[1]Thermo-Mechanical Response Predictions for Metal Matrix Composite Laminates

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ABSTRACT

An analytical, micromechanical model is employed for prediction of the stress-strain response of metal matrix composite laminates subjected to thermo-mechanical loading. The predicted behavior of laminates is based upon knowledge of the thermo-mechanical response of the transversely isotropic, elastic fibers and the elastic-viscoplastic, work-hardening matrix.

The method is applied to study the behavior of silicon carbide/titanium metal matrix composite laminates. The response of laminates is compared with that of unidirectional lamina. The results demonstrate the effect of cooling from a stress-free temperature and the mismatch of thermal and mechanical properties of the constituent phases on the laminate's subsequent mechanical response. Typical results are presented for a variety of laminates subjected to monotonic tension, monotonic shear and cyclic tensile/compressive loadings.

INTRODUCTION

As engineers and scientists strive to improve the performance of structures and materials for applications in extreme environments, it is natural that their attention has turned to metal matrix fibrous composite materials. Metal matrix composites (MMC) offer a broad range of potential advantages depending upon the application. Some of the most important advantages include high stiffness, high strength, dimensional stability, fatigue resistance, and lower weight. The choice of fiber and matrix usually depends upon the load requirements, environmental conditions, and cost.

A significant advantage of metal matrix composites is the ability to use them at elevated temperatures where resin matrices decompose and burn. Consequently, modeling the inelastic behavior of MMC unidirectional laminae and laminates is of a significant importance to the technical community. Several available elastic-plastic models of fibrous composite materials can be found in the review article by Bahei-El-Din and Dvorak /1/. More recent works include those by Bahei-El-Din /2/, Fujita et al /3/, Dvorak et al /4/ and Sun et al /5/.

The method of cells is a micromechanical model of periodic array of fibers which was recently reviewed by Aboudi /6/. This micromechanical approach is analytical and requires minimal computational effort while offering the ability to predict the behavior of composites under a wide variety of thermomechanical loading conditions.

The prediction of thermo-mechanical yield surfaces of metal matrix composites was presented previously by Herakovich et al /7/. In the present paper the micromechanical method is generalized for the prediction of the non-linear, thermo-mechanical response of laminates. This extension enables the study of the effect of cooling from a stress-free temperature and mismatch of thermal expansion properties of the fiber and matrix phases on a laminate's subsequent stress-strain response. Thus the residual stresses induced in the matrix material due to a temperature change can be accounted for in the micromechanical predictions and used in the prediction of laminate response.

The laminate problem exhibits an additional level of sophistication over the lamina. In addition to the micromechanical mismatch effects, there are the effects of mismatch in effective properties of different layers in a laminate. Further, individual layers of the laminate may "yield" at different thermo-mechanical load states and this must be modelled when predicting the effective nonlinear stress-strain response of the laminate.

In the following, typical results are presented for monotonic tension, monotonic shear, and cyclic tensile/compressive thermo-mechanical loadings of unidirectional lamina and laminates.

FORMULATION

As mentioned previously, the micromechanical method of cells was reviewed in /6/. The method models a unidirectional fibrous composite as a repeating fiber cell imbedded regularly in the matrix (Fig. 1a). The fibers have a square cross section $h^2{}_1$ and are arranged at a distance h_2 apart. A typical representative cell (Fig. 1b) consists of a fiber subcell and three matrix subcells. The micromechanics analysis of the representative cell consists of the application of the continuity of displacements and tractions at the interfaces between the subcells and between neighboring cells on an average basis together with equilibrium considerations.

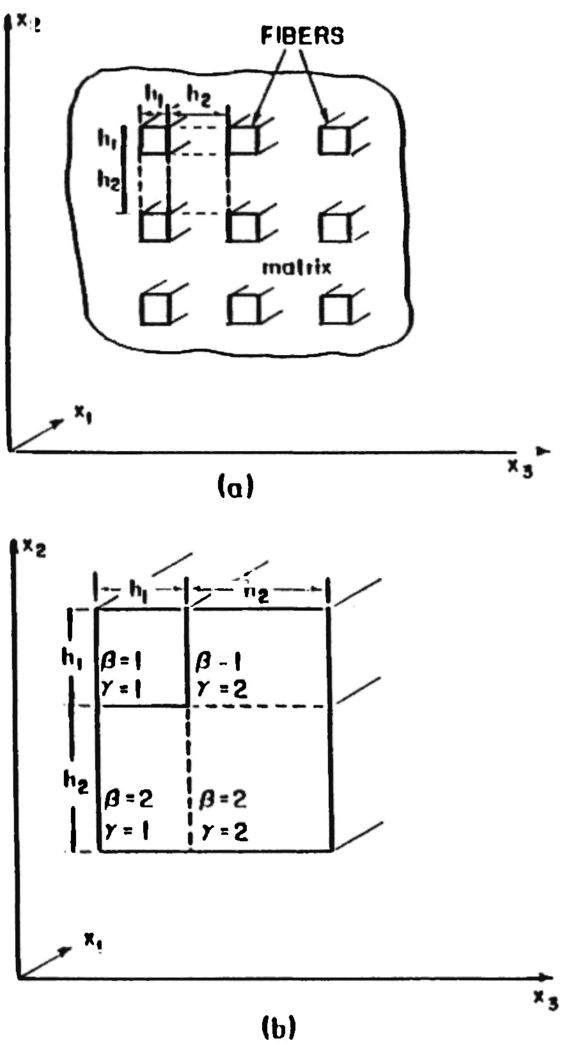

Fig. 1: Method of cells model

Consider an inelastic metal matrix reinforced by unidirectional fibers. The matrix is assumed to be an isotropic elastic-viscoplastic material. Its viscoplastic behavior is described by the Bodner and Partom /8/ unified theory of plasticity. Thus, the constitutive laws governing the matrix material behavior can be expressed in the form

$$\sigma^{(m)} = C^{(m)}(\varepsilon^{(m)} - \varepsilon^{PL(m)}) - \Gamma^{(m)}\Delta T \tag{1}$$

where $\sigma^{(m)}$ and $\varepsilon^{(m)}$ denote the stress and strain in the matrix, $C^{(m)}$ is the stiffness matrix and $\varepsilon^{PL(m)}$ is the plastic strain. The thermal term is represented by $\Gamma^{(m)}\Delta T$ where ΔT is the temperature deviation from a reference temperature.

The plastic strain rate of the viscoplastic materials is given by

$$\dot{\varepsilon}^{PL(m)} = \Lambda_m \hat{\sigma}^{(m)} \tag{2}$$

where $\hat{\sigma}^{(m)}$ is the deviatoric stress tensor. In eqn. (2), Λ_m is the flow rule function which is given, according to Bodner and Partom unified theory by

$$\Lambda_m = D_0 \exp\left\{-\hat{n}\left[Z^2/(3J_2)\right]^n\right\}/J_2^{1/2} \tag{3}$$

with $\hat{n} = 0.5(n+1)/n, J_2 = \hat{\sigma}_{ij}\hat{\sigma}_{ij}/2$, and D_0 and n being inelastic parameters. In (3) Z is a state variable which in the case of isotropic hardening can be determined from

$$Z = Z_1 + (Z_0 - Z_1)\exp[-m\,W_p/Z_0]$$

where W_p is the plastic work, and Z_0, Z_1, m are additional plastic parameters.

The fibers are considered to be elastic and transversely isotropic, i.e.,

$$\sigma^{(f)} = C^{(f)}\varepsilon^{(f)} - \Gamma^{(f)}\Delta T \qquad (4)$$

where $C^{(f)}$ and $\Gamma^{(f)}$ are the corresponding quantities which describe the fiber material behavior.

It can be shown /6/ that the micromechanics analysis leads to the following constitutive law which controls the overall behavior of the unidirectional metal matrix composite

$$\bar{\sigma} = E(\bar{\varepsilon} - \bar{\varepsilon}^{PL}) - U\Delta T \qquad (5)$$

Here, σ and ε denote the average stresses and strains in the composite, $\bar{\varepsilon}^{PL}$ is the average plastic strain, E is the effective stiffness matrix and U ΔT represents the thermal term in which ΔT is the deviation of the temperature from a reference temperature at which the material is stress-free and the elastic strain is zero. The explicit form of the elements of E and U in terms of the fiber and matrix properties and reinforcement ratio can be found in /6/, where the evolution law of $\bar{\varepsilon}^{PL}$ is also given.

The constitutive law (5) can be employed to predict the nonlinear behavior of unidirectional fibrous composites from the knowledge of the material properties of the constituents. By utilizing the classical lamination theory one can establish the following constitutive equations of metal matrix composite laminates.

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} - \begin{bmatrix} N^{PL} \\ M^{PL} \end{bmatrix} - \begin{bmatrix} N^T \\ M^T \end{bmatrix} \qquad (6)$$

In eqn. (6), N and M denote the stress resultant and moments, and ε^0, κ are the laminate midplane strains and curvatures. The terms N^{PL}, M^{PL} denote plastic resultants and moments, and N^T, M^T correspond to the thermal resultants and moments. Finally, A, B and D are the extensional stiffness matrix, the coupling stiffness matrix, and bending stiffness matrix, respectively. Note that while eqn. (5) is formulated in terms of material coordinate system (x_1, x_2, x_3) in which the fibers are oriented in the x_1 direction, eqn. (6) is given in terms of the laminate coordinate system (x, y, z) in which z is directed in the direction of the normal to the middle surface.

TYPICAL RESULTS

The derived constitutive relations which govern the response of metal matrix laminates are employed herein to predict the nonlinear response of continuous SCS_6 silicon carbide fiber, Ti-15-3 titanium matrix composites. All laminates investigated had a symmetric stacking sequence. Stress-strain response predictions were determined for the stress-free residual state ($\Delta T = 0$) and for thermo-mechanical loadings in which the temperature was decreased (corresponding to a fabrication process) followed by mechanical loading. In this analysis, all material properties were considered temperature independent. The material properties assumed for each constituent material are presented in Table 1 and the stress-strain curves for the two constituents are presented in Fig. 2. As indicated in the figure, the fiber is linear elastic and the matrix is elastic, nearly perfectly plastic with a maximum stress of approximately 135 ksi. A fiber volume fraction $v_f = 0.4$ was chosen for all predictions of composite response.

Unidirectional Lamina

The thermo-mechanical stress-strain responses for four different load histories are shown in Fig. 3. The four curves correspond to temperature changes of 0, -1000, -3000, and -5000° F followed by mechanical loading ε_{11} = 2.0%. The axial contraction with zero average stress under the thermal loading and subsequent nonlinear response under mechanical loading are clearly evident in the figure for all four load histories. It was

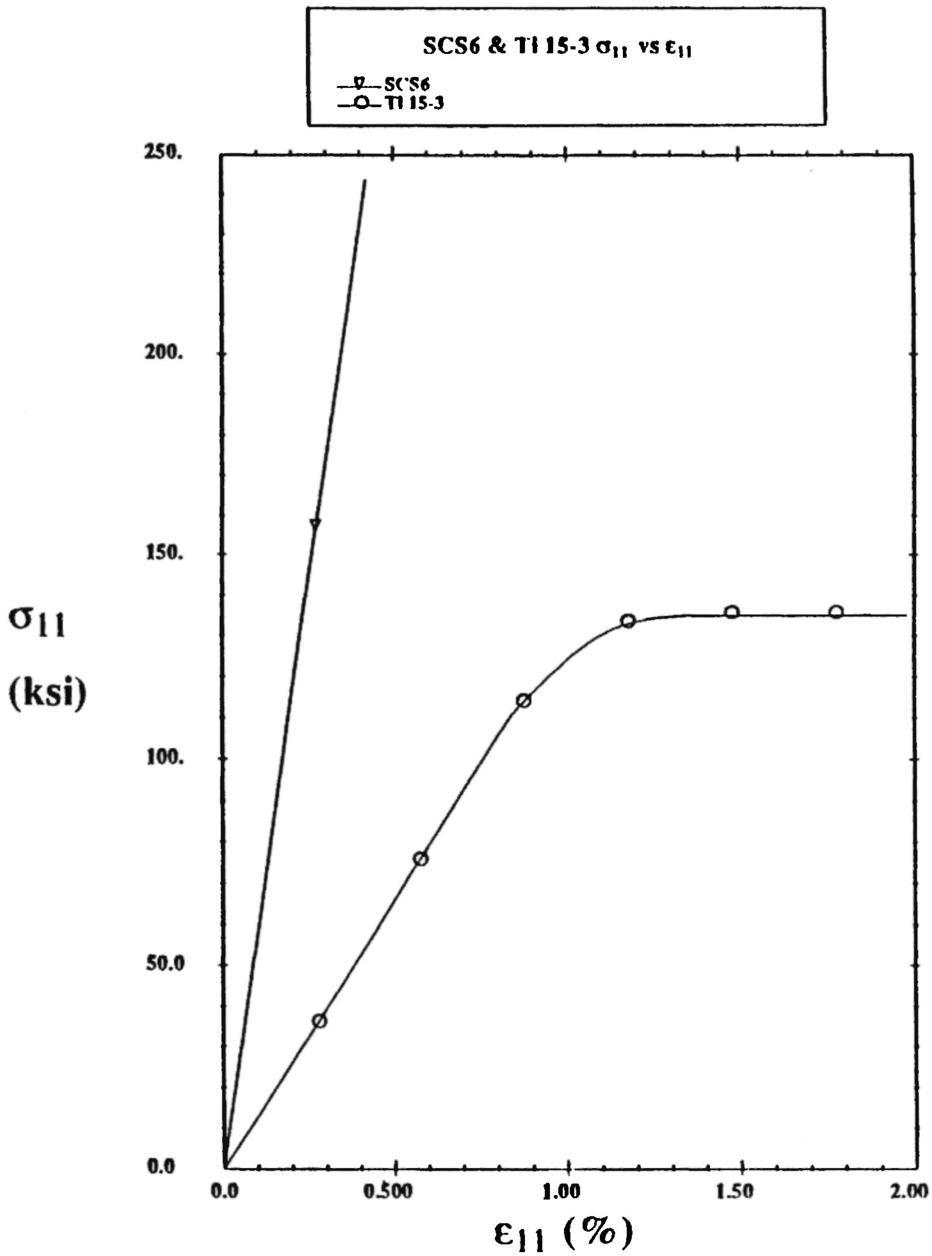

Fig. 2: Constituent responses

determined in reference /7/ that thermal yielding of a subcell initiated at -4230° F. Thus the response for ΔT = -5000 includes yielding during the thermal loading. A temperature change of -5000° F is not realistic for SCS_6/Ti-15-3 as the material cannot withstand the high temperatures required for such a temperature change. Nevertheless, the predictions are presented to demonstrate the effects of thermal yielding followed by mechanical loading for MMC.

The effects of the residual thermal stresses on the axial stress-strain response for the different thermal loadings is demonstrated more clearly in Fig. 4. In this figure the curves have been shifted such that the mecha-

Table 1 MATERIAL PROPERTIES OF SCS_6 FIBERS AND TI-15-3 MATRIX

SCS_6	E_A(Msi)$^+$	ν_A	E_T(Msi)	ν_T
	58.0	0.25	58.0	0.25
	G_A(Msi)	Y (ksi)	$\alpha_A(10^{-6}/°F)$	$\alpha_T(10^{-6}/°F)$
	23.2	---	2.77	2.57
Ti-15-3	E(Msi)	ν	G(Msi)	n
	13.2	0.36	4.83	7.0
	D_0 (sec)	Z_0 (ksi)	Z_1 (ksi)	m
	10^4	140	170	1700
	Y (ksi)	$\alpha(10^{-6}/°F)$		
	110	5.14		

nical loadings all initiate at the origin. The figure clearly shows a monotonically decreasing proportional limit with increasing magnitude of ΔT. The stiffness of the response after complete yielding of the matrix is identical for all four load cases as exhibited by the parallel final slopes of the stress-strain curves. The final stiffness corresponds very closely to the rule of mixtures prediction with zero contribution from the matrix.

Transverse responses for three thermal-mechanical loadings are shown in Figs. 5 and 6. It is evident from Fig. 5 that the transverse response under mechanical loading exhibits a much sharper nonlinearity than does the axial response. This is expected since the transverse response is more of a matrix dominated response. The results in Fig. 6 where the mechanical responses have been shifted to initiate at the origin exhibit some surprising results. The proportional limit is not a monotonic function of the temperature change. The proportional limit for ΔT = -1000° F is higher than that for ΔT = 0, but that for ΔT = -3000° F is lower than for ΔT = 0. Also, while initial yielding at low stress levels is evident for ΔT = -3000, complete yielding at this temperatures occurs at a higher final stress than the lower ΔT cases. Indeed, after complete yielding of the matrix, the stress-strain curves are in the order of the temperature changes with the largest temperature change correspon-

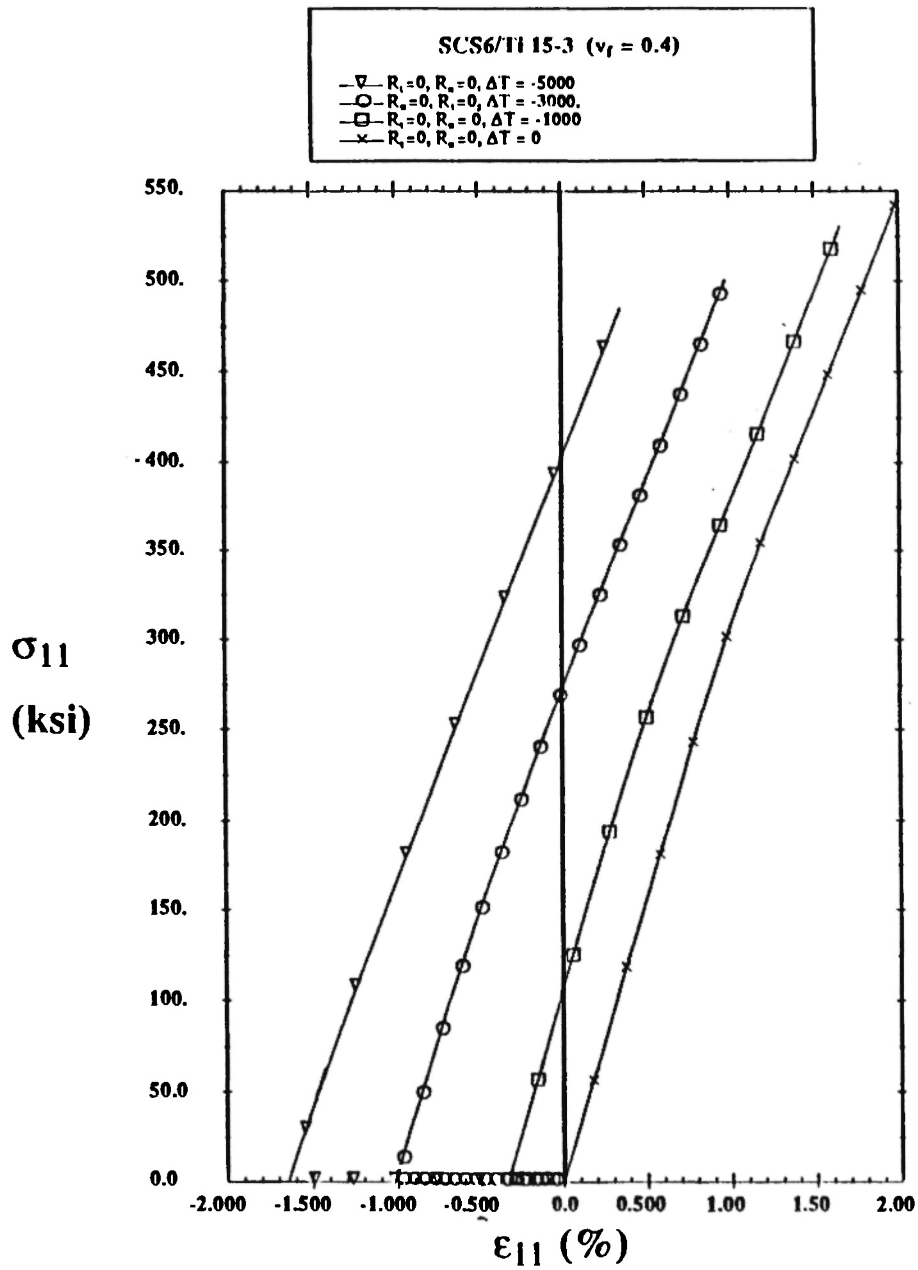

Fig. 3: Thermo-mechanical σ_{11} vs ε_{11}

ding to the most "strain-hardening". To be sure, the changes with thermal loading are relatively small, but nevertheless evident.

The inplane shear response of the unidirectional lamina for three different thermal loading conditions is shown in Fig. 7. The proportional limit in shear decreases with increasing ΔT with all curves attaining approximately the same fully yielded shear stress.

These results for thermo-mechanical response of unidirectional laminae (Figs. 3-7) demonstrate the triaxial nature of the residual stress state. Subsequent yielding due to mechanical loading is dependent on the type of mechanical loading and the degree of thermal loading.

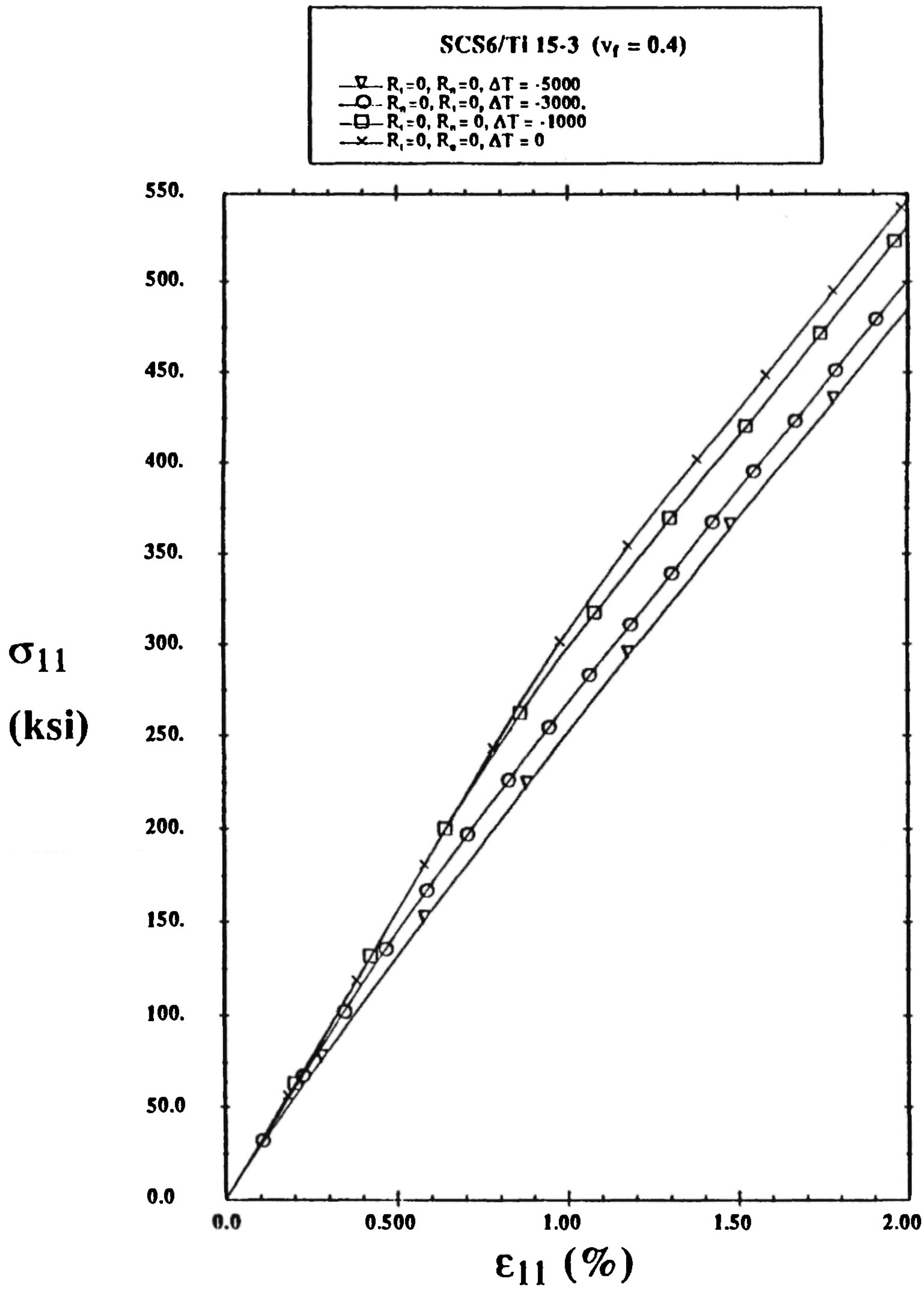

Fig. 4: Mechanical σ_{11} vs ε_{11}

Laminates

Three different laminates have been chosen to demonstrate the nonlinear thermo-mechanical response of laminated MMC, and also to demonstrate the versatility of the method of cells. The presented results are typical of results for a larger number of laminates which were studied during the course of the investigation. Additional results are not presented due to space limitations and the lack of any significant differences in

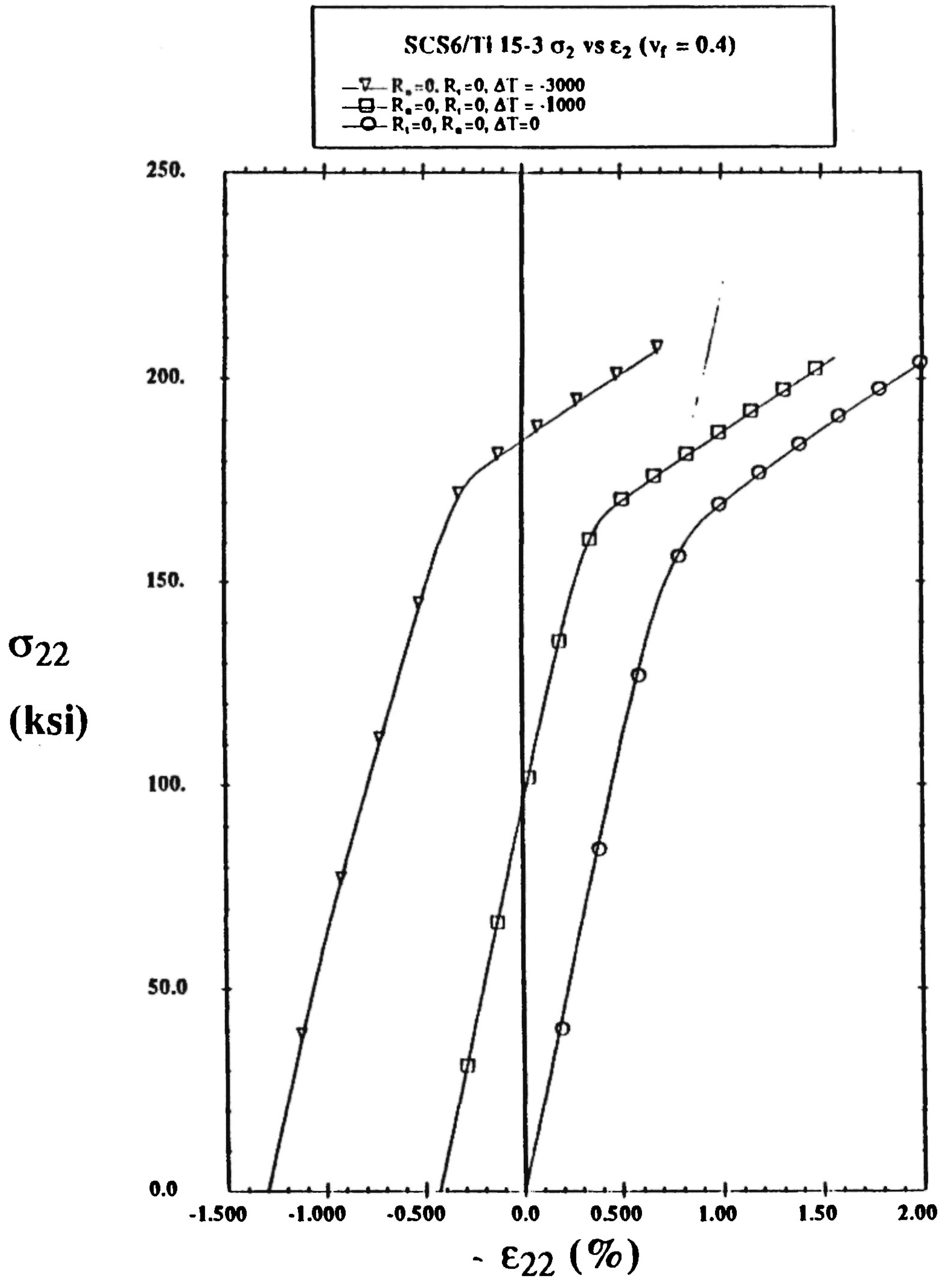

Fig. 5: Thermo-mechanical σ_{22} vs ε_{22}

response. In all of the following laminate results, the curves have been shifted such that the mechanical loading initiates from the origin.

Fig. 8 shows the tensile response of a $[0/\pm15]_s$ laminate for three different thermal conditions. Clearly the proportional limit decreases monotonically with increasing magnitude of temperature change and the final stiffness is the same for all three thermal conditions. The shear response for this same laminate is shown in

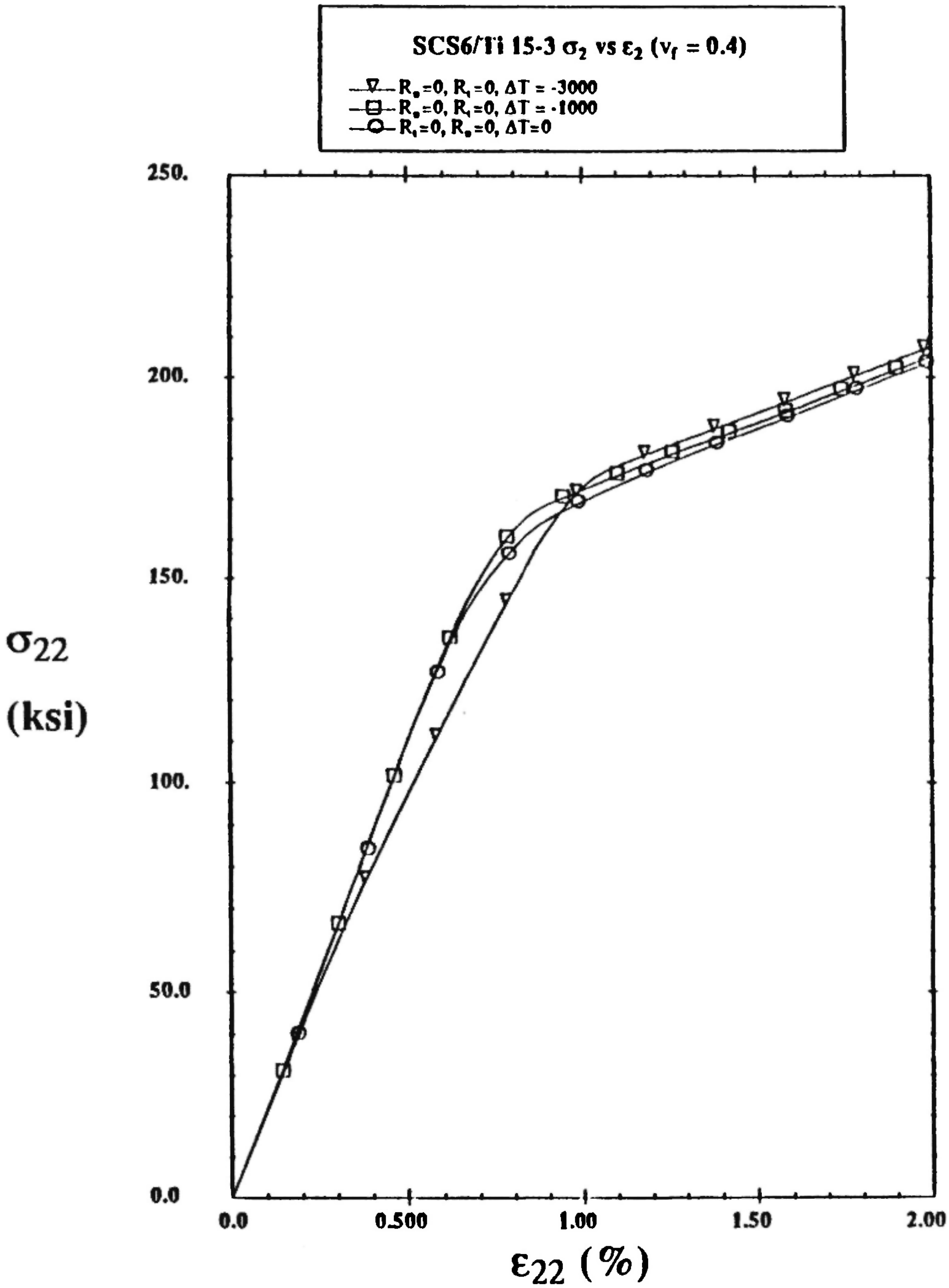

Fig. 6: Mechanical σ_{22} vs ε_{22}

Fig. 9. These results show that the shear response is affected very little by the residual thermal stresses. Comparison of the axial responses (Fig. 8) and shear responses (Fig. 9) indicates that residual stresses have more influence on the axial response than on the shear response.

Cyclic tensile-compressive thermo-mechanical loading of a $[0/\pm 15]_s$ laminate is shown for two thermal conditions in Fig. 10. As in the previous results for this laminate, the residual stresses do not affect significantly the overall response either for the loading or unloading mechanical phases. The thermal stresses have

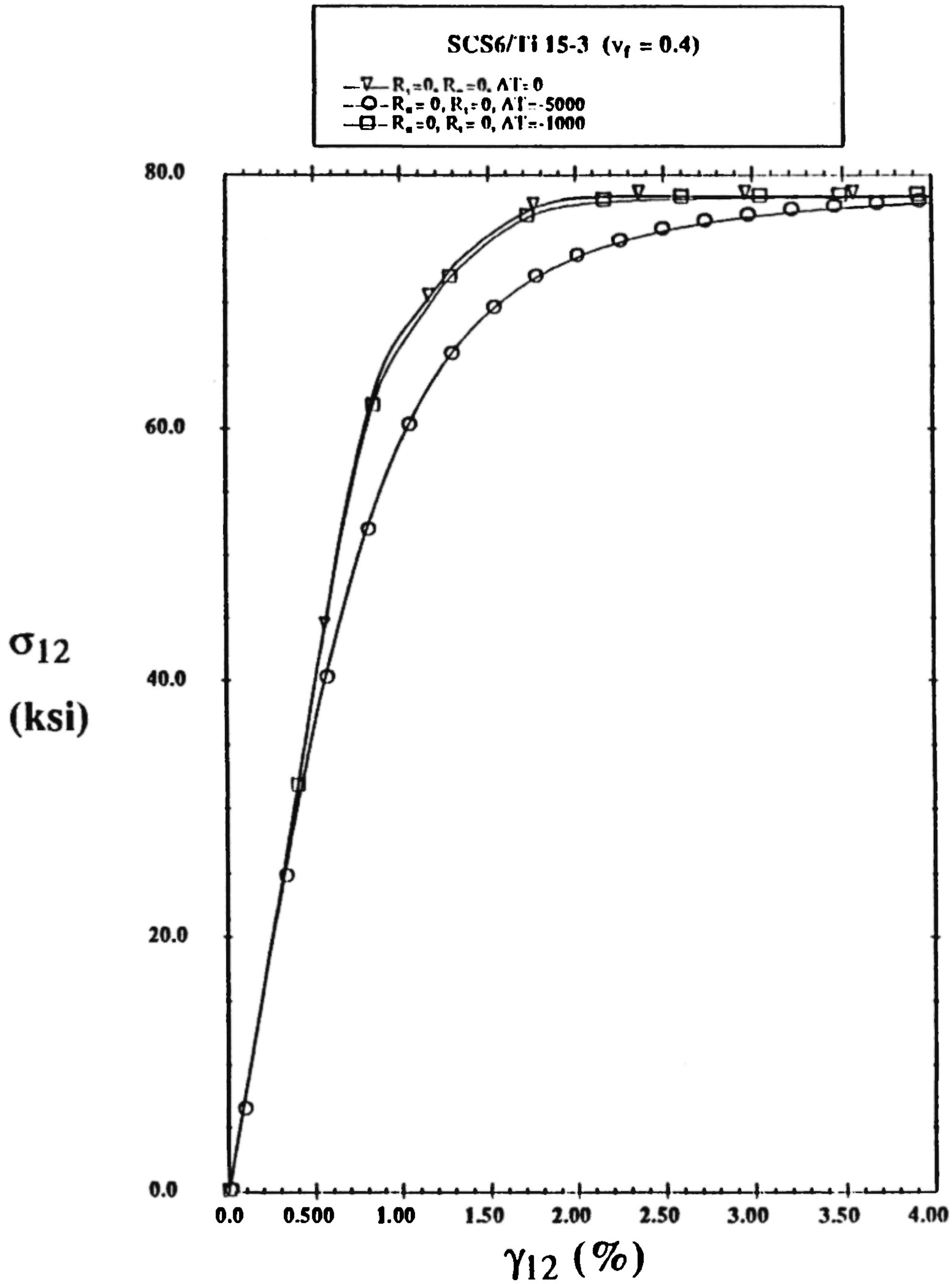

Fig. 7: Mechanical σ_{12} vs γ_{12}

the effect of translating the stress-strain response without significantly altering the size or shape of the curves. One difference in the curves is the lower initial proportional limit in the presence of thermal stresses. These results clearly demonstrate the ability of the model to track completely the thermo-mechanical, cyclic response of the laminates.

The shear response of a $[0/\pm45]_s$ laminate is shown in Fig. 11 for three different thermal conditions. The shear response is almost identical for all three conditions. The cyclic tensile-compressive response of this

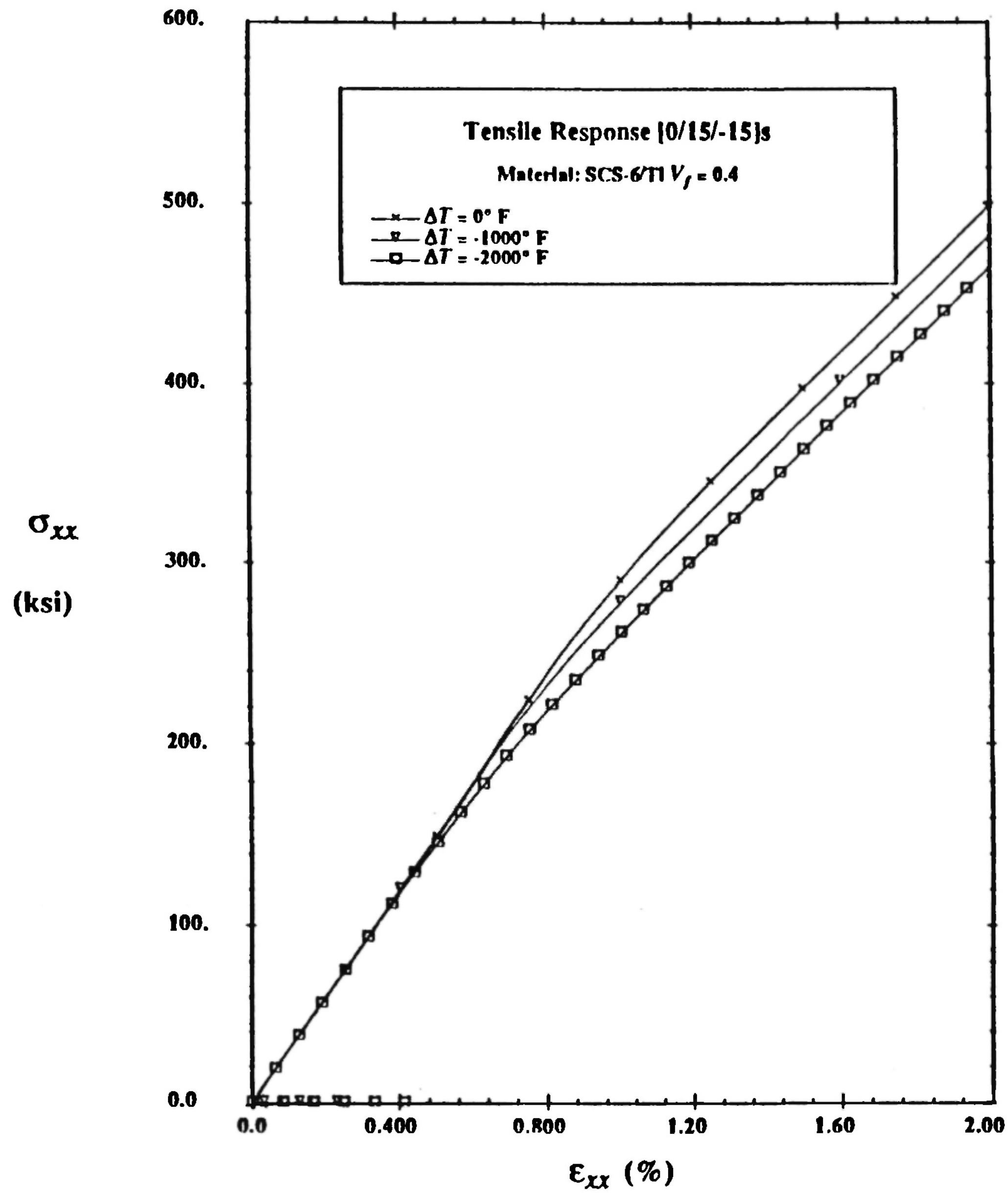

Fig. 8: Mechanical σ_{xx} vs ε_{xx} = $[0/\pm15]_s$

laminate for two thermal states is shown in Fig. 12. Comparison of these results with those of Fig. 10 for the $[0/\pm15]_s$ laminate indicates the expected lower yield stress and greater ductility for the $[0/\pm45]_s$ laminate. Otherwise the responses are quite similar with the thermal stresses again shifting the response but not otherwise altering its size of shape. The results in Fig. 12 do show clearly the different linear segments of the stress-strain response corresponding to "yielding" of the different layers of the laminate.

The final laminate to be considered is the $[0/\pm45/90]_s$. The axial response (Fig. 13) and the shear response (Fig. 14) for three thermal-mechanical load conditions exhibit the same type of response that was exhibited by the other laminates. The proportional limit

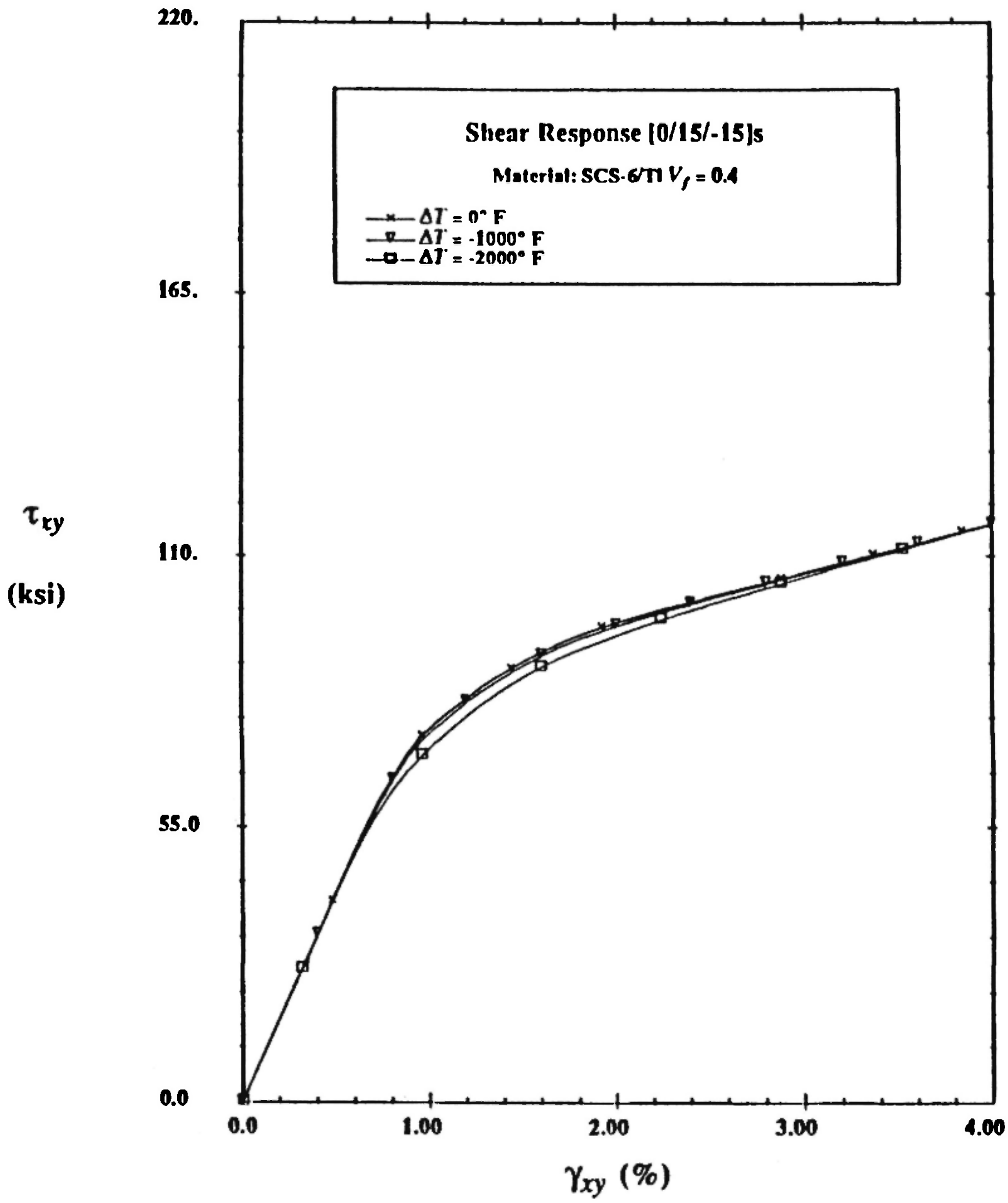

Fig. 9: Mechanical τ_{xy} vs γ_{xy} = $[0/\pm15]_s$

for axial response decreases with increasing temperature change, but the shear response is affected only minimally by residual stresses.

CONCLUSIONS

The method of cells has been shown to be very versatile for predicting the thermo-mechanical response of laminated composites. It has been shown that thermal stresses do not have a major influence on either the axial or shear response of laminates. The primary effect of thermal stresses in laminates is a relatively small shift in the stress-strain response. It has also been shown that the effects of thermal stresses is more significant in unidirectional lamina than in laminates.

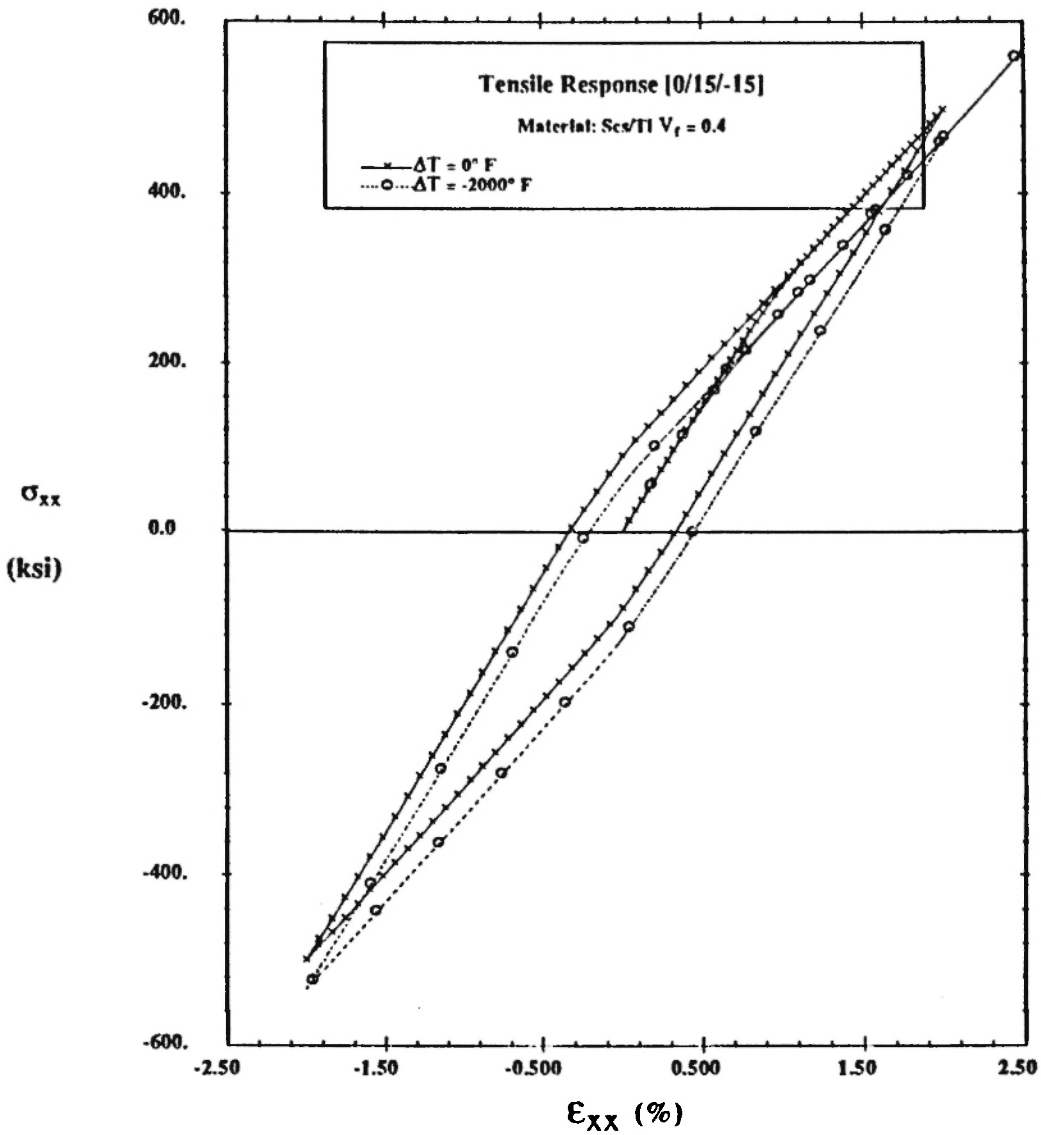

Fig. 10: Cyclic mechanical σ_{xx} vs ε_{xx} = $[0/\pm15]_s$

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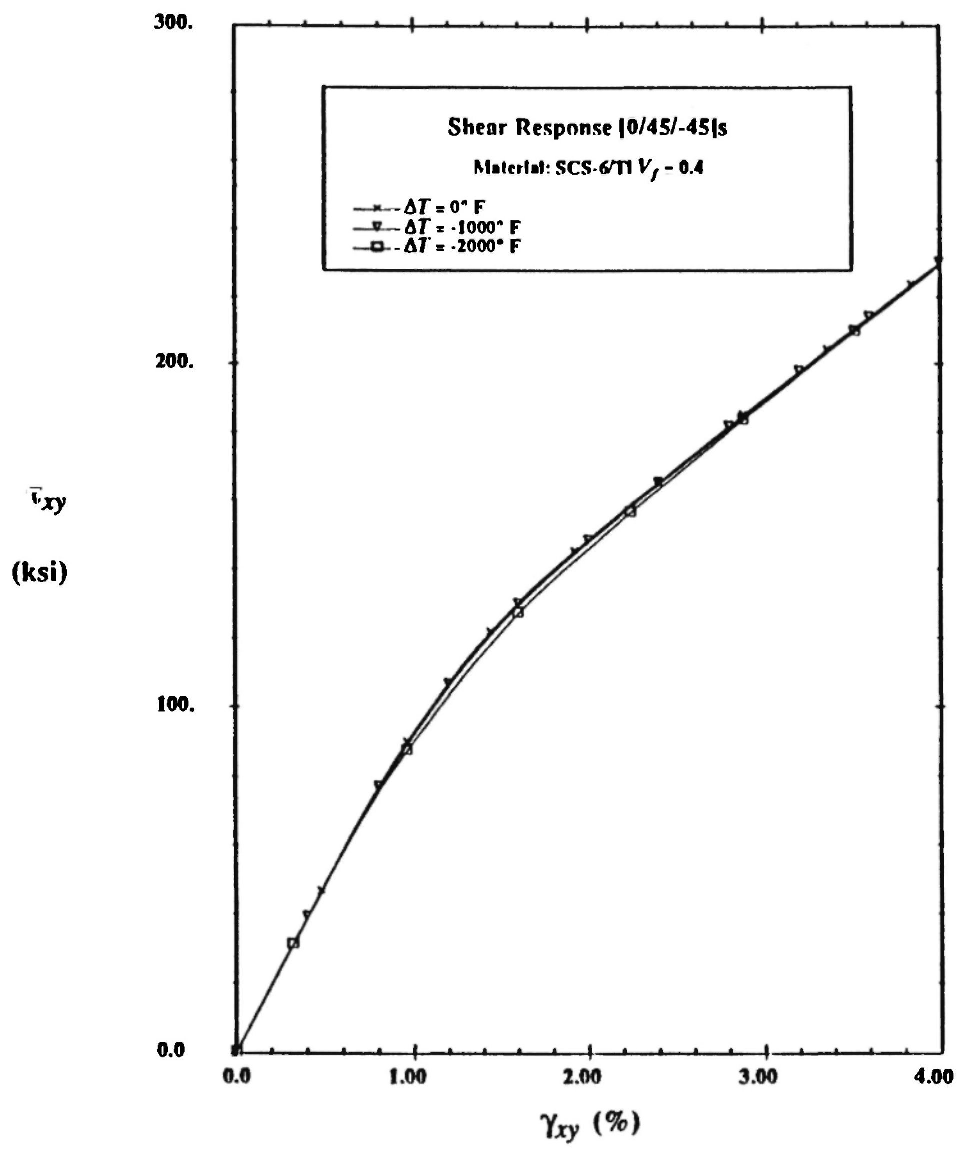

Fig. 11: Mechanical τ_{xy} vs γ_{xy} = $[0/\pm 45]_s$

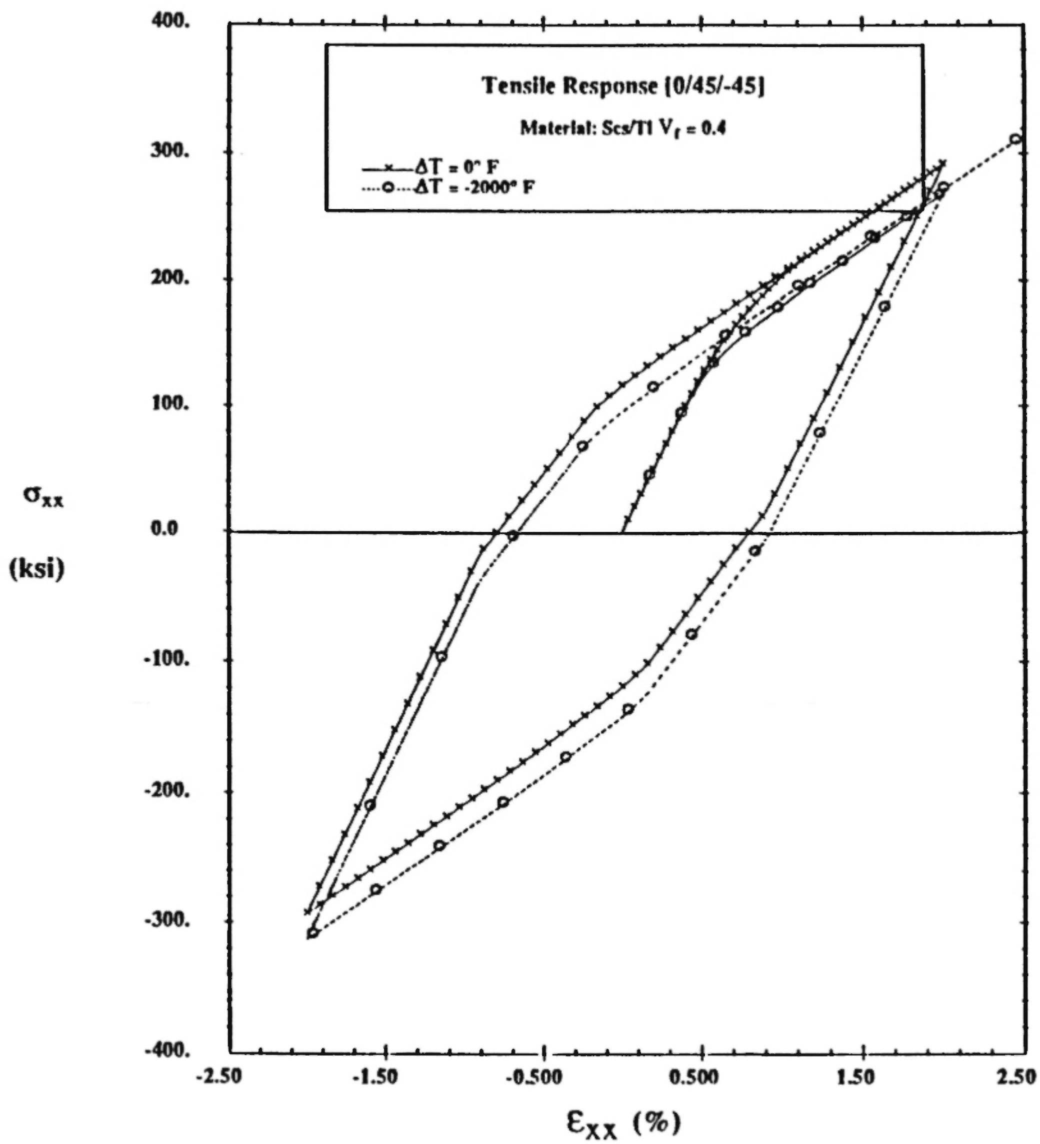

Fig. 12: Cyclic mechanical σ_{xx} vs ε_{xx} = $[0/\pm45]_s$

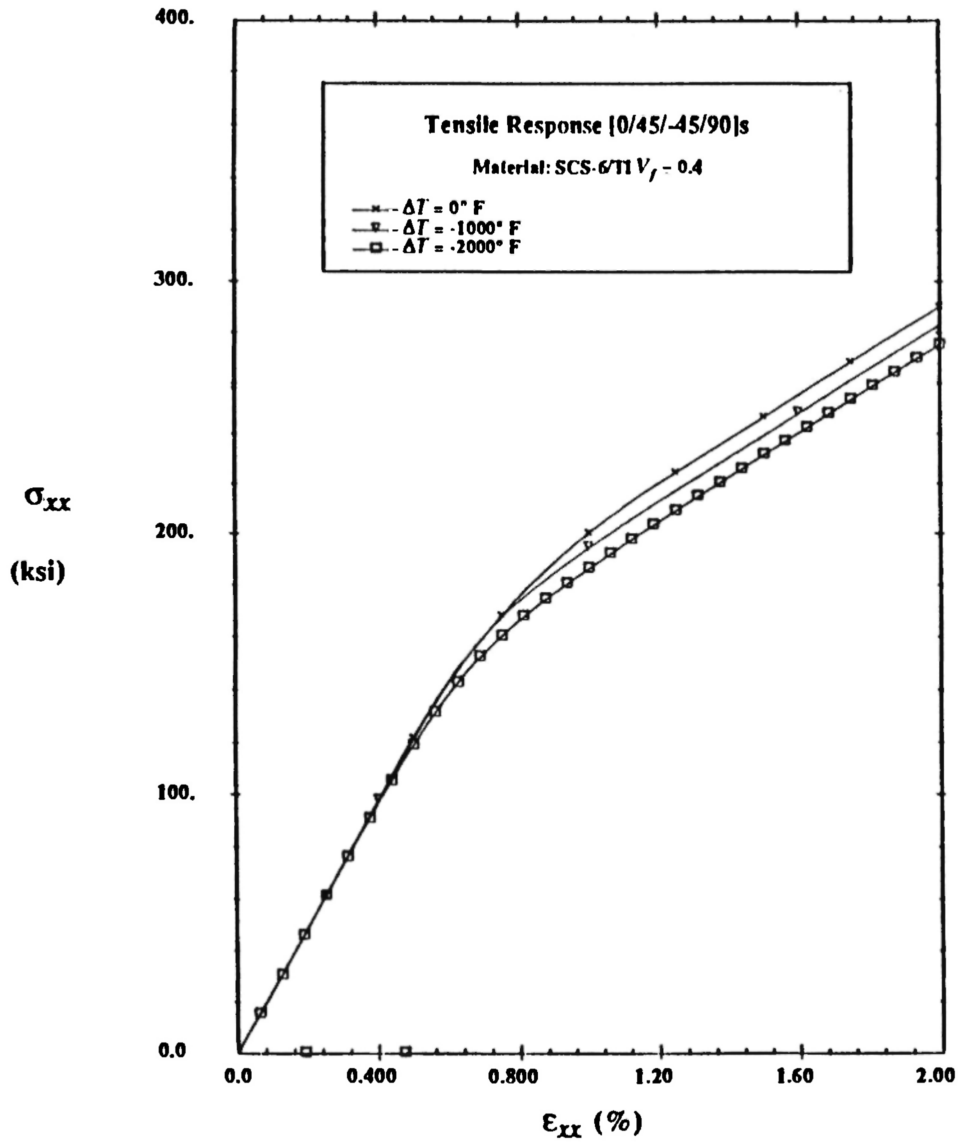

Fig. 13: Mechanical σ_{xx} vs ε_{xx} = $[0/\pm45/90]_s$

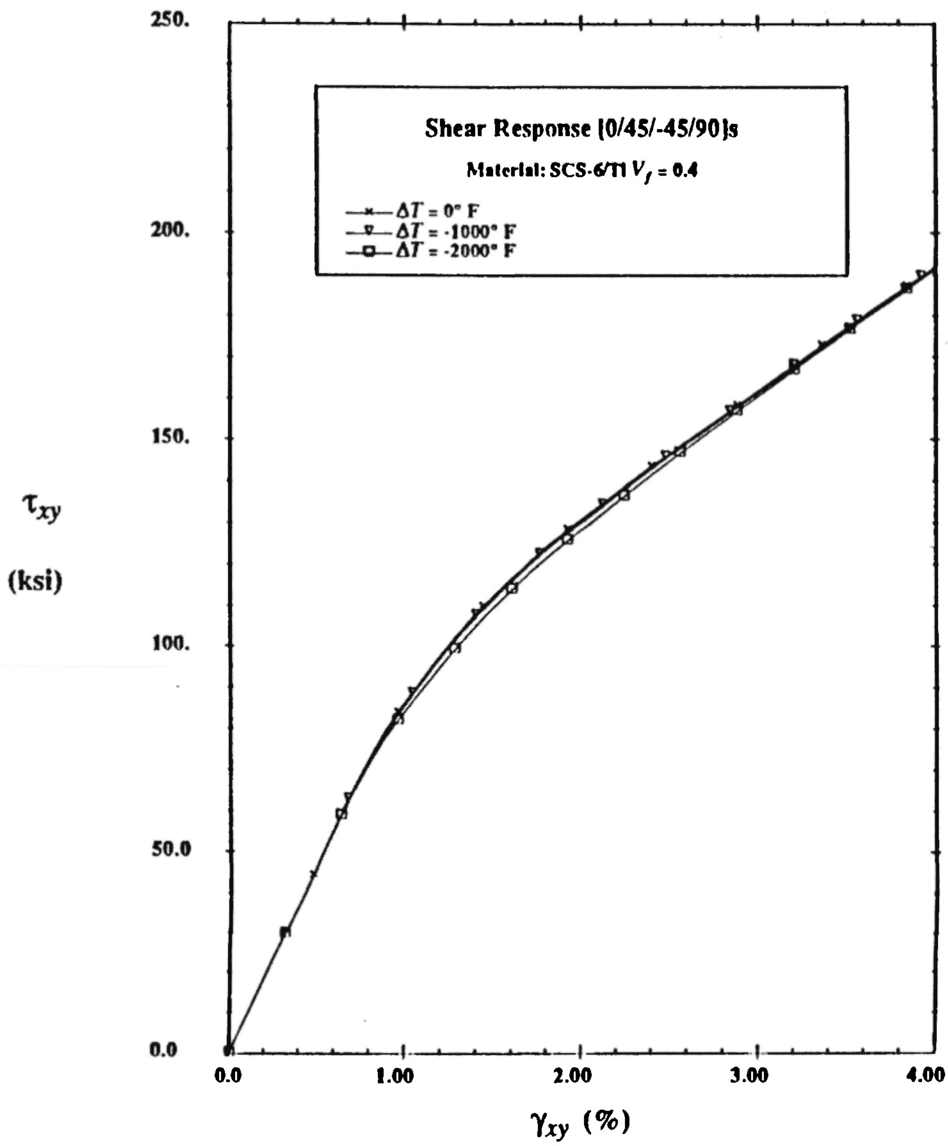

Fig. 14. Mechanical $\bar{\tau}_{xy}$ vs $\bar{\gamma}_{xy}$ - $[0/\pm45/90]_s$

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