

DEA Cross-Efficiency Evaluation Method Based on Good Relationship

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Abstract Benevolent and aggressive cross-efficiency evaluation methods are the improvement of the cross-efficiency evaluation method. They merely maximize or minimize the efficiency of the composite unit constructed by evaluated decision-making units while maintaining the optimal efficiency of the decision-making unit under evaluation. The two methods completely ignore the self-evaluation efficiency of evaluated unit and the good relationship among decision-making units. To solve the above drawbacks, the authors consider the efficiency score of the decision-making unit as an interval number and propose a more reasonable interval number. On the basis of the interval efficiency, the authors provide the benevolent and aggressive DEA cross-efficiency evaluation models based on the good relationship among all decision-making units. Finally, a numerical example is provided to illustrate the proposed method.

Keywords DEA; the benevolent cross-efficiency; the aggressive cross-efficiency; interval efficiency; good relationship

1 Introduction

Data envelopment analysis (DEA) is a non-parametric method for evaluating the relative efficiency of homogeneous decision making units (DMUs) with multiple inputs and multiple outputs. Since Charnes et al.^[1] proposed CCR model, DEA method has been widely applied to various fields, for example, management science, decision analysis and technical evaluation. Traditional CCR model is a pure self-evaluation method. A DMU is permitted to select the most

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favorable weights to achieve its best efficiency but completely neglect other DMUs. However, the nature of self-evaluation leads to the optional weights on input and output too much flexibility. And the efficiency score of more than one DMU is frequently equivalent to 1 at the same time to result in lacking discrimination power for traditional CCR model as well. To overcome self-evaluation flaws in traditional DEA model, Sexton et al.^[2] first proposed the DEA cross-efficiency evaluation method to improve the discrimination power.

DEA cross-efficiency method is a methodology to extend the self-evaluation of DEA model into the peer-evaluation. The average of self-evaluation efficiency and peer-evaluation efficiency of each DMU is regarded as the overall cross-efficiency score of the DMU. The cross-efficiency method can rank all DMUs utilizing cross-efficiency values and remove the unrealistic weights in DEA model, which enhances the discrimination power among DMUs. Due to its discrimination power among DMUs, the cross-efficiency evaluation method has been applied to various fields. For example, Chen^[3] used cross-efficiency to evaluate the resource utilization efficiency for electricity distribution sector in Taiwan. Wu et al.^[4] utilized cross-efficiency to rank the candidates in a preference voting setting considering the competition factor. Wu et al.^[5] applied cross-efficiency to confirming the rankings of participating nations in Olympics, Yu et al.^[6] examined how to select the most appropriate information sharing for the supply chain participant. Falagario et al.^[7] used cross-efficiency to measure the different offers in public procurement tenders through the “Most Economically Advantageous Tender” (MEAT) criterion. Lim et al.^[8] utilized cross-efficiency to choose the portfolio in Korean stock market.

However, the traditional cross-efficiency evaluation still exist some drawbacks. As the original DEA model existing generally multiple optimal weights, the cross-efficiency based on the original DEA model cannot be unique. To solve the problem, Doyle and Green^[9] introduced the secondary goals based on traditional cross-efficiency model to eliminate the non-uniqueness and proposed aggressive and benevolent cross-efficiency models. The idea of aggressive cross-efficiency model is to minimize the average efficiency of other DMUs while keeping the optimal efficiency of DMU under evaluation. The benevolent model, on the contrary, is to maximize the average efficiency of other DMUs. Furthermore, many other scholars also studied and improved the drawbacks, such as Liang et al.^[10], Wu et al.^[11], Yang et al.^[12], Wu et al.^[4], Wang et al.^[13], Wu et al.^[14], Ramon et al.^[15].

Nevertheless, though the aggressive and benevolent cross-efficiency evaluation method can solve the multiple solutions problem of the cross-efficiency model to some extent, the formulations still need to be improved. In the peer-evaluation process, a special DMU only cares for maximizing its own efficiency value, but ignores efficiency values of other DMUs. Sometimes it is unfair and unreasonable to other DMU based on a special preference such as benevolent or aggressive method. To overcome the limit, Wang and Chin^[13] proposed a neutral DEA cross-efficiency evaluation model without choosing between the different models. Instead, each DMU can decide its weights space from its own point of view with no need for being aggressive or benevolent to the other DMUs. Furthermore, Yang et al.^[12] utilized the interval DEA cross-efficiency to improve the problem. They considered the efficiency values of the traditional aggressive and benevolent models as the lower and upper limit of the interval. Hence, there is no need to choose between benevolent and aggressive models. But the interval is not suitable

for this paper. We propose a more applicable interval in this paper.

Moreover, benevolent and aggressive methods view the evaluated DMUs as an entirety while completely ignoring the good relationship among DMUs. We define the good relationship for the degree of friendship that a DMU experiences when evaluated by other DMUs. Due to the different evaluation criterion of each DMU, sometimes evaluated DMUs experience different friendly degrees of the DMU under evaluation to them, although they have the same cross-efficiency values. The social exchange theory (Homans)^[16] suggests that individual expects to derive the return from others as much as they pay out in their social interactions, aiming to achieve the balance. Hence, we know others for their good relationship determines their good relationship to others. In addition, according to the human relations theory (Hart)^[17], we realize that a DMU not only focuses on its own efficiency values, but also cares about the good relationship of other DMUs to it. Hence, it is unreasonable to average the efficiency score of cross-efficiency only, which regards the evaluated DMUs as an entirety and ignores the good relationship among DMUs. In this paper, we present a cross-efficiency evaluation model based on good relationship to improve traditional aggressive and benevolent cross-efficiency methods. As a result, the efficiency value derived by proposed method avoids the extreme case that the evaluation bias to certain DMU excessively.

The rest of the paper is organized as follows. Section 2 introduces the aggressive and benevolent cross-efficiency models. Section 3 proposes the cross-efficiency model based on friendly relationship. A numerical example is provided in Section 4, and conclusions are presented in Section 5.

2 The Cross-Efficiency Evaluation Model

Assume we have n independent homogeneous DMUs, denoted by $DMU_j (j = 1, 2, \dots, n)$. Each DMU consumes m inputs $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})^T$ to produce s outputs $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})^T$. Then the optimal CCR efficiency score of $DMU_k (k = 1, 2, \dots, n)$ can be obtained by model (1):

$$\begin{aligned}
 E_{kk}^* = \max \quad & \sum_{r=1}^s u_{rk} y_{rk} \\
 \text{s.t.} \quad & \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0 \\
 & \sum_{i=1}^m v_{ik} x_{ik} = 1 \\
 & u_{rk} \geq 0, \quad v_{ik} \geq 0 \\
 & r = 1, 2, \dots, s, \quad i = 1, 2, \dots, m
 \end{aligned} \tag{1}$$

where u_{rk}, v_{ik} are weights attached to outputs and inputs of DMU_k . Then optimal weights and optimal efficiency can be expressed as $(u_{1k}^*, u_{2k}^*, \dots, u_{sk}^*, v_{1k}^*, v_{2k}^*, \dots, v_{mk}^*)$ and E_{kk}^* , respectively. Utilizing the optimal weight set of DMU_k , we can compute the cross-efficiency of DMU_j as follows:

$$E_{kj} = \frac{\sum_{r=1}^s u_{rk}^* y_{rj}}{\sum_{i=1}^m v_{ik}^* x_{ij}}, \quad j = 1, 2, \dots, n, \quad j \neq k \tag{2}$$

We can obtain n sets of optimal weights from model (1), each time for one different DMU. Then, each DMU is evaluated by n sets of optimal weights. A cross-efficiency matrix shown in Table 1 contains one CCR efficiency value and $(n - 1)$ cross-efficiency values for each DMU.

Table 1 The cross-efficiency matrix

DMU	1	2	3	...	n
1	E_{11}	E_{12}	E_{13}	...	E_{1n}
2	E_{21}	E_{22}	E_{23}	...	E_{2n}
3	E_{31}	E_{32}	E_{33}	...	E_{3n}
...	...	...	...	...	...
n	E_{n1}	E_{n2}	E_{n3}	...	E_{nn}
Averaged cross-efficiency	$\bar{E}_1$	$\bar{E}_2$	$\bar{E}_3$	...	$\bar{E}_n$

The values of the diagonal are CCR efficiency values in cross-efficiency matrix, and others are peer-evaluation efficiency values. Then, the cross-efficiency of DMU $_j$ is defined as the average of all E_{kj} :

$$\bar{E}_j = \frac{1}{n} \sum_{k=1}^n E_{kj}, \quad j = 1, 2, \dots, n \quad (3)$$

It is noticed that the optimal solutions in model (1) are non-unique. As a result, the cross-efficiency defined in model (2) is multiple. To resolve the multiple solutions problem, Doyle and Green^[9] presented aggressive and benevolent models to recognize the optimal weights based on the traditional cross-efficiency. The two models not only maximize the efficiency of a special DMU under evaluation, but also minimize or maximize the average efficiency of other DMUs. The benevolent formulation is shown as follows:

$$\begin{aligned}
\max \quad & \sum_{r=1}^s u_{rk} \left(\sum_{j=1, j \neq k}^n y_{rj} \right) \\
\text{s.t.} \quad & \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0 \\
& \sum_{i=1}^m v_{ik} \left(\sum_{j=1, j \neq k}^n x_{ij} \right) = 1 \\
& \sum_{r=1}^s u_{rk} y_{rk} - E_{kk}^* \sum_{i=1}^m v_{ik} x_{ik} = 0 \\
& u_{rk} \geq 0, \quad v_{ik} \geq 0 \\
& r = 1, 2, \dots, s, \quad i = 1, 2, \dots, m \\
& j = 1, 2, \dots, n, \quad j \neq k
\end{aligned} \quad (4)$$

Hence, the aggressive formulation can be represented as follows:

$$\begin{aligned}
\min \quad & \sum_{r=1}^s u_{rk} \left(\sum_{j=1, j \neq k}^n y_{rj} \right) \\
\text{s.t.} \quad & \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0 \\
& \sum_{i=1}^m v_{ik} \left(\sum_{j=1, j \neq k}^n x_{ij} \right) = 1 \\
& \sum_{r=1}^s u_{rk} y_{rk} - E_{kk}^* \sum_{i=1}^m v_{ik} x_{ik} = 0 \\
& u_{rk} \geq 0, \quad v_{ik} \geq 0 \\
& r = 1, 2, \dots, s, \quad i = 1, 2, \dots, m \\
& j = 1, 2, \dots, n, \quad j \neq k
\end{aligned} \tag{5}$$

Model (4) is the benevolent model which is to maximize the cross-efficiency of the composite unit composed by others evaluated DMUs while keeping the best self-evaluation efficiency of DMU under evaluation. To the contrary, model (5) interprets the aggressive model to minimize the cross-efficiency of the composite unit.

3 The Cross-Efficiency Evaluation Based on Good Relationship

The benevolent and aggressive cross-efficiency evaluation methods only maximize or minimize the cross-efficiency of composite unit composed by others evaluated DMUs while maintaining the self-evaluation efficiency of the DMU under evaluation. The two methods both view all evaluated DMUs without discrimination while completely ignoring self-evaluation efficiencies of evaluated DMUs and the good relationship among DMUs. However, each DMU has its own friendly evaluation criterion in reality. That is, although the efficiency values are equal for different evaluated DMUs, they experience different friendly degrees of the DMU under evaluation to them. The DMU not only focuses on its own efficiency value, but also cares about the good relationship of other DMUs to it. And others for their good relationship determine their degree of friendship to others. Traditional benevolent and aggressive evaluation methods neglect the problem. In this paper, we present a cross-efficiency evaluation model based on good relationship to improve the evaluation result.

3.1 Interval Efficiency

The efficiency value of a DMU in traditional CCR model is computed by maximum of ratio its weighted sum of outputs to weighted sum of inputs. It is unable to reflect the situation of the DMU all-round and objective on account of being the optimistic efficiency value derived by the method. As a result, many researchers began to study the interval efficiency. Entani et al.^[18] studied the efficiency values of DMUs from the point of pessimism and optimism. They utilized the pessimistic efficiency value and optimism efficiency value as lower limit and upper limit to compose efficiency interval, respectively. From another perspective, Wang and Luo^[19] introduced anti-ideal DMU to compute the fresh lower limit by linear programming model. Yang

et al.^[12] considered the efficiency values of the traditional aggressive and benevolent models as the lower and upper limit of the interval. However, as we consider the good relationship, the lower limits derived by above literatures are unfit in current paper. In this paper, we consider the minimum efficiency value of peer-evaluation as the lower limit which is calculated by model (6). Hence, assume that efficiency interval of DMU_j is defined as $E_j \in [\min E_j, \max E_j]$, we derive the $\min E_j$ and $\max E_j$ from the model (6) and model (1) respectively. We define the $\min E_j = \min(\theta_{1j}, \theta_{2j}, \dots, \theta_{j-1j}, \theta_{j+1j}, \dots, \theta_{nj})$. The smaller the lower limit is, the more reasonable, and the more suitable for this paper.

$$\begin{aligned}
 \theta_{kj} = \min \quad & \sum_{r=1}^s u_{rk} y_{rj} \\
 \text{s.t.} \quad & \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0 \\
 & \sum_{i=1}^m v_{ik} x_{ij} = 1 \\
 & \sum_{r=1}^s u_{rk} y_{rk} - E_{kk}^* \sum_{i=1}^m v_{ik} x_{ik} = 0 \\
 & u_{rk} \geq 0, \quad v_{ik} \geq 0 \\
 & r = 1, 2, \dots, s, \quad i = 1, 2, \dots, m \\
 & j = 1, 2, \dots, n, \quad j \neq k
 \end{aligned} \tag{6}$$

3.2 Good Relationship

The current paper proposes the concept of the good relationship among DMUs on basis of the interval efficiency. The good relationship is also called the satisfaction degree of evaluated DMUs. The good relationship of DMU under evaluation to evaluated DMUs is not only related to the peer-evaluation efficiency, but also self-evaluation efficiency. Thus, the good relationship of DMU_k to DMU_j is defined as:

$$R_{kj} = \frac{E_{kj} - \min E_j}{\max E_j - \min E_j}, \quad j = 1, 2, \dots, n, \quad j \neq k \tag{7}$$

Obviously, R_{kj} is between 0 and 1, namely $R_{kj} \in [0, 1]$. When $R_{kj} = 1$, that is, $E_{kj} = \max E_j$, DMU_k is the most friendly to DMU_j in peer-evaluation process. Otherwise, $R_{kj} = 0$, namely $E_{kj} = \min E_j$, DMU_k is the least friendly to DMU_j. Friendship is reciprocal known by the human relations theory. So it is logical to maximize or minimize the overall satisfaction.

3.3 Cross-Efficiency Evaluation Method Based on Good Relationship

In this paper, we introduce the good relationship based on aggressive and benevolent formulations proposed by Doyle and Green^[9]. That is, while maintaining the optimum efficiency of DMU under evaluation maximizes or minimizes the sum of satisfaction of other evaluated DMUs. The model based on the aggressive formulation of Doyle and Green^[9] is developed as

follows:

$$\begin{aligned}
& \max \quad \sum_{j=1}^n R_{kj} \\
& \text{s.t.} \quad \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0 \\
& \quad \sum_{r=1}^s u_{rk} y_{rk} - E_{kk}^* \sum_{i=1}^m v_{ik} x_{ik} = 0 \\
& \quad E_{kj} = \frac{\sum_{r=1}^s u_{rk} y_{rj}}{\sum_{i=1}^m v_{ik} x_{ij}} \\
& \quad R_{kj} = \frac{E_{kj} - \min E_j}{\max E_j - \min E_j} \\
& \quad 0 \leq R_{kj} \leq 1, \quad \forall j \neq k \\
& \quad u_{rk} \geq 0, \quad v_{ik} \geq 0 \\
& \quad r = 1, 2, \dots, s, \quad i = 1, 2, \dots, m \\
& \quad j = 1, 2, \dots, n, \quad j \neq k
\end{aligned} \tag{8}$$

Similarly, the model based on the benevolent formulation of Doyle and Green^[9] can be obtained as follows:

$$\begin{aligned}
& \min \quad \sum_{j=1}^n R_{kj} \\
& \text{s.t.} \quad \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0 \\
& \quad \sum_{r=1}^s u_{rk} y_{rk} - E_{kk}^* \sum_{i=1}^m v_{ik} x_{ik} = 0 \\
& \quad E_{kj} = \frac{\sum_{r=1}^s u_{rk} y_{rj}}{\sum_{i=1}^m v_{ik} x_{ij}} \\
& \quad R_{kj} = \frac{E_{kj} - \min E_j}{\max E_j - \min E_j} \\
& \quad 0 \leq R_{kj} \leq 1, \quad \forall j \neq k \\
& \quad u_{rk} \geq 0, \quad v_{ik} \geq 0 \\
& \quad r = 1, 2, \dots, s, \quad i = 1, 2, \dots, m \\
& \quad j = 1, 2, \dots, n, \quad j \neq k
\end{aligned} \tag{9}$$

By using Charnes and Cooper^[20] transformation, model (8) can be converted into the linear model (10) as follows:

$$\begin{aligned}
& \max \quad \sum_{j=1}^n \left[\sum_{r=1}^s u_{rk} y_{rj} - (\min E_j) \sum_{i=1}^m v_{ik} x_{ij} \right] \\
& \text{s.t.} \quad \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0
\end{aligned}$$

$$\begin{aligned}
& \sum_{r=1}^s u_{rk} y_{rk} - E_{kk}^* \sum_{i=1}^m v_{ik} x_{ik} = 0 \\
& \sum_{j=1}^n \left[\sum_{i=1}^m v_{ik} x_{ij} (\max E_j - \min E_j) \right] = 1 \\
& 0 \leq R_{kj} \leq 1, \quad \forall j \neq k \\
& u_{rk} \geq 0, \quad v_{ik} \geq 0 \\
& r = 1, 2, \dots, s, \quad i = 1, 2, \dots, m \\
& j = 1, 2, \dots, n, \quad j \neq k
\end{aligned} \tag{10}$$

Hence, model (9) can be converted into

$$\begin{aligned}
\min \quad & \sum_{j=1}^n \left[\sum_{r=1}^s u_{rk} y_{rj} - (\min E_j) \sum_{i=1}^m v_{ik} x_{ij} \right] \\
\text{s.t.} \quad & \sum_{r=1}^s u_{rk} y_{rj} - \sum_{i=1}^m v_{ik} x_{ij} \leq 0 \\
& \sum_{r=1}^s u_{rk} y_{rk} - E_{kk}^* \sum_{i=1}^m v_{ik} x_{ik} = 0 \\
& \sum_{j=1}^n \left[\sum_{i=1}^m v_{ik} x_{ij} (\max E_j - \min E_j) \right] = 1 \\
& 0 \leq R_{kj} \leq 1, \quad \forall j \neq k \\
& u_{rk} \geq 0, \quad v_{ik} \geq 0 \\
& r = 1, 2, \dots, s, \quad i = 1, 2, \dots, m \\
& j = 1, 2, \dots, n, \quad j \neq k
\end{aligned} \tag{11}$$

4 A Numerical Example

In this section, we provide a numerical example generated randomly by Matlab software to illustrate the comprehensive feasibility and superiority of the proposed method. We also analyze roundly the different results of benevolent and aggressive cross-efficiency models under new strategy and old strategy. There are $n = 10$ DMUs and each DMU has $m = 3$ inputs denoted as X_1, X_2 and X_3 , and $s = 3$ outputs denoted as Y_1, Y_2 and Y_3 . Table 2 shows the inputs and outputs data sets of ten DMUs.

Via model (1), we derive the CCR efficiency value of each DMU as the upper limit of interval efficiency and calculate the model (6) to obtain lower limit of interval efficiency. Then the interval efficiency of each DMU derived by the proposed method is shown in Table 3.

The cross-efficiency values of benevolent and aggressive cross-efficiency models under new strategy and old strategy as shown in Table 4 can be obtained by computing model (4), model (5), model (10) and model (11), respectively.

Table 2 The input and output data of each DMU

DMU	X_1	X_2	X_3	Y_1	Y_2	Y_3
DMU ₁	4	6	10	1	10	8
DMU ₂	3	8	8	3	1	3
DMU ₃	2	2	5	9	3	5
DMU ₄	4	4	3	9	5	8
DMU ₅	9	2	8	2	4	9
DMU ₆	1	1	2	7	7	1
DMU ₇	4	5	10	2	6	3
DMU ₈	1	6	6	6	5	5
DMU ₉	6	2	6	8	8	10
DMU ₁₀	4	6	7	7	1	1

Table 3 Interval efficiency of each DMU

DMU	Lower limit	Upper limit	Interval efficiency
DMU ₁	0.0238	0.8447	[0.0238, 0.8447]
DMU ₂	0.0179	0.3281	[0.0179, 0.3281]
DMU ₃	0.1714	1	[0.1714, 1]
DMU ₄	0.1786	1	[0.1786, 1]
DMU ₅	0.0317	0.9000	[0.0317, 0.9000]
DMU ₆	0.1875	1	[0.1875, 1]
DMU ₇	0.0571	0.3761	[0.0571, 0.3761]
DMU ₈	0.1190	1	[0.1190, 1]
DMU ₉	0.1950	1	[0.1905, 1]
DMU ₁₀	0, 0239	0.2857	[0.0239, 0.2857]

Results in Table 4 show that ranks of some DMUs are identical under new strategy and old strategy, but the cross-efficiency values of all DMUs are not the same absolutely. For certain undoubtedly DMU, ranking results are invariable for all strategy, for example DMU₁₀, DMU₂ and DMU₇. By using the cross-efficiency, DMUs that have identical CCR efficiencies can be ranked completely. The result shows that the objective function after introducing the good relationship considers the self-efficiency of evaluated DMU and various friendly standards of different DMUs. Thus, it leads to the not identical common weights under different strategy. Compared with the efficiency under different strategy, some phenomena are shown as follows.

The benevolent cross-efficiency value of most DMUs under new strategy is not more than the old strategy. To maximize the sum of the evaluated DMU's efficiency under old strategy, it makes the DMU with larger efficiency to be close to the upper limit. However, the DMU with larger upper limit has larger difference between the maximum efficiency and minimum efficiency under new strategy. The denominator of the efficiency value of well-behaved DMU increases.

Therefore, the benevolent cross-efficiency value under new strategy is not more than the one under the old strategy.

Table 4 CCR efficiency and cross-efficiency of each DMU under different strategy and rank

DMU	CCR	Ranking	The average cross-efficiency under old strategy				The average cross-efficiency under new strategy			
			benevolent ranking		aggressive ranking		benevolent ranking		aggressive ranking	
DMU ₁	0.8447	7	0.6973	6	0.3702	7	0.6916	6	0.3913	7
DMU ₂	0.3281	9	0.2553	9	0.1600	9	0.2498	9	0.1749	9
DMU ₃	1	1	0.9062	2	0.6700	3	0.9062	3	0.6997	2
DMU ₄	1	1	0.9205	1	0.6664	4	0.9205	1	0.6664	3
DMU ₅	0.9000	6	0.5421	7	0.4507	6	0.5520	7	0.3964	6
DMU ₆	1	1	0.8624	4	0.7824	1	0.8624	4	0.7824	1
DMU ₇	0.3761	8	0.3105	8	0.1849	8	0.3092	8	0.1918	8
DMU ₈	1	1	0.8393	5	0.5403	5	0.8082	5	0.6663	4
DMU ₉	1	1	0.8966	3	0.7343	2	0.9103	2	0.6482	5
DMU ₁₀	0.2857	10	0.1172	10	0.1401	10	0.1164	10	0.1468	10

In the same way, the aggressive cross-efficiency value of most DMUs under new strategy is not less than that of old strategy. Because of the introduced friendliness under the new strategy, the denominator of the efficiency of badly-behaved DMU is increased which due to the higher cross-efficiency under the new strategy.

It is not appropriate to say which methods are right or wrong. What matters is merely which strategy is more suitable for the evaluation environment of the DMU from a different aspect and more easily accepted by all DMUs. Based on the good relationship, this paper does not only consider the self-evaluation efficiency and interval efficiency, but also it illustrates the rationality to maximize or minimize the overall satisfaction. The efficiency value derived by our proposed method avoids the extreme case that bias to certain DMU excessively. But beyond that, the proposed method has overall advantages of the evaluation method under old strategy. Thus, the method proposed in this paper may have some superiority.

5 Conclusions

As the cross-efficiency model under old strategy does not well integrate the self-evaluation with peer-evaluation, this paper proposes the cross-efficiency model based on friendly relationship to overcome this flaw. Then, a numerical example is used to illustrate the superiority of the proposed method. Compared to the old strategy, the proposed method not only considers the self-evaluation efficiency and interval efficiency, but it also introduces the self-evaluation efficiency and interval efficiency into objective function. The proposed method can avoid some extreme cases such as the overly friend or overexposed bad common weights computed by benevolent and aggressive cross-efficiency. Empirical findings indicate that the method of this paper is more objective and fair and it is more applicative to the real life all DMUs.

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