Journal of Systems Science and Information
Aug., 2015, Vol. 3, No. 4, pp. 321–333
DOI: 10.1515/jssi-2015-0321

Detecting the Structural Breaks in GARCH Models Based on Bayesian Method: The Case of China Share Index Rate of Return

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Abstract This paper investigates the detection for structural breaks in GARCH models based on Bayesian method. The authors firstly introduce the background and significance of this problem, then present the current situation and recent developments in this field. Because the rates of return have heavy tails, the authors present GARCH models. In this paper, the authors innovatively suppose that the error term follows standard student t distribution with degree of freedom v instead of standard normal distribution. The authors give the specific description of estimation using Bayesian method, including a single structural break situation and multiple structural breaks situation when the number of breaks is unknown. In an application, the authors empirically research the volatility of stock market in China. The authors estimate GARCH models with structural breaks for the Shanghai A-share index and Shenzhen A-share index rate of return over the period of January 4, 2000–September 30, 2011. The authors explain the breaks together with the nearby big political and economic events. Empirical results show that the detecting method used in this paper is feasible.

Keywords volatility; GARCH model; structural break; Bayesian method; China share index rate of return

1 Introduction

Volatility index has important applications in the financial market, including risk management, portfolio management and the pricing of derivative securities. Volatility modeling has

Received October 10, 2014, accepted November 18, 2014
Corresponding author: Liming WANG
Supported by the National Statistical Research Fund of China (2011LZ035), the Natural Science Fund of Shandong Province (2014AL006), the Statistical Research Key Project in Shandong Province and the Graduate Innovation Fund of Shanghai University of Finance and Economics (CXJJ-2014-445)

become an important tool in many areas of both economics and finance. Due predominantly to the prevalence of volatility clustering in asset return, the generalized autoregressive conditional heteroscedastic (GARCH) model, GARCH(1,1) model in particular, has become the major framework through which to model and forecast volatility.

As an emerging financial market in the world, it is not a long time since China built stock market, due to the imperfect marketing mechanism, incomplete legal system and institutions and investors' immature mentality, stock market in China would be affected by the external factors to become fluctuating. The external factors include some big events, especially political events and economic events. It has great importance to study stock market in China deeply. As a basic variable describing the global character of stock market, the share index rate of return contains much information about the market, especially the volatility of stock price. We introduce the detection of structural breaks of volatility in China share index rate of return in order to systematically study the structural change of fluctuation in China stock market, analyze the character of change points that has been estimated, further investigate other problems caused by structural breaks. It has great significance in better understanding the fluctuation law of China stock market, guiding reasonable investment, promoting the healthy and steady development of stock market, and effectively playing barometer role of stock market in economy.

Financial data generally span a long time period. The variance of rate of return in financial time series vary with time, which means it is time-varying and is well known to be subject to structural breaks. Thus we should make inference of structural breaks in GARCH models. Failure to take into account the structural breaks in financial data results in many undesirable consequences. For example, [1] found that the persistence in variance of stock-return data may be overstated if deterministic structural shifts are ignored in the model. Furthermore, [2, 3] pointed out structural breaks also have implications about the existence of higher order unconditional moments, such as kurtosis or the tail index, in financial time series. Also, [4] found that forecasting based on an estimated model with structural breaks being ignored will be unreliable. [1, 5] had proved that there exists structural breaks in financial rate of return and volatility.

[6] firstly proposed a cumulative sum of squares (CUSUM) algorithm to estimate the multiple structural breaks. [7] greatly improved Brown's CUSUM algorithm, they called the improved algorithm as iterative cumulative sums of squares (ICSS). [8] proposed a Bayesian approach for models with multiple change points. Methods for the computation of Bayesian factors are also developed. [9] proposed a Bayesian algorithm to estimate the structural breaks in GARCH/ARCH models, in which the error term follows the standard normal distribution. When the number of structural breaks is unknown, the iterative Bayesian algorithm can also estimate the whole structural breaks, she used USD/CAD data to get a desirable outcome. In China, [10] put forward a measurement model of change points based on Bayesian theory, and applied the model to Shanghai and Shenzhen stock index time series. [11] investigated the policy impact factors on the volatility of Shanghai and Shenzhen stock market in China.

This paper focuses on estimation of structural breaks for GARCH model based on Bayesian method and empirical analysis of change points in China stock market. Our paper is organized as follows. We firstly introduce the background and research significance, then present the current situation and recent developments in this field, including the research contents and methods. Moreover, we illustrate the method and framework of this paper. Next we present GARCH model in this paper and detect the change points based on Bayesian method. As we all know, the rates of return have heavy tails. So we innovatively suppose that the error term follows standard student t distribution with degree of freedom υ instead of standard normal distribution. Then we give the specific description of estimation using Bayesian method, including a single structural break situation and multiple structural breaks situation when the number of breaks is unknown. In an application, we estimate a GARCH model with structural breaks for the Shanghai A-share index and Shenzhen A-share index rate of return, which are both from January 4, 2000 to September 30, 2011. We explain the breaks together with the nearby big political and economic events.

2 Methodology

In this section, we will detect the structural breaks in time series by Monte Carlo simulation. We discuss two cases respectively, a single structural break and multiple structural breaks.

2.1 A Single Structural Break

2.1.1 GARCH(1,1) Model with t Distribution

GARCH(1,1) model is widely used in analysis of financial time series. Because the rates of return have heavy tails, in our GARCH(1,1) model we suppose that the error term follows standard student t distribution with degree of freedom υ instead of standard normal distribution. This is more reasonable. Assume that we observe $y = (y_1, y_2, \cdots, y_T)$, $\{y_t : t = 1, 2, \cdots, T\}$ follows GARCH(1,1) model with a single change point, the error term $\varepsilon_t \sim \text{student}(0, 1, \upsilon)$, i.e.

$$y_t = \sigma_t \varepsilon_t, \quad t = 1, 2, \cdots, T \tag{1}$$

$$\varepsilon_t \sim \text{student}(0, 1, \upsilon) \tag{2}$$

$$\sigma_t = \begin{cases} \omega_1 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2, & 1 \le t \le k \\ \omega_2 + \alpha_2 y_{t-1}^2 + \beta_2 \sigma_{t-1}^2, & k < t \le T \end{cases} \tag{3}$$

where $\text{student}(0, 1, \upsilon)$ is standard student t distribution, the degree of freedom is $\upsilon(\upsilon > 2)$, σ_t^2 is conditional variance and is subject to structural instability.

For the conditional volatility to be strictly positive, constants must be placed on the parameters: e.g. $\omega_i > 0, \alpha_i, \beta_i \ge 0, i = 1, 2$. For covariance stationarity, the GARCH parameters must satisfy $0 < \alpha_i + \beta_i < 1, i = 1, 2$.

2.1.2 Conditional Posterior and Gibbs Sampling

Let $\theta = (\omega_1, \alpha_1, \beta_1, \omega_2, \alpha_2, \beta_2, \upsilon, k)$ be the parameter vector of the model, given a sample of T observations $\{y_t : t = 1, 2, \cdots, T\}$, let $y = (y_1, y_2, \cdots, y_T)$, the posterior density of θ is

$$\varphi(\theta|y) \propto l(y|\theta)\varphi(\theta) \tag{4}$$

where $\varphi(\theta)$ is the prior density of θ, and $l(y|\theta)$ is the likelihood function.

Given the change point k, the likelihood function is

$$l(y|\theta) = \prod_{t=1}^{k} \frac{\Gamma(\frac{\upsilon+1}{2})}{\Gamma(\frac{\upsilon}{2})\sqrt{(\upsilon-2)\pi}} \frac{1}{\sigma_t} \left[1 + \frac{y_t^2}{(\upsilon-2)\sigma_t^2}\right]^{-\frac{\upsilon+1}{2}} \cdot \prod_{t=k+1}^{T} \frac{\Gamma(\frac{\upsilon+1}{2})}{\Gamma(\frac{\upsilon}{2})\sqrt{(\upsilon-2)\pi}} \frac{1}{\sigma_t} \left[1 + \frac{y_t^2}{(\upsilon-2)\sigma_t^2}\right]^{-\frac{\upsilon+1}{2}} \tag{5}$$

where the likelihood depends on the parameter θ through σ_t^2.

Firstly, we need to choose priors for the model parameters. Here, let us assume each of the model parameters follows a uniform distribution, which assigns equal probability to each possible value. This is called uninformative prior and is very useful in practice since we may not know much about the model parameter before performing estimation.

$$f(\omega_i) \sim U(0,\infty), \quad f(\alpha_i) \sim U(0,1), \quad f(\beta_i) \sim U(0,1), \quad f(k) \sim U(1,T), \quad i=1,2$$

Given our model specification and the prior choice, it is very difficult to derive a closed form for the joint posterior distribution $\varphi(\theta|y)$ over the whole set of parameters. However, the Gibbs sampling procedure allows us to draw each parameter from the conditional posterior density given other parameters fixed, and after many iterations, joint distribution of the parameters will still converge to the true joint distribution.

Given the initial value $\theta^{(0)} = (\omega_1^{(0)}, \alpha_1^{(0)}, \beta_1^{(0)}, \omega_2^{(0)}, \alpha_2^{(0)}, \beta_2^{(0)}, \upsilon^{(0)}, k^{(0)})$, Gibbs sampling procedure is as follows:

1) Draw $\omega_1^{(1)}$ from $\varphi(\omega_1|y, \alpha_1^{(0)}, \beta_1^{(0)}, \omega_2^{(0)}, \alpha_2^{(0)}, \beta_2^{(0)}, \upsilon^{(0)}, k^{(0)})$;
2) Draw $\alpha_1^{(1)}$ from $\varphi(\alpha_1|y, \omega_1^{(1)}, \beta_1^{(0)}, \omega_2^{(0)}, \alpha_2^{(0)}, \beta_2^{(0)}, \upsilon^{(0)}, k^{(0)})$;
3) Draw $\beta_1^{(1)}$ from $\varphi(\beta_1|y, \omega_1^{(1)}, \alpha_1^{(1)}, \omega_2^{(0)}, \alpha_2^{(0)}, \beta_2^{(0)}, \upsilon^{(0)}, k^{(0)})$;
4) Draw $\omega_2^{(1)}$ from $\varphi(\omega_2|y, \omega_1^{(1)}, \alpha_1^{(1)}, \beta_1^{(1)}, \alpha_2^{(0)}, \beta_2^{(0)}, \upsilon^{(0)}, k^{(0)})$;
5) Draw $\alpha_2^{(1)}$ from $\varphi(\alpha_2|y, \omega_1^{(1)}, \alpha_1^{(1)}, \beta_1^{(1)}, \omega_2^{(1)}, \beta_2^{(0)}, \upsilon^{(0)}, k^{(0)})$;
6) Draw $\beta_2^{(1)}$ from $\varphi(\beta_2|y, \omega_1^{(1)}, \alpha_1^{(1)}, \beta_1^{(1)}, \omega_2^{(1)}, \alpha_2^{(1)}, \upsilon^{(0)}, k^{(0)})$;
7) Draw $\upsilon^{(1)}$ from $\varphi(\upsilon|y, \omega_1^{(1)}, \alpha_1^{(1)}, \beta_1^{(1)}, \omega_2^{(1)}, \alpha_2^{(1)}, \beta_2^{(1)}, k^{(0)})$;
8) Draw $k^{(1)}$ from $\varphi(k|y, \omega_1^{(1)}, \alpha_1^{(1)}, \beta_1^{(1)}, \omega_2^{(1)}, \alpha_2^{(1)}, \beta_2^{(1)}, \upsilon^{(1)})$;

$\vdots$

By cycling repeatedly through draws of each parameter conditional on the remaining ones, we obtain a Markov chain $\{\theta^{(m)} : m = 1, 2, \cdots, M\}$, which has equilibrium distribution $\varphi(\theta|y)$ under quite mild conditions.

The conditional posterior density $\varphi(\omega_1|y, \alpha_1^{(0)}, \beta_1^{(0)}, \omega_2^{(0)}, \alpha_2^{(0)}, \beta_2^{(0)}, \upsilon^{(0)}, k^{(0)})$ has a kernel given by

$$\kappa(\omega_1|y, \theta_{-\omega_1}) = \left[\frac{\Gamma(\frac{\upsilon+1}{2})}{\Gamma(\frac{\upsilon}{2})\sqrt{(\upsilon-2)\pi}}\right]^T \prod_{t=1}^{k} \frac{1}{\sigma_t} \left[1 + \frac{y_t^2}{(\upsilon-2)\sigma_t^2}\right]^{-\frac{\upsilon+1}{2}} \cdot \prod_{t=k+1}^{T} \frac{1}{\sigma_t} \left[1 + \frac{y_t^2}{(\upsilon-2)\sigma_t^2}\right]^{-\frac{\upsilon+1}{2}} \times \lambda \exp(-\lambda(\upsilon-\delta)) I_{\{\upsilon>\delta\}} \tag{6}$$

where $\theta_{-\omega_1}$ denotes the remaining parameters except ω_1.

The kernels of the conditional posterior densities $\varphi(\omega_2|y, \theta_{-\omega_2})$, $\varphi(\alpha_i|y, \theta_{-\alpha_i})$ and $\varphi(\beta_i|y, \theta_{-\beta_i})$, $i = 1, 2$ bear the same expression, but subject to the stationary restrictions.

The change point k is a discrete variable, and it takes value from 2 to $T-1$. Since we assume that the prior of k is uniform distribution on $[0, T-1]$ (discrete version), then there is equal probability assigned to each possible value of k, $p(k=t)=\frac{1}{T-2}$. The conditional posterior density for the break point k is given by

$$p(k=t|y,\theta_{-\tau})=\frac{\prod_{\tau=1}^{t}\frac{1}{\sigma_\tau}\left[1+\frac{y_\tau^2}{(\upsilon-2)\sigma_\tau^2}\right]^{-\frac{\upsilon+1}{2}}\prod_{\tau=t+1}^{T}\frac{1}{\sigma_\tau}\left[1+\frac{y_\tau^2}{(\upsilon-2)\sigma_\tau^2}\right]^{-\frac{\upsilon+1}{2}}}{\sum_{s=2}^{T-1}\prod_{\tau=1}^{s}\frac{1}{\sigma_\tau}\left[1+\frac{y_\tau^2}{(\upsilon-2)\sigma_\tau^2}\right]^{-\frac{\upsilon+1}{2}}\prod_{\tau=s+1}^{T}\frac{1}{\sigma_\tau}\left[1+\frac{y_\tau^2}{(\upsilon-2)\sigma_\tau^2}\right]^{-\frac{\upsilon+1}{2}}} \quad (7)$$

It is hard to make random draws from the above densities, due to their complex functional forms. Existing methodologies regarding making random draws from densities of any functional form generally fall into two categories, either using acceptance/rejection sampling algorithm or applying nonparametric approximations to the inverse of the cumulative distribution function (CDF). The first method involves proposing a candidate density which we can easily draw from, and using some acceptance and rejection criteria to determine whether to keep the current draw. The second approach features Griddy-Gibbs sampler proposed in [12], where the conditional posterior kernel is evaluated at prespecified grid points, and then numerical integration method can be used to approximate the CDF which can be directly used to make random draws. For example, let $F(\cdot)$ be the CDF and y be a draw from $U(0,1)$, then $x=F^{-1}(y)$ is a random draw from the CDF $F(\cdot)$. Not like the acceptance/rejection sampler, the Griddy-Gibbs sampler does not involve any candidate density, and thus no tuning parameter is required.

Given the value of change point k, the Griddy-Gibbs sampling method for $\omega_1,\alpha_1,\beta_1,\omega_2,\alpha_2,\beta_2$ follows the work of [13] because Griddy-Gibbs sampler doesn't need any candidate density. However, for the sampling of change point k, we adopt acceptance/rejection sampling algorithm.

The Griddy-Gibbs sampling algorithm for GARCH model:

Let us take the sampling method for ω_1 as an example. The conditional posterior density $\varphi(\omega_1|y,\alpha_1,\beta_1,\omega_2,\alpha_2,\beta_2,\upsilon,k)$ has a kernel given by (4). And the conditional posterior densities for $\alpha_1,\beta_1,\omega_2,\alpha_2,\beta_2$ are of the same form, thus the sampling methods are the same.

Step 1 Set the initial value$\theta^{(0)}=(\omega_1^{(0)},\alpha_1^{(0)},\beta_1^{(0)},\omega_2^{(0)},\alpha_2^{(0)},\beta_2^{(0)},\upsilon^{(0)},k^{(0)})$.

Step 2 (a) Set grid $(w_1,w_2,\cdots,w_G)$ for ω_1. For α and β, the grids have to be confined on $[0,1]$ to meet the condition for the stationarity of a GARCH process. According to [13], we choose the number of grids $G=33$;

(b) Compute the values $G_\kappa=(\kappa_1,\kappa_2,\cdots,\kappa_G)$, where $\kappa_j=\kappa(w_j|y,\theta_{-\omega_1}^{(0)})$, for $j=1,2,\cdots,G$;

(c) Compute the values $G_\Phi=(0,\Phi_2,\cdots,\Phi_G)$ by a deterministic integration rule, where

$$\Phi_j=\int_{w_1}^{w_j}\kappa(\omega_1|y,\theta_{-\omega_1}^{(0)})\mathrm{d}\omega_1,\quad j=2,3,\cdots,G$$

Also compute the normalized pdf value $G_\varphi=\frac{G_\kappa}{\Phi_G}$. Calculate $E(\omega_1|y,\theta_{-\omega_1}^{(0)})$ and $\mathrm{Var}(\omega_1|y,\theta_{-\omega_1}^{(0)})$ by the same integration rule as above, and store the values;

(d) Draw u from a uniform distribution on $[0,\Phi_G]$. Invert Φ_j by numerical interpolation to get a draw $\omega_1^{(1)}$.

For large data set, the value of $\kappa(w_1|y,\theta_{-\omega_1}^{(0)})$ might become a very large number or close to zero, exceeding the capability of the computer. [9] used the following modified procedure (b′) and (c′) to handle such a case instead of using the procedure (b) and (c):

(b′) Define $LG_\kappa = (\log\kappa_1, \log\kappa_2, \cdots, \log\kappa_G,)$, and $LG^* = \max(\log\kappa_1, \log\kappa_2, \cdots, \log\kappa_G)$;

(c′) Calculate $G_\Phi = (0, \Phi_2, \cdots, \Phi_G)$ according to the following rule:

$$\begin{aligned}\Phi_j &= \exp(-LG^*)\int_{w_1}^{w_j}\exp(\log(\kappa(\omega_1|y,\theta^{(0)}_{-\omega_1})))\mathrm{d}\omega_1\\ &= \int_{w_1}^{w_j}\exp(\log(\kappa(\omega_1|y,\theta^{(0)}_{-\omega_1})) - LG^*)\mathrm{d}\omega_1, \quad j = 2, 3, \cdots, G\end{aligned}$$

The added constant $\exp(-LG^*)$ doesn't change the shape of the kernel. The redefined kernel is uniformly bounded, i.e. $0 < \exp(\log(\kappa(\omega_1|y,\theta^{(0)}_{-\omega_1} - LG^*) \leq 1$.

Step 3 Sample $\alpha_1^{(1)}, \beta_1^{(1)}, \omega_2^{(1)}, \alpha_2^{(1)}, \beta_2^{(1)}$ follow the same procedure in Step 2.

Step 4 By cycling repeatedly through draws of each parameter conditional on the remaining ones, we obtain a Markov chain $\{\theta^{(m)} : m = 1, 2, \cdots, M\}$, which has equilibrium distribution $\varphi(\theta|y)$ under quite mild conditions.

2.1.3 The Sampling Method of Change Point k

The change point k is a discrete variable. For the sampling of change point k, we adopt acceptance/rejection sampling algorithm. Let $p_t = P(k = t)$. Further suppose that a discrete random variable has probability mass function $q_t = \frac{1}{T-2}, t = 2, 3, \cdots, T-1$, compute $c = \max_t\{p_t/q_t\}$. The sampling procedure:

Step 1 Generate random variables $U_1, U_2 \sim U(0, 1)$ and set $z = [U_1(T-2)] + 1$. If $z = 1$ or $z = T$, repeat Step 1;

Step 2 If $U_2 < \frac{p_z}{cq_t}$, then set $k = z$; otherwise return to Step 1.

2.2 Multiple Structural Breaks (the Number of Change Points Is Unknown)

In the presence of multiple breaks, an important issue arises — How to determine the number of breaks. Many works that addresses multiple breaks problem assume fixed number of breaks, then compare models with different number of breaks by model selection tools such as Bayesian factor method. To overcome this difficulty, we suggest to use the "estimating one at a time" method proposed by [14] instead of estimate the multiple break points simultaneously. For a general regression model, Bai shows that the each sequentially estimated change point is consistent for one of the true breaks, and the convergence rate is T, the same as in the simultaneous estimation of all break dates together. [9] proved that sequentially estimation is still robust to misspecification of the number of breaks. We state the methodology formally as following:

Suppose the process $\{y_t\}_{t=1}^T$ follows a true data generating process with structural breaks at time $(k_1, k_2, \cdots, k_M)$, and the fraction representation is $(\lambda_1, \lambda_2, \cdots, \lambda_M)$, where $\lambda_m = \frac{k_m}{T}, m = 1, 2, \cdots, M$, but we don't know the number of M.

Step 1 Assume there is only one structural break, and estimate k_1 using the full sample. $\frac{k_1}{T}$ may not necessarily be consistent for λ_1, but it is consistent for one of the $(\lambda_1, \lambda_2, \cdots, \lambda_M)$, say, λ_m, depending on the break magnitude between the adjacent regimes;

Step 2 Divide the sample into two subsamples, the pre-k_1 one and the post-k_1 one. Repeat Step 1 for each of them;

Step 3 Repeat the procedure until the Bayesian algorithm fails to detect any more breaks.

[9] proved the consistency of the sequential estimates holds for the GARCH/ARCH family models. Again, our Bayesian algorithm provides a natural way and powerful tool to make inference for GARCH/ARCH family models with structural breaks.

3 Empirical Analysis (the Case of China Share Index Rate of Return)

3.1 Data Source and Processing

This paper uses data from the daily closing prices of Shanghai Stock Exchange Renminbi ordinary stocks (referred to as the Shanghai A-share index, abbr. SH-A) and Shenzhen Stock Exchange Renminbi ordinary stocks (referred to as the Shenzhen A-share index, abbr. SZ-A) over the period of January 4, 2000–September 30, 2011, both 2, 840 data. Data are from the Wind Information Financial Terminal. In this section, we detect the structural breaks in GARCH (1,1) model based on Bayesian method. We estimate its parameters and the dates of change points.

Let daily closing price is p_t, $\{r_t\}$ is daily return time series, where

$$r_t = [\ln(p_t) - \ln(p_{t-1})] \times 100 = \ln\left(\frac{p_t}{p_{t-1}}\right) \times 100$$

3.2 Statistical Analysis of Daily Return Time Series Data

Table 1 below gives descriptive statistics of daily return time series data.

Table 1 Descriptive statistics of daily return time series data

	Mean	Variance	Median	Extreme deviation	Interquantile range	Skewness	Kurtosis
SH-A	0.0177	2.8180	0.0750	18.6606	1.6371	−0.0993	3.9771
SZ-A	0.0303	3.2787	0.1388	18.1657	1.8192	−0.3711	3.2027

From Table 1, we can find the variance of the Shanghai and Shenzhen A-share index rate of return are both large. The China share index rate of return is volatile, which can also be seen from the extreme deviation which are both smaller than zero, which indicates that the two stock markets have more days whose rates of return are lower than the mean. Judging from the kurtosis values, both over three, higher kurtosis means that the conditional variances for the two stock markets have more extreme values, which imply strong volatility. Combined skewness with kurtosis values, the two A-share index rates of return show a heavy tail, pinnacle and non-normality, which show student t distribution is more appropriate to describe the error term.

3.3 Parameter Estimation and Detection of Change Points

To implement the proposed Bayesian method in practice, the number of change points is unknown, we use the estimating one at a time method proposed by [14] to detect the structure breaks and determine its number. Based on Griddy-Gibbs sampling and acceptance/rejection sampling algorithm introduced above, we get the following change points, where Griddy-Gibbs sampling iterates 2000 times. We abandon the first 1000 iterative values and only use the latter 1000 iterative values in parameter estimation. We found that the Shanghai and Shenzhen A-share index rate of return both have 11 change points. Concrete results are shown in Table 2, Table 3 below.

Table 2 Parameter estimation of GARCH(1,1) model of the SH-A

	Full sample (1–2839)	Subsample 1 (1–881)	Subsample 2 (1–774)	Subsample 3 (42–774)	Subsample 4 (42–364)	Subsample 5 (365–774)
$\hat{\omega}_1$	0.3291 (0.0634)	0.2793 (0.0403)	0.6205 (0.1011)	0.1620 (0.0360)	0.3148 (0.0791)	0.5081 (0.0768)
$\hat{\alpha}_1$	0.1993 (0.0401)	0.0496 (0.0187)	0.1814 (0.0539)	0.1970 (0.0350)	0.1714 (0.0540)	0.2243 (0.0391)
$\hat{\beta}_1$	0.7214 (0.0579)	0.7741 (0.0676)	0.7342 (0.0735)	0.6393 (0.0360)	0.6077 (0.0666)	0.6367 (0.0365)
$\hat{\omega}_2$	0.1214 (0.0275)	0.2808 (0.0711)	0.1047 (0.0177)	0.2666 (0.0554)	0.1875 (0.0424)	0.3101 (0.0470)
$\hat{\alpha}_2$	0.0709 (0.0103)	0.1057 (0.0282)	0.1986 (0.0203)	0.1838 (0.0396)	0.1405 (0.0499)	0.1007 (0.0390)
$\hat{\beta}_2$	0.8986 (0.3014)	0.8286 (0.0864)	0.7281 (0.0158)	0.6549 (0.0431)	0.5449 (0.0543)	0.6089 (0.0398)
$\hat{\upsilon}$	45 (20.3608)	28 (2.8604)	31 (4.5743)	31 (4.4832)	37 (9.6092)	41 (14.4423)
$\hat{k}$	9/11/2003 (881 days)	4/4/2003 (774 days)	3/16/2000 (41 days)	7/18/2001 (364 days)	5/31/2000 (90 days)	6/25/2002 (585 days)
Unconditional variance before change point	4.1503	1.5844	7.3534	0.9898	1.4251	3.6542
Unconditional variance after change point	3.9866	4.2768	1.4297	1.6529	0.5959	1.0679

	Subsample 6 (586–774)	Subsample 7 (882–2839)	Subsample 8 (882–2625)	Subsample 9 (1660–2625)	Subsample 10 (1660–2207)
$\hat{\omega}_1$	0.2543 (0.0558)	0.2086 (0.0378)	0.2851 (0.0414)	0.4967 (0.0872)	0.4748 (0.0826)
$\hat{\alpha}_1$	0.0722 (0.0545)	0.0648 (0.0186)	0.0566 (0.0197)	0.0792 (0.0133)	0.1460 (0.0414)
$\hat{\beta}_1$	0.4946 (0.0665)	0.8265 (0.1963)	0.7646 (0.0968)	0.8394 (0.0619)	0.7198 (0.0491)
$\hat{\omega}_2$	0.3444 (0.0632)	0.1898 (0.0403)	0.1528 (0.0356)	0.3297 (0.0474)	0.4096 (0.0634)
$\hat{\alpha}_2$	0.0821 (0.0445)	0.0576 (0.0182)	0.0830 (0.0162)	0.0522 (0.0155)	0.0434 (0.0242)

Table 2 (continued) Parameter estimation of GARCH(1,1) model of the SH-A

	Subsample 6	Subsample 7	Subsample 8	Subsample 9	Subsample 10
$\hat{\beta}_2$	0.6735	0.8442	0.8965	0.8104	0.7659
	(0.0465)	(0.2298)	(0.2411)	(0.1370)	(0.0337)
$\hat{\upsilon}$	32	36	42	40	38
	(5.4016)	(6.3104)	(14.9409)	(13.4836)	(11.0135)
$\hat{k}$	11/1/2002	11/19/2010	11/29/2006	3/4/2009	9/1/2009
	(672 days)	(2625 days)	(1659 days)	(2207 days)	(2332 days)
Unconditional variance before change point	0.5869	1.9176	1.5949	6.1018	3.5393
Unconditional variance after change point	1.4092	1.9325	7.4183	2.3992	2.1478

Remark 1 For the parameter estimation of GARCH(1,1) model in the paper, we adopt $\sum_{n=1}^{N} E(\omega_1|\alpha_1^{(n)}, \beta_1^{(n)}, \omega_2^{(n)}, \alpha_2^{(n)}, \beta_2^{(n)}, \upsilon^{(n)}, k^{(n)}, y)/N$ instead of $\sum_{n=1}^{N} \omega_1^{(n)}/N$. [13] pointed out that the variance of the former is smaller than that of the latter in spite that they are both root-N consistent.

Remark 2 The values in brackets are standard errors, similarly hereinafter.

Table 3 Parameter estimation of GARCH(1,1) model of the SZ-A

	Full sample	Subsample 1	Subsample 2	Subsample 3	Subsample 4	Subsample 5
	(1–2839)	(19–2839)	(19–962)	(19–726)	(42–726)	(367–726)
$\hat{\omega}_1$	0.5926	0.1631	0.1450	0.5480	0.1128	0.5386
	(0.0967)	(0.0320)	(0.0682)	(0.0917)	(0.0253)	(0.0829)
$\hat{\alpha}_1$	0.2005	0.2306	0.1746	0.2126	0.2051	0.2281
	(0.0538)	(0.0291)	(0.0305)	(0.0567)	(0.0285)	(0.0417)
$\hat{\beta}_1$	0.7420	0.7003	0.7603	0.7214	0.6825	0.6445
	(0.0919)	(0.0326)	(0.0407)	(0.0907)	(0.0265)	(0.0435)
$\hat{\omega}_2$	0.0484	0.0924	0.1958	0.0761	0.3069	0.2594
	(0.0104)	(0.0141)	(0.0476)	(0.0170)	(0.0708)	(0.0597)
$\hat{\alpha}_2$	0.1007	0.0892	0.1441	0.2137	0.1948	0.1837
	(0.0061)	(0.0066)	(0.0363)	(0.0208)	(0.0442)	(0.0447)
$\hat{\beta}_2$	0.8729	0.9062	0.6883	0.7457	0.7012	0.6534
	(0.1056)	(0.2556)	(0.0424)	(0.0152)	(0.0494)	(0.0441)
$\hat{\upsilon}$	34	32	31	35	35	28
	(7.1087)	(5.0192)	(4.9981)	(7.7684)	(7.4550)	(4.3573)
$\hat{k}$	2/14/2000	1/12/2004	1/17/2003	3/16/2000	7/20/2001	6/25/2002
	(18 days)	(962 days)	(726 days)	(41 days)	(366 days)	(585 days)

Table 3 (continued) Parameter estimation of GARCH(1,1) model of the SZ-A

	Full sample	Subsample 1	Subsample 2	Subsample 3	Subsample 4	Subsample 5
Unconditional variance before change point	10.2978	2.3574	2.2280	8.3039	1.0034	4.2259
Unconditional variance after change point	1.8338	20.0235	1.1682	1.8786	2.9523	1.5922
	Subsample 6 (586–726)	Subsample 7 (963–2839)	Subsample 8 (963–1643)	Subsample 9 (1644–2839)	Subsample 10 (2208–2839)	
$\hat{\omega}_1$	0.2395 (0.0547)	0.3445 (0.0565)	0.3791 (0.0590)	0.5151 (0.0834)	0.5280 (0.0825)	
$\hat{\alpha}_1$	0.0788 (0.0556)	0.0732 (0.0258)	0.0533 (0.0223)	0.1213 (0.0180)	0.1157 (0.0319)	
$\hat{\beta}_1$	0.5252 (0.06675)	0.7628 (0.1087)	0.7560 (0.0291)	0.7997 (0.0235)	0.7387 (0.0389)	
$\hat{\omega}_2$	0.3545 (0.0831)	0.1767 (0.0405)	0.2746 (0.0804)	0.2834 (0.0401)	0.3120 (0.0565)	
$\hat{\alpha}_2$	0.1078 (0.0505)	0.0829 (0.0180)	0.0948 (0.0573)	0.0694 (0.0141)	0.0576 (0.0262)	
$\hat{\beta}_2$	0.7586 (0.0953)	0.8865 (0.2805)	0.6224 (0.0696)	0.8185 (0.1322)	0.7897 (0.1031)	
$\hat{\upsilon}$	35 (8.7242)	33 (6.5655)	29 (4.4237)	30 (5.7355)	32 (6.0270)	
$\hat{k}$	11/4/2002 (6732 days)	11/7/2006 (1643 days)	8/15/2006 (1588 days)	3/4/2009 (2207 days)	1/21/2011 (2669 days)	
Unconditional variance before change point	0.6048	2.1012	1.9873	6.5259	3.6271	
Unconditional variance after change point	2.6530	5.7685	0.9710	2.5288	2.0435	

3.4 Analysis of Change Points

Using Bayesian method introduced above, we get 11 change points in Shanghai A-share index rate of return, the same number of change points in Shenzhen A-share index rate of return. Now we summarize them together in order to compare and explain.

Table 4 The change points of SH-A and SZ-A

Change points of SH-A	Change points of SZ-A
	2/14/2000
3/16/2000	3/16/2000
5/31/2000	
7/18/2001	7/20/2001
6/25/2002	6/25/2002
11/1/2002	11/4/2002
	1/17/2003
4/4/2003	
9/11/2003	
	1/12/2004
	8/15/2006
11/29/2006	11/7/2006
3/4/2009	3/4/2009
9/1/2009	
11/19/2009	
	1/21/2011

As can be seen from Table 4, on the same sample interval (from January 4, 2000 to September 30, 2011), there are 11 change points in Shanghai A-share index rate of return, the same number of change points in Shenzhen A-share index rate of return, and the dates of structural breaks are relatively close, even some on the same days, which shows that they are highly correlated. As we all know, the changes of variance structure must have their reasons, which are closely linked to the dates of major political and economic events. Major political events refers to the major events at home and abroad affecting the stock price, as well as government policies, measures, statutes, and so on. The government's social-economic development plans, changes in economic policy, new decrees and regulations will affect the stock price. And important economic events include macro-economic policies, and the government's policies to the stock market. Therefore, we need to explore the causes of change points.

In an efficient market, changes of stock prices are influenced by different types of information. China is the biggest developing country in the world. As far as an emerging market is concerned, it is widely believed that the structural volatility of stock markets is caused by many factors, such as certain market imperfection, global equity market, policy factors and so on. In China, some domestic research has shown that it is especially influenced by policy factors. We collect and summarize the important political and economic events which cause the change points in stock market in China. Major events are selected. We focus on the China stock market instead of single SH-A or SZ-A, so when the change points are close in SH-A and SZ-A, we put them together to explain. The results are in Table 5.

In conclusion, corresponding with the events at the same point, there are generally changes in variance, which illustrates that the change points in China stock market are associated with some important events. These important political and economic events caused the fluctuations in stock market, resulted in the generation of change points. Empirical results show that Bayesian methods detecting the change points presented in this paper is feasible, and the results are satisfactory.

Table 5 Change points and corresponding political and economic events

SH-A	SZ-A	Important political and economic events
	2/14/2000	China securities regulatory commission reformed the way of stock offering on February 14, 2000.
3/16/2000	3/16/2000	Stock issue approval process was proclaimed in March, 2000.
7/18/2001	7/20/2001	The state council issued officially the temporary management methods of reducing the state-owned shares and raising the social security funds on July 12, 2001.
6/25/2002	6/25/2002	The state council decided to stop reducing the state-owned shares through domestic securities market on June 24, 2002.
11/1/2002	11/4/2002	China's qualified foreign institutional investors in securities investment management interim measure was promulgated in November, 2002.
4/4/2003	1/17/2003	The SARS broke out in China.
9/11/2003		Shanghai Stock Exchange enabled block trading system on August 20, 2003; The China central bank declared that the deposit reserve ratio would be increased by 1 percent in September, 2003.
	1/12/2004	The state council issued "several suggestions about advancing the reform and opening and the stable development of the capital market" in January, 2004.
	8/15/2006	The China central bank declared that the deposit reserve ratio would be increased by 0.5 percent on July 5, 2006.
11/29/2006	11/7/2006	State administration of foreign exchange declared that China's foreign exchange reserve exceeded $1 trillion mark on November 6, 2006.
3/4/2009	3/4/2009	In March, 2009, the government work report made by Primer Jiabao WEN reported that we should promote the reform of capital market, maintain the stability of stock market, develop and standardize the bond market, develop steadily the futures market.
9/1/2009		Growth enterprises markets are listed.
11/19/2009		The China banking regulatory commission issued the pilot administrative measures for commercial banks to make equity investment in insurance companies in November, 2009.
	1/21/2011	Authorities imposed further housing purchase restriction; The China central bank increased the deposit reserve ratio in January, 2011.

4 Conclusions

This paper develops an effective Bayesian method to estimate and make inference about structural breaks for the time-varying volatility GARCH models. It is feasible to detect the structural breaks in GARCH model through Bayesian method. The Bayesian algorithm we proposed needn't to assume the number of change points, this method is still feasible even with the wrong assumption of change-point number.

Because the rates of return have heavy tails, we present GARCH models, in this paper, we suppose that the error term follows standard student t distribution with degree of freedom υ instead of standard normal distribution in order to capture the characteristics of heavy tail in financial time series data.

First, we introduce the detection of a single structural break in GARCH model, then introduce the detection of multiple structural breaks when the number of breaks is unknown. We use the estimating one at a time method proposed by [14].

In an application, taking China stock market as an example, we estimate a GARCH model with structural breaks for the Shanghai A-share index and Shenzhen A-share index rate of return, which are both from January 4, 2000 to September 30, 2011. We explain the breaks together with the nearby big political and economic events. Empirical results show that the method proposed in the paper is feasible in the detection of change points.

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