

Linear Quadratic Nash Game of Stochastic Singular Time-Delay Systems with Multiple Decision Makers

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Abstract This paper discusses linear quadratic Nash game of stochastic singular time-delay systems governed by Itô's differential equation. Sufficient condition for the existence of Nash strategies is given by means of linear matrix inequality for the first time. Moreover, in order to demonstrate the usefulness of the proposed theory, stochastic H_2/H_∞ control with multiple decision makers is discussed as an immediate application.

Keywords singular system; time-delay system; Nash game; stochastic H_2/H_∞ control

1 Introduction

Singular systems, also known as descriptor systems, generalized state-space systems or implicit systems, are described by differential-algebraic equations. Singular systems have been extensively studied over the past decades due to the fact that they can describe a great many natural phenomena in physical systems such as microelectronic circuits, economics, demography and so on (see, e.g., [1–3]). A great number of fundamental notions and results in control

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and system theory based on state-space systems have been extended successfully to singular systems (see, e.g., [4–8] and the references therein).

In the past few decades, stochastic systems governed by Itô's differential equations have received a great deal of research attention [9, 10]. Although a variety of results for optimal control of linear stochastic systems have been reported, the dynamic games of such systems have received comparatively little attention. Moreover, to the best of our knowledge, stochastic Nash games for singular time-delay systems have not been fully investigated. Since delays appear in many practical plants, the design of such strategy is an important issue that remains open.

This paper is concerned with the problem of Nash game of stochastic singular time-delay systems with multiple decision makers. In terms of a set of linear matrix inequalities, we present a sufficient condition for the existence of both Nash strategies and the upper bound of the cost function for the first time. Moreover, in order to demonstrate the effectiveness of the proposed theory, stochastic H_2/H_∞ control with multiple decision makers is discussed as an immediate application.

The rest of this paper is organized as follows. In Section 2, some preliminaries are given. In Section 3, the main results are given, which generalizes the results of [11]. Section 4 discusses the stochastic H_2/H_∞ control with multiple decision makers by using the obtained results. Section 5 ends this paper with some comments.

Notations: Throughout this paper, unless otherwise specified, we will employ the following notations. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ is a complete probability space with Ω being a sample space, $\mathcal{F}$ being a σ -field, $\{\mathcal{F}_t\}_{t \geq 0}$ being a natural filtration and $\mathbb{P}$ being a probability measure. A^T is the transpose of a matrix A ; A^{-1} is inverse of a matrix A ; $A > 0$ means that A is positive definite; I_n denotes the $n \times n$ identity matrix; $\text{rank}(A)$ denotes the rank of A ; $\deg(\det(sI - A))$ denotes degree of determinant $sI - A$. $\mathbf{R}^n$ is the n -dimensional Euclidean space; $E[\cdot]$ denotes the expectation operator.

2 Preliminary Results

Consider the following time-invariant stochastic singular time-delay systems

$$\begin{cases} E dx(t) = [Ax(t) + A_1 x(t - \delta) + Bu(t)]dt + \sum_{p=1}^M A_p x(t) dw_p(t) \\ x(t) = \varphi(t), \quad t \in [-\delta, 0] \end{cases} \quad (1)$$

where $x(t) \in \mathbf{R}^n$ is the state vector; $u(t) \in \mathbf{R}^m$ is the control vector; E is a known singular matrix with $\text{rank}(E) = r \leq n$; $w_p(t) \in \mathbf{R}$ is a one-dimensional standard Wiener process defined in the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$. Without loss of generality, it is assumed that $w_r(t)$ and $w_s(t)$ are mutually independent for all $r, s = 1, 2, \dots, M$, and $E[w(t)w^T(t)] = I_M$, where $w(t) := [w_1(t), w_2(t), \dots, w_M(t)]^T$. Here, the scalar $\delta > 0$ is the time delay of the system. $\varphi(t)$ is a real-valued initial function. A, A_1, B, A_p are given real matrices of suitable sizes.

Let us consider the following stochastic linear quadratic control problem subject to (1):

$$\begin{aligned} \text{minimize } J(u(\cdot), x(0)) &= \mathbb{E} \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)] dt \\ Q = Q^T &\geq 0, \quad R = R^T > 0 \end{aligned} \quad (2)$$

The following definitions are similar with Definition 3.1 which was introduced in [12].

Definition 1 The system (1) is called mean-square stable if there exists a linear state feedback law $u(t) = Kx(t)$, $K \in \mathbf{R}^{m \times n}$, such that the resultant closed-loop system is asymptotically stable in mean-square, i.e., its trajectories satisfy $\lim_{t \rightarrow \infty} \mathbb{E} \|x(t)\|^2 = 0$, for any $\varphi(0)$.

Definition 2

- (I) The system (1) is said to be regular if $\det(sE - A)$ is not identically zero;
- (II) The system (1) is said to be impulse-free if $\deg(\det(sE - A)) = \text{rank}(E)$;
- (III) The system (1) is said to be mean-square admissible, if the system is regular, impulse-free and mean-square stable.

The following lemma plays a key technical role in this paper.

Lemma 1 Assume that for any $u(t)$, the closed-loop system is mean-square admissible. Suppose that there exists two real symmetric matrix $P > 0$ and $W > 0$, such that

$$\mathcal{V}(P) = \begin{bmatrix} \Xi & E^T P A_1 \\ A_1^T P E & -W \end{bmatrix} \leq 0 \quad (3)$$

where

$$\Xi = E^T P A + A^T P E + \sum_{p=1}^M A_p^T P A_p - E^T P B R^{-1} B^T P E + Q + W$$

Then the optimal feedback strategy for the stochastic linear quadratic control problem is

$$u^*(t) = K^* x(t) = -R^{-1} B^T P E x(t) \quad (4)$$

Moreover, the optimal value of cost function

$$J(u^*(\cdot), x(0)) \leq \mathbb{E}[x^T(0)E^T P E x(0)] + \mathbb{E} \left[\int_{-\delta}^0 \varphi^T(\tau) W \varphi(\tau) d\tau \right] \quad (5)$$

Proof Its proof can be demonstrated by using the square completion technique. First, define the following quadratic function

$$V(t) = x^T(t)E^T P E x(t) + \int_{t-\delta}^t x^T(\tau) W x(\tau) d\tau \quad (6)$$

where $P = P^T > 0$, $W = W^T > 0$.

Applying Itô's formula to the stochastic system (1), we have

$$\begin{aligned} d[V(t)] &= \left\{ x^T(t) \left[E^T P A + A^T P E + \sum_{p=1}^M A_p^T P A_p + W \right] x(t) + 2u^T(t)B^T P E x(t) \right. \\ &\quad \left. + 2x^T(t)E^T P A_1 x(t-h) - x^T(t-h)W x(t-h) \right\} dt + \{\dots\} dw_p(t) \end{aligned} \quad (7)$$

where $\{\dots\}$ does not affect the calculation results and can be omitted.

Integrating (7) from 0 to ∞ , taking expectations $E[\cdot]$ on both sides, one gets

$$\begin{aligned} & E[V(\infty)] - E[V(0)] \\ &= E \int_0^\infty \left\{ x^T(t) \left[E^T P A + A^T P E + \sum_{p=1}^M A_p^T P A_p + W \right] x(t) + 2u^T(t) B^T P E x(t) \right. \\ & \quad \left. + 2x^T(t) E^T P A_1 x(t-h) - x^T(t-h) W x(t-h) \right\} dt \end{aligned} \quad (8)$$

Under the assumption that the closed-loop system is mean-square admissible, we get $E[V(\infty)] = 0$. Thus, adding this to (2) and, using the square completion technique, we have

$$\begin{aligned} & J(u(\cdot), x(0)) - E[V(0)] \\ &= E \int_0^\infty \eta^T(t) \Upsilon(P) \eta(t) dt + E \int_0^\infty [u(t) + R^{-1} B^T P E x(t)]^T R [u(t) + R^{-1} B^T P E x(t)] dt \\ &\geq E \int_0^\infty \eta^T(t) \Upsilon(P) \eta(t) dt \\ &= J(u^*(\cdot), x(0)) \end{aligned} \quad (9)$$

where $\eta^T(t) = [x^T(t) \ x^T(t-\delta)]$.

Thus, the feedback control (4) is the optimal control. On the other hand,

$$J(u^*(\cdot), x(0)) - E[V(0)] = E \int_0^\infty \eta^T(t) \Upsilon(P) \eta(t) dt \leq 0 \quad (10)$$

Thus, if (3) holds, then the desired result is obtained.

3 Main Results

In this section, we will utilize the obtained results of stochastic linear quadratic optimal control to derive the results of stochastic Nash games.

3.1 Problem Formulation

Consider the following stochastic singular time-delay systems with N decision makers involving state-dependent noise

$$\begin{cases} Edx(t) = \left[Ax(t) + A_1 x(t-\delta) + \sum_{i=1}^N B_i u_i(t) \right] dt + \sum_{p=1}^M A_p x(t) dw_p(t) \\ x(t) = \varphi(t), \quad t \in [-\delta, 0] \end{cases} \quad (11)$$

where A, A_1 and A_p are $n \times n$ real matrices, $u_i(t) \in \mathbf{R}^{m_i}$, $i = 1, 2, \dots, N$, is the i -th control input, which represents the player i 's control strategy of this game, $B_i, i = 1, 2, \dots, N$, are $n \times m_i$ real matrices.

The cost function for each decision maker is defined by

$$J_i(u_1(\cdot), u_2(\cdot), \dots, u_N(\cdot), x(0)) = E \int_0^\infty [x^T(t) Q_i x(t) + u_i^T(t) R_i u_i(t)] dt \quad (12)$$

where $i = 1, 2, \dots, N$, $Q_i = Q_i^T \geq 0$, $R_i = R_i^T > 0$.

It should be noted that $u_j(\cdot)$, $j \neq i$ does not appear in the cost function. However, since they are included in the stochastic systems (11), they must have impacts on the cost functions (12).

Without loss of generality, the strategies in this paper are restricted as linear state feedback strategies, such as $u_i(t) = F_i x(t)$, where F_i is a constant matrix of suitable sizes.

Let $\mathbf{F}_N$ denote the set of all $(F_1 x(t), F_2 x(t), \dots, F_N x(t))$ such that the following closed-loop stochastic system

$$E dx(t) = \left(A + \sum_{i=1}^N B_i F_i \right) x(t) dt + A_1 x(t - \delta) dt + \sum_{p=1}^M A_p x(t) dw_p(t)$$

is mean-square admissible.

Our problem is to look for a strategy set $(u_1^*(\cdot), u_2^*(\cdot), \dots, u_N^*(\cdot))$ which is called the stochastic Nash equilibrium strategy set for the game, if for each $i = 1, 2, \dots, N$, the following inequality holds:

$$J_i(u_1^*(\cdot), \dots, u_N^*(\cdot), x(0)) \leq J_i(u_1^*(\cdot), \dots, u_{i-1}^*(\cdot), u_i(\cdot), u_{i+1}^*(\cdot), \dots, u_N^*(\cdot), x(0)) \quad (13)$$

for all $x(0)$ and $(F_1^* x(t), F_2^* x(t), \dots, F_N^* x(t))$ that satisfy $(F_1^* x(t), F_2^* x(t), \dots, F_N^* x(t)) \in \mathbf{F}_N$.

3.2 Solution to Stochastic Nash Games

The following theorem generalizes the existing results of [11].

Theorem 1 Assume that for all $u_i(t)$, $i = 1, 2, \dots, N$, the resultant closed-loop system is mean-square admissible. Suppose that N real symmetric matrices $P_i > 0$ and N real symmetric matrices $W_i > 0$ exist such that

$$\mathcal{R}_i(P_1, \dots, P_N) = \begin{bmatrix} \Xi_i & E^T P_i A_1 \\ A_1^T P_i E & -W_i \end{bmatrix} \leq 0 \quad (14)$$

where $i = 1, 2, \dots, N$,

$$\Xi_i = E^T P_i A_{-i} + A_{-i}^T P_i E + \sum_{p=1}^M A_p^T P_i A_p - E^T P_i B_i R_i^{-1} B_i^T P_i E + Q_i + W_i$$

$$A_{-i} = A - \sum_{j=1, j \neq i}^N B_j R_j^{-1} B_j^T P_j E$$

Define the strategy set $(F_1^* x(t), F_2^* x(t), \dots, F_N^* x(t))$ by

$$u_i^*(t) = F_i^* x(t) = -R_i^{-1} B_i^T P_i E x(t), \quad i = 1, 2, \dots, N \quad (15)$$

Then, $(F_1^* x(t), F_2^* x(t), \dots, F_N^* x(t)) \in \mathbf{F}_N$, and this strategy set denotes the stochastic Nash equilibrium. Furthermore, the optimal value of cost function

$$J_i(F_1^* x(t), F_2^* x(t), \dots, F_N^* x(t), x(0)) \leq E[x^T(0) E^T P_i E x(0)] + E \left[\int_{-\delta}^0 \varphi^T(\tau) W_i \varphi(\tau) d\tau \right] \quad (16)$$

Proof Now, let us consider the following problem in which the cost functional (17) is minimal at $u_i(t) = u_i^*(t)$.

$$\phi(u_i(\cdot), x(0)) = E \int_0^\infty [x^T(t) Q_i x(t) + u_i^T(t) R_i u_i(t)] dt \quad (17)$$

where $x(t)$ follows from

$$\begin{cases} E dx(t) = [A_{-i}x(t) + A_1x(t - \delta) + B_i u_i(t)]dt + \sum_{p=1}^M A_p x(t) dw_p(t) \\ x(t) = \varphi(t), \quad t \in [-\delta, 0] \end{cases} \quad (18)$$

Note that the function ϕ coincides with function $J(u(\cdot), x(0))$ in Lemma 1. Applying Lemma 1 to this optimization problem as

$$A_{-i} \Rightarrow A, \quad B_i \Rightarrow B, \quad Q_i \Rightarrow Q, \quad R_i \Rightarrow R$$

yields the fact that the function ϕ is minimal at

$$u_i^*(t) = F_i^* x(t) = -R_i^{-1} B_i^T P_i E x(t) \quad (19)$$

Moreover, the optimal value of cost function is less than or equal to

$$E[x^T(0)E^T P_i E x(0)] + E \left[\int_{-\delta}^0 \varphi^T(\tau) W_i \varphi(\tau) d\tau \right]$$

This completes the proof.

Remark 1 Note that when $\text{rank}(E) = r = n$, i.e., $E = I$, the inequality (14) is a normal matrix inequalities. This type of matrix inequalities was proposed in [11]. In this section, it is extended to the stochastic singular time-delay system case and it has more universality than the stochastic delay system.

Remark 2 Nash strategy $F_i x(t)$ of (15) can be obtained by solving the matrix inequalities (14). It should be noted that the matrix inequalities (14) can be assessed by applying the Newton's iterative method, which was proposed in [11].

4 Application to Stochastic H_2/H_∞ Control

Over the last decade, stochastic control problems governed by Itô's differential equation have attracted considerable research interest. Recently, stochastic linear quadratic and H_∞ control problems with state- and control-dependent noise have been investigated (see, e.g., [13, 14]). They have received much attention and have been widely used in various fields. In particular, the stochastic H_2/H_∞ control with state dependent noise and state, control and disturbance-dependent noise have been addressed (see, e.g., [10, 15, 16]), but up to present, stochastic H_2/H_∞ control with multiple decision makers have not been reported, and the design of such strategy is an issue that remains to be considered.

Now, we apply the above proposed theory to solve some problems related to stochastic H_2/H_∞ control with multiple decision makers.

Consider the following stochastic controlled system with state-dependent noise, which in-

volve N -decision makers

$$\begin{cases} \text{Ed}x(t) = \left[Ax(t) + A_1x(t - \delta) + Bv(t) + \sum_{i=1}^N B_i u_i(t) \right] dt + A_p x(t) dw(t) \\ z_i(t) = \begin{bmatrix} C_i x(t) \\ D_i u_i(t) \end{bmatrix}, \quad z(t) = \begin{bmatrix} Cx(t) \\ D_1 u_1(t) \\ \vdots \\ D_N u_N(t) \end{bmatrix} \\ x(t) = \varphi(t), \quad t \in [-\delta, 0] \end{cases} \quad (20)$$

where $D_i^T D_i = I_{m_i}$, $C = [C_1^T \ C_2^T \ \dots \ C_N^T]^T$, $x(t) \in \mathbf{R}^n$, $z(t) \in \mathbf{R}^m$, $v(t) \in \mathbf{R}^l$, $u_i(t) \in \mathbf{R}^{m_i}$ stand for the system state, controlled output, exogenous disturbance signal and i -th control input, respectively. All coefficient matrices are assumed to be real constant.

Given a disturbance attenuation level $\gamma > 0$, define performance functions

$$J_0(u_1, u_2, \dots, u_N, v) = \mathbb{E} \int_0^\infty [\gamma^2 v^T(t) v(t) - z^T(t) z(t)] dt \quad (21)$$

and

$$J_i(u_1, u_2, \dots, u_N, v) = \mathbb{E} \int_0^\infty z_i^T(t) z_i(t) dt \quad (22)$$

The infinite-time horizon stochastic H_2/H_∞ control with multiple decision makers of system (20) can be stated as follows.

Definition 3 For any given disturbance attenuation level $\gamma > 0$, find if possible strategies $u_i^*(t) \in \mathbf{R}^{m_i}$, $i = 1, 2, \dots, N$ such that:

(i) $u_i^*(t)$ makes system (20) mean-square admissible, i.e., when $v(t) = 0$ and $u_i(t) = u_i^*(t)$, the closed-loop system is regular, impulse-free and mean-square stable.

(ii)

$$\|L_{u_i^*}\|^2 = \sup_{\substack{v \neq 0, x(0) = \varphi(t) \equiv 0, \\ t \in [-\delta, 0]}} \frac{\mathbb{E} \int_0^\infty [\|Cx(t)\|^2 + \sum_{i=1}^N \|u_i^*(t)\|^2] dt}{\mathbb{E} \int_0^\infty [\|v(t)\|^2] dt} < \gamma^2 \quad (23)$$

(iii) When the worst case disturbance $v^*(t) \in \mathbf{R}^l$, if it exists, is applied to (20), $u_i^*(t)$ minimizes the output energy

$$J_i(u_1, u_2, \dots, u_N, v^*) = \mathbb{E} \int_0^\infty \|z_i(t)\|^2 dt = \mathbb{E} \int_0^\infty (\|C_i x(t)\|^2 + \|u_i(t)\|^2) dt \quad (24)$$

If the above $(u_1^*, u_2^*, \dots, u_N^*, v^*)$ exist, we say that the infinite-time horizon stochastic H_2/H_∞ control with multiple decision makers is solvable. Obviously, $(u_1^*, u_2^*, \dots, u_N^*, v^*)$ are the Nash equilibria of the two functions (21) and (22), which satisfy

$$J_0(u_1^*, u_2^*, \dots, u_N^*, v^*) \leq J_0(u_1^*, u_2^*, \dots, u_N^*, v), \quad \forall v \in \mathbf{R}^l \quad (25)$$

$$J_i(u_1^*, u_2^*, \dots, u_N^*, v^*) \leq J_i(u_1^*, u_2^*, \dots, u_{i-1}^*, u_i, u_{i+1}^*, \dots, u_N^*, v^*), \quad \forall u_i \in \mathbf{R}^{m_i} \quad (26)$$

According to Theorem 1 discussed in Section 3, a solution to the stochastic H_2/H_∞ control can be obtained straightly.

Theorem 2 Assume that for all $u_i(t), i = 1, 2, \dots, N$, the resultant closed-loop system is mean-square admissible. Suppose that $N + 1$ real symmetric matrices $(P_0, P_1, \dots, P_N)$ with $P_i > 0$, and $N + 1$ real symmetric matrices $(W_0, W_1, \dots, W_N)$ with $W_i > 0$ exist such that

$$\Upsilon_0(P_0, P_1, \dots, P_N) = \begin{bmatrix} \Xi_0 & E^T P_0 A_1 \\ A_1^T P_0 E & -W_0 \end{bmatrix} \leq 0 \quad (27)$$

$$\Upsilon_i(P_0, P_1, \dots, P_N) = \begin{bmatrix} \Xi_i & E^T P_i A_1 \\ A_1^T P_i E & -W_i \end{bmatrix} \leq 0 \quad (28)$$

where $i = 1, 2, \dots, N$,

$$\begin{aligned} \Xi_0 &= E^T P_0 A_{-F} + A_{-F}^T P_0 E + A_p^T P_0 A_p + W_0 - C^T C - \sum_{j=1}^N K_j^T K_j - \gamma^{-2} E^T P_0 B B^T P_0 E \\ \Xi_i &= E^T P_i A_{-i} + A_{-i}^T P_i E + A_p^T P_i A_p + C_i^T C_i + E^T W_i E - E^T P_i B_i B_i^T P_i E \\ A_{-F} &= A + \sum_{i=1}^N B_i K_i, A_{-i} = A + B F + \sum_{j=1, j \neq i}^N B_j K_j \\ F &= -\gamma^{-2} B^T P_0 E, K_i = -B_i^T P_i E \end{aligned}$$

If system (20) is mean-square admissible, then the set $(u_1^*, u_2^*, \dots, u_N^*)$ with

$$u_i^*(t) = K_i x(t) = -B_i^T P_i E x(t), \quad i = 1, 2, \dots, N \quad (29)$$

denotes the infinite-time horizon stochastic H_2/H_∞ control. Moreover, the worst case disturbance

$$v^*(t) = F x(t) = -\gamma^{-2} B^T P_0 E x(t) \quad (30)$$

5 Conclusions

In the present paper, we have dealt with the Nash game for stochastic singular time-delay systems with multiple decision makers in infinite-time horizon. In terms of a set of linear matrix inequalities, sufficient condition for the existence of Nash strategies is given for the first time. Moreover, the infinite-time horizon stochastic H_2/H_∞ control with multiple decision makers are treated by using these obtained results.

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