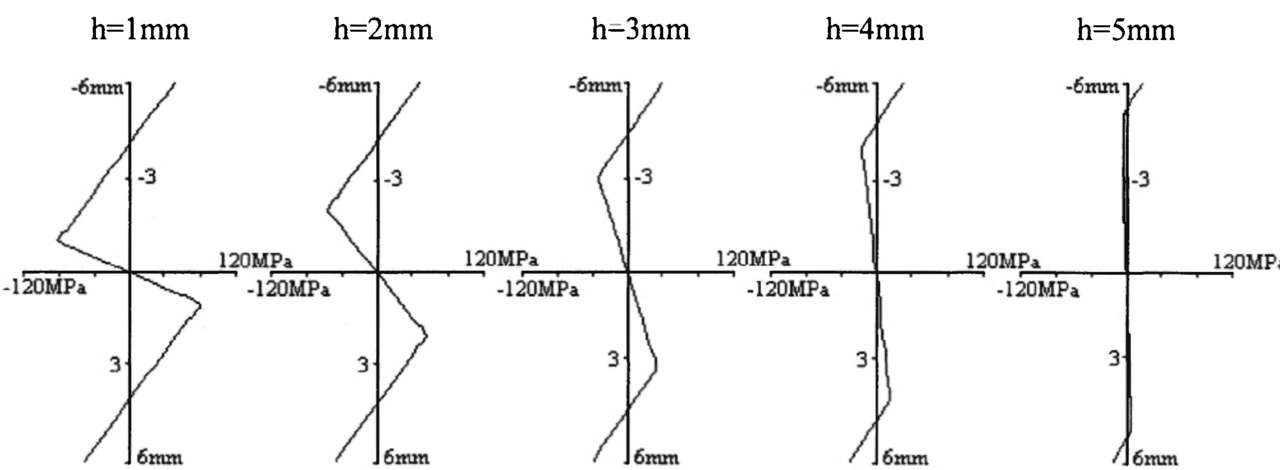

Fig. 8: The distribution of the residual stress component of σ_x for the $[45°/-45°]_s$ orientation.

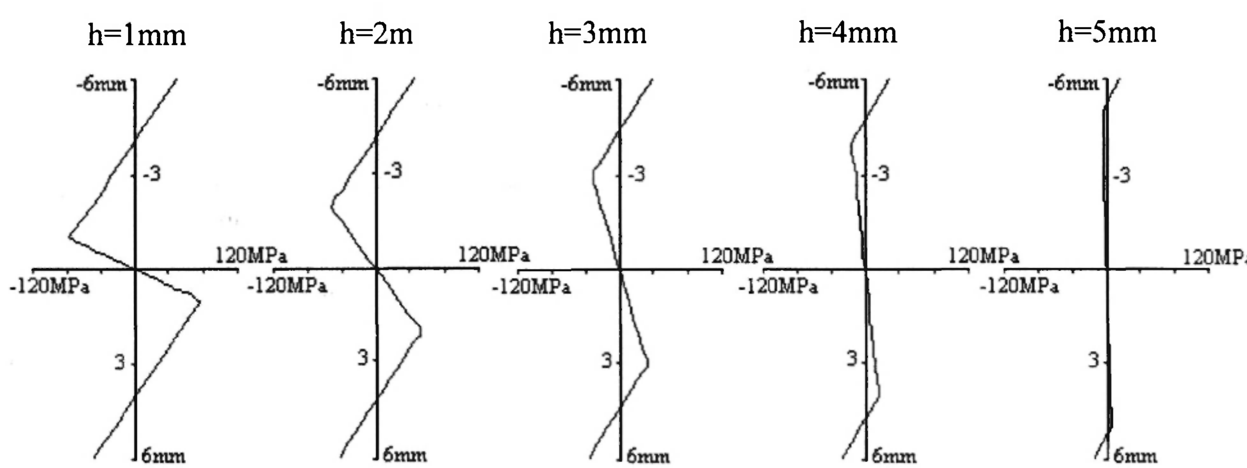

Fig. 9: The distribution of the residual stress component of σ_x for the $[60°/-60°]_s$ orientation.

angle-ply laminated beam.

- The magnitude of the residual stress component of σ_x is the highest at the upper and lower surfaces for all the other orientations, except for the $[90°/0°]_s$ orientation. When the plastic region is further increased, it becomes the largest at the boundary of the elastic and plastic regions for both angle-ply and cross-ply laminated beams.
- The magnitude of the residual stress component of σ_x is the greatest for the $[90°/0°]_s$ orientation angle in comparison with $[30°/-30°]_s$, $[45°/-45°]_s$ and $[60°/-60°]_s$ orientation angles.
- The magnitude of the equivalent plastic strain is found to be highest for the $[90°/0°]_s$ orientation.

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