On The Analytical Modelling Of Elastic Properties For Some Particle - Reinforced Aluminum Matrix Composites

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ABSTRACT:

A study was made to evaluate the applicability of several micromechanics models that predict the variation of the elastic modulus as a function of reinforcement nature and volume fraction for some particulate-reinforced metal matrix composites.

The composites were made by Vortex casting and based on a commercial aluminum alloy, with either single-phase or two-phase (hybrid) reinforcement. The reinforcing particles were graphite and silicon carbide, in volume fraction concentrations of up to 10%.

The three evaluated models were: the Paul model for cube-shaped inclusions, the Paul estimation of modulus upper and lower bounds and the Halpin-Tsai model. Some generalized expressions are proposed in this paper for the first and the third of the cited models in order to be applied to hybrid particulate-reinforced composites. The predictive results were overall compared with the corresponding experimentally determined composite moduli.

The predictive moduli are generally situated above the experimental data, but the precision of the evaluation increases for the composites with prevailing ceramic particle reinforcement. One can say that the three models are all acceptable for hybrid particulate-reinforced metal matrix composites. However, the expression corresponding to the Paul model seems to exceed some fitting parameters, in order to increase the accuracy of the resulting moduli predictions.

INTRODUCTION

The opportunity to tailor material characteristics to match performance requirements is a key feature of composite materials. With this aim in mind, metallurgists and material scientists try various combinations of reinforcements and processing to make new materials and then test them experimentally, in order to see if the properties are improved. It would be of great benefit to the materials development process if micromechanics models were available to help understand a priori the effect of changes in the material microstructure on the composite's mechanical properties.

There are many models of this kind presented in the literature and recommended for evaluating the elastic modulus of particle-reinforced composites. It is interesting to evaluate the applicability of these models for metal matrix composites and even more for the composites with two-phase (hybrid) particulate reinforcement.

The micromechanics models presented in this paper result in predictions of Young's modulus for discontinuously reinforced composites, as a function of reinforcement volume fraction concentration. These models will be examined for the case of some aluminum based composites, by comparing the predictive results with experimental data for single-phase or hybrid reinforcements.

MICROMECHANICS MODELS

Three rather simple models form the basis of our analytical predictions. These models were chosen because they are simple and straightforward, since they do not use empirical constants or unmeasurable material constants, which serve as fitting parameters for other micromechanics models.

The Paul Model

Paul presented a simple strength-of-materials approximation /1/ for the composite modulus (E_C) of a material with cube-shaped inclusions. In the model it was assumed that the inclusion was of finite length, the matrix and inclusion were subjected to the same strain and had the same Poisson's ratios.

Paul's equation may be written as:

$$E_C = E_m \frac{E_m + (E_p - E_m)\sqrt[3]{V_p^2}}{E_m + (E_p - E_m)\sqrt[3]{V_p^2}\left(1 - \sqrt[3]{V_p}\right)} \quad (1)$$

where

E_m, E_p = Young's modulus of the matrix and the particulate material, respectively,

V_p = volume fraction of the particulate reinforcement.

In order to use this model for a composite with two-phase inclusion, one could try to extend the above equation by including some terms, corresponding to the features of the second reinforcing phase. Therefore, Paul's equation for a hybrid composite could be written as follows:

$$E_C = E_m \frac{E_m + (E_{p1} - E_m)\sqrt[3]{V_{p1}^2} + (E_{p2} - E_m)\sqrt[3]{V_{p2}^2}}{E_m + (E_{p1} - E_m)\sqrt[3]{V_{p1}^2}\left(1 - \sqrt[3]{V_{p1}}\right) + (E_{p2} - E_m)\sqrt[3]{V_{p2}^2}\left(1 - \sqrt[3]{V_{p2}}\right)} \quad (2)$$

The Paul Model for Moduli Bounds Evaluation

Paul also proposed a relationship /2/ for determining the bounds of elastic modulus for a composite with an "n"-phase reinforcement, i.e. with reinforcing particles in "n" different materials.

By keeping the above mentioned notations, one can write:

$$\frac{1}{\frac{V_m}{E_m} + \frac{V_{p1}}{E_{p1}} + \cdots + \frac{V_{pn}}{E_{pn}}} \le E_C < E_m \cdot V_m + E_{p1} \cdot V_{p1} + \cdots + E_{pn} \cdot V_{pn} \quad (3)$$

or, in an equivalent form:

$$E_L \le E_C \le E_U \quad (4)$$

It is obvious that (V_m) means the matrix volume fraction, so that:

$$V_m = 1 - (V_{p1} + V_{p2} + \ldots + V_{pn}) \quad (5)$$

The bounds of the composite's modulus (E_C), as obtained from Eqs. 3 and 4, represent the extremes of behavior of the studied material, in the sense that the modulus can never exceed the upper bound (E_U), nor fall below the lower bound (E_L). In the present paper, these limiting values will be calculated from the corresponding parts of Eq. 3, and then compared with the experimental data.

The Halpin-Tsai Model

This model was originally developed to predict the elastic moduli and Poisson's ratios of continuous fiber-reinforced composites, but it can be applied to particulate reinforcements, using a Halpin - Tsai - Kardos type expression /3/, which may be written as follows:

$$E_C = E_m \frac{1 + 2s \cdot q \cdot V_p}{1 - q \cdot V_p} \quad (6)$$

where (s) stands for the aspect ratio (L / d) of the discontinuous reinforcement, and the (q) factor is

$$q = \left(\frac{E_p}{E_m} - 1\right) \frac{1}{\frac{E_p}{E_m} + 2s} \quad (7)$$

It would appear that the (q) factor does not depend on the particles volume fraction, but it is fairly dependent of the elastic moduli of matrix and reinforcement. This fact is of great importance for the following estimations, because the (q) factor will be *constant*, for particles of a certain nature, being independent of the possible hybrid character of the particulate reinforcement.

It is important to point out the fact that this model was not developed for hybrid composites, but the appearance of the above cited expressions leads to the idea of extending their applicability on the (theoretical) supposition that the two types of reinforcing particles are introduced into the matrix structure not simultaneously, but successively. As a consequence, it is possible to estimate, firstly, the elastic modulus for the composite containing only particles of the *A* type, by using an expression of type (6). That estimated value could be used afterwards to predict, from another expression (6), the modulus of the hybrid composite, which also includes particles of the *B* type. In this way, one can write the following *generalized expression* of type (6) for an estimated elastic modulus of a hybrid composite:

$$E_C = E_m \frac{1+2s_A \cdot q_A \cdot V_{pA}}{1-q_A \cdot V_{pA}} \cdot \frac{1+2s_B \cdot q_B \cdot V_{pB}}{1-q_B \cdot V_{pB}} \tag{8}$$

where the subscript symbols *A* and *B* are obviously referring to the two types of reinforcing particles, and the corresponding *q* factors must be calculated from expressions of type (7), as follows:

$$q_{A,B} := \left(\frac{E_{pA,B}}{E_m} - 1\right) \frac{1}{\frac{E_{pA,B}}{E_m} + 2s_{A,B}} \tag{9}$$

Using these expressions, the elastic moduli of some hybrid-reinforced composites will be predicted and then compared with the corresponding experimental results.

MATERIALS

The investigated hybrid composite was manufactured by Vortex casting in laboratory conditions /4/. The unreinforced matrix alloy was a commercial ATSi7Cu3Mg, with a nominal composition of 2.93% Cu, 6.6% Si, 0.51% Fe, 0.36% Mg, 0.15% Ti, with aluminum as the balance.

The reinforcing particles were graphite (Gr) – with an average diameter of 63μm, and silicon carbide (SiC) – 40μm in size.

Each of the composite samples used for mechanical testing was heat-treated by solubilizing (for 4 hours, at 520÷530°C), followed by artificial aging (for 8 hours, at 160÷170°C).

Among the composite's mechanical properties, this paper focuses on the elastic modulus, determined by tensile tests, made in laboratory air at room temperature according to corresponding standards for metals.

EVALUATION RESULTS AND COMMENTS

The predictions for Young's moduli for the studied composites are based, in the above micromechanics models, on some characteristics of matrix and particulate materials.

Firstly, it is necessary to know the elastic modulus of the three materials:

- for the matrix $E_m = E_{Al} = 51$ GPa (obtained by averaging the experimental results for the base alloy);
- for the reinforcement $E_{SiC} = 410$ GPa; $E_{Gr} = 27$ GPa (for example, according to Ref. 5).

Subsequently it is necessary to specify, for each type of composite, the volume content of reinforcing particles, because during the casting process one refers to the mass content (V_{pm}). For this purpose, the density values for the three materials are needed. For this specific case, these are obtained as follows: $\rho_{Al} = 2.75$; $\rho_{SiC} = 3.20$; $\rho_{Gr} = 2.10$ [g/cm^3]. These values are in good agreement with the corresponding limits indicated in literature (see Ref. 5). The resulting volume content values, (V_m) and (V_p), are presented below, in Tables 1 and 2.

Application of the Halpin-Tsai model comprises the calculation of (q) factors for the two types of reinforcing materials.

For the studied composites, considering the approximate spherical shape of the reinforcing particles, together with the specific method that was used for standardizing the particles' size, an aspect ratio of s=1 was assumed for use in the expressions (8) and (9).

Using also the above values of the elastic moduli, the expressions of type (9) yield (q) factor values as follows:

- for silicon carbide particles $q_{SiC} = 0.70117$
- and for graphite particles $q_{Gr} = -0.18605$.

Table 1

The Paul and the Halpin-Tsai predictions versus the experimental values of Young's modulus, for hybrid composites with 10% SiC and a variable content (0-5%) of Gr particles

Reinforcing particles content				Predictions		Experimental
				Paul	Halpin	
V_{pm} - SiC	V_p - SiC	V_{pm} - Gr	V_p - Gr	E_C [GPa]		E_C [GPa]
0.1	0.1042	-	-	72.69	63.06	62.3
0.1	0.1021	0.03	0.0201	71.17	62.11	60.9
0.1	0.1008	0.05	0.0331	70.86	61.49	59.1

Table2

The Paul and the Halpin-Tsai predictions versus the experimental values of Young's modulus, for hybrid composites with 5% Gr and a variable content (0-9%) of SiC particles

Reinforcing particles content				Predictions		Experimental
				Paul	Halpin	
V_{pm} - SiC	V_p - SiC	V_{pm} - Gr	V_p - Gr	E_C [GPa]		E_C [GPa]
-	-	0.05	0.0370	50.08	49.95	34.7
0.03	0.0325	0.05	0.0356	58.44	53.49	38.2
0.05	0.0530	0.05	0.0350	62.46	55.78	43.8
0.07	0.0727	0.05	0.0341	66,05	58.10	51.9
0.09	0.0916	0.05	0.0334	69,31	60.36	58.3

One can observe that the second factor is negative, corresponding to the effect of soft particles on the composite's elastic modulus.

All these values allow application of the expressions (2), (3) and (8), in order to predict or evaluate the moduli for the composites that are proposed to be studied.

Predictions based on the Paul model

By replacing the cited real values in Eq. 2, one can obtain for the composite's moduli the results that are summarized in Tables 1 and 2 below. The predictions based on this model are also presented in Figure 1.

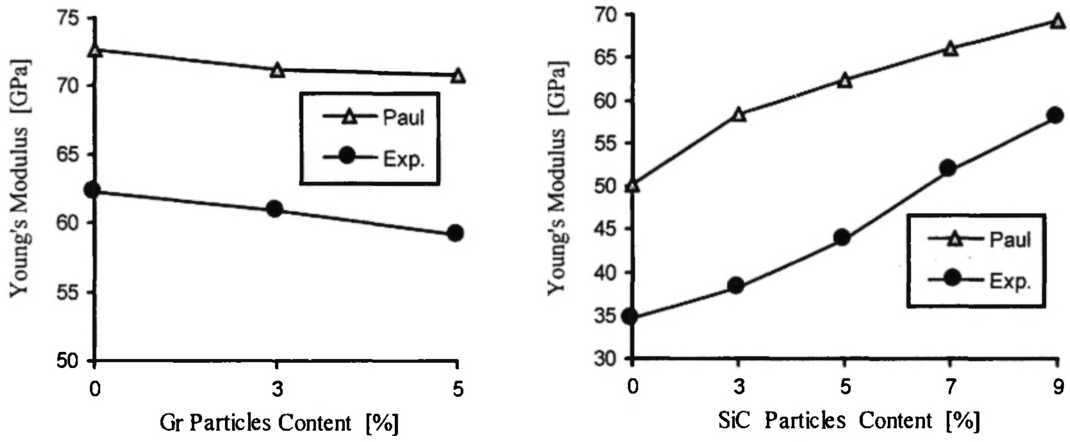
This model gives a good prediction of the sense of evolution for the composite modulus as a function of the variable particle content for all of the studied material systems, so the generalized expressions are suitable for hybrid composites.

Nevertheless, the predicted values are overall greater than the experimentally determined moduli. This fact seems to be caused by the approximations of the Paul model. It may be necessary to use some fitting parameters in order to decrease the overestimation of the composite modulus.

Predictions based on the Halpin-Tsai model

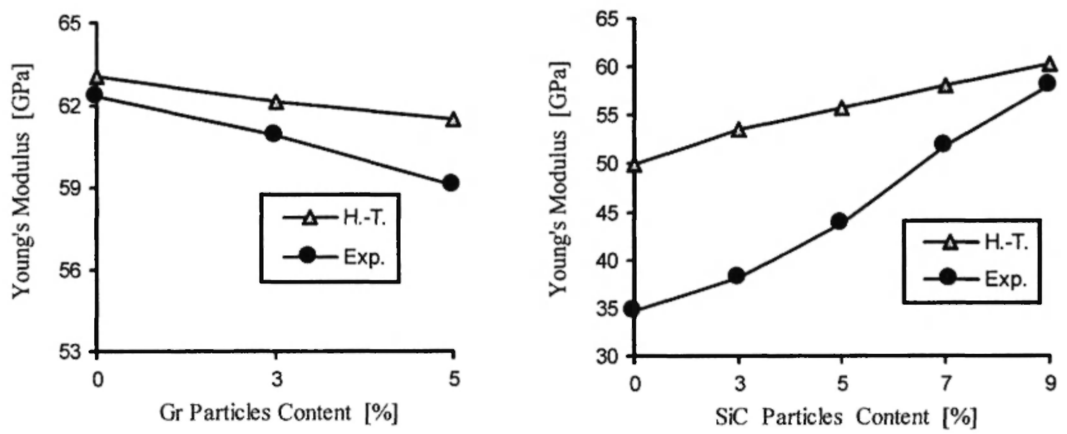
Using expressions like Eq. 8, and the above input data, one can evaluate the Young's modulus for hybrid composites. The predicted moduli are listed in Tables 1 and 2 for each of the studied material systems, together with the corresponding average experimental values.

These results are also compared in Figure 2 in order



(a) (b)

Fig. 1: Comparison of experimental and predicted (from the Paul model) composite's moduli (a) 10% SiC and a variable content of Gr particles; (b) 5% Gr and a variable content of SiC particles.



(a) (b)

Fig. 2: Comparison of experimental and predicted (from the Halpin-Tsai model) composite's moduli (a) 10% SiC and a variable content of Gr particles; (b) 5% Gr and a variable content of SiC particles.

to evaluate the effective accuracy of predictions for the two types of hybrid composites.

The result is that the predicted tendency of modulus evolution, in dependence of the reinforcement content, is in good agreement with that indicated by the experimental results for both categories of materials. However, several features are to be emphasized.

Thus, in the case of a variable graphite content, the predicted values are within 6% of the experimental data, being very close for 0% Gr (only 10% SiC particle reinforcement). The accuracy of evaluation seems to decrease with increasing content of Gr particles, because the real data tend to fall faster than the predictions.

For the other case, as can be observed in Figure 2b, the micromechanics model gives an appreciable overestimation of Young's modulus for the composite with only soft reinforcing particles (0% SiC content), but the predicted values are then closer to the experimental ones, as the content of ceramic particles increases.

Evaluation of Young's modulus bounds

With expressions of type (3), one can evaluate the upper (E_U) and the lower (E_L) bounds of the composite's modulus. Tables 3 and 4, together with Figure 3, give a synthesis of the results that are obtained for the studied composites.

The prediction of bounds is very good, especially for the lower bound, for the composites with an important SiC particle content (*Fig. 3*). The effect of soft particles in decreasing the composite modulus seems to be underestimated.

CONCLUSIONS

- Three simple micromechanics models are used for predicting the evolution of the composite elastic modulus as a function of reinforcement nature and content to some particulate-reinforced MMCs.
- One can propose generalized expressions of the Paul and Halpin-Tsai micromechanics models, in order to apply them for hybrid reinforced MMCs.
- The predicted moduli are overall situated above the experimental ones, but the precision of evaluation increases for the composites with prevailing ceramic particle reinforcement.
- One can say that the cited models lead to good predictions of the influence exerted by some hard reinforcing particles on the hybrid composite's modulus. On the other hand, the effect of graphite particles in weakening the composite's elastic properties seems to be underestimated.

Table 3

The Paul evaluation of Young's modulus bounds, in comparison with the experimental values, for hybrid composites with 10% SiC and a variable content (0-5%) of Gr particles

Volumic fraction concentration			Evaluated bounds		Experimental
V_p - SiC	V_p - Gr	V_m	E_L [GPa]	E_U [GPa]	E_C [GPa]
0.1042	-	0.8958	56.12	88.42	62.3
0.1021	0.0201	0.8777	54.93	87.19	60.9
0.1008	0.0331	0.8661	54.19	86.39	59.1

Table 4

The Paul evaluation of Young's modulus bounds, in comparison with the experimental values, for hybrid composites with 5% Gr and a variable content (0-9%) of SiC particles

Volumic fraction concentration			Evaluated bounds		Experimental
V_p - Gr	V_p - SiC	V_m	E_L [GPa]	E_U [GPa]	E_C [GPa]
0.0370	-	0.9630	49.38	50.11	34.7
0.0356	0.0325	0.9319	50.84	61.81	38.2
0.0350	0.0530	0.9120	51.79	69.19	43.8
0.0341	0.0727	0.8932	52.76	76.30	51.9
0.0334	0.0916	0.8750	53.71	83.08	58.3

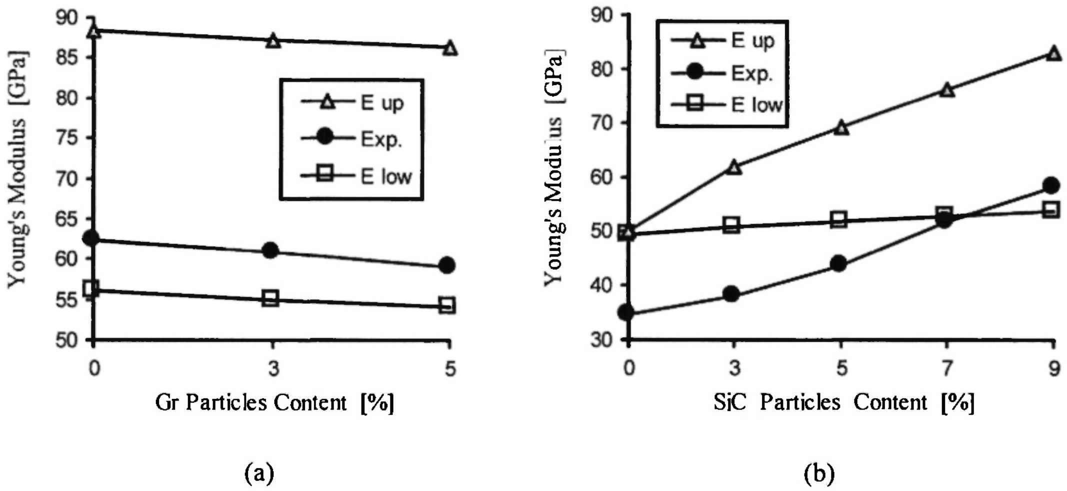


Fig. 3: Comparison of experimental composite's moduli and the evaluated (Paul model) bounds (a) 10% SiC and a variable content of Gr particles; (b) 5% Gr and a variable content of SiC particles.

- It is possible to explain this by the fact that the micromechanics models were developed for composites of the classical type, where the reinforcement is harder than the unreinforced matrix.

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