Numerical Investigation of Hydraulic Fracture Propagation

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ABSTRACT

The present paper includes new results in the following scopes: development of thermodynamically correct mathematical models of damageable thermoelastoviscoplastic medium (microfracture); development of methods for determination of "nonstandard" constants of medium models, connected with microfracture of material; numerical investigation of hydraulic fracturing of petroleum layers.

1. INTRODUCTION

Thermomechanical processes, which proceed in deformable solids under intensive dynamic loading, consist of mechanical, thermal and structural ones, which are linked. The structural processes involve formation, motion and interaction of defects in metallic crystals, phase transitions, breaking of bonds between molecules in polymers, accumulation of micro structural damages (pores, cracks), etc. Irreversible deformations, zones of adiabatic shear microfractures are caused by these processes. Dynamic fracture is a complicated multistage process, which includes appearance, evolution and confluence of microdefects and formation of embryonic microcracks, pores that can grow to the break-up of bodies with formation of free surfaces.

This paper presents a 2D problem of dynamical deforming and destruction of an oil-holding layer in hydraulic fracturing. The governing equations of the state of the layer were taken from the model of thermoviscoelastoplastic media with micro-defects (micro-pores) /1/, filled with another phase: liquid or gas. The micropores can change their size in the process of dynamical deforming. They can expand or collapse. The model was created using the main thermodynamic principles and therefore it conforms to all the basic laws of mechanics and thermodynamics. All the processes – deformation, microdamaging, heating – are linked with each other. The criterion of destruction is based on critical level of specific dissipated energy and the numerical method for the Lagrangean approach to hydraulic fracturing problem using explicit distinguishing of the fracture slopes and adaptive grid. The similar type models and methods were also presented in /2-7/.

2. STATEMENT OF HYDRAULIC FRACTURING PROBLEM

One of the most effective methods for enhancing oil recovery is hydraulic fracturing. Hydraulic fracturing is a complex scientific and technological problem. The major part of the scientific component of the problem relates to continua mechanics.

The hydraulic fracturing problem is very close to the problem of low velocity penetration of a jet into a dense medium from the point of view of physical and geometrical statement of the problem. At present the problems of penetration are among the best developed. The recent book on the problem [8] published in Russia contains an extensive bibliography. Thus, hydraulic fracturing problems can be solved using mathematical models and numerical methods developed in mechanics of penetration and fracture.

Let us consider the following 2D test problem of an oil layer hydraulic fracturing. There is a rectangular region ABCD sized

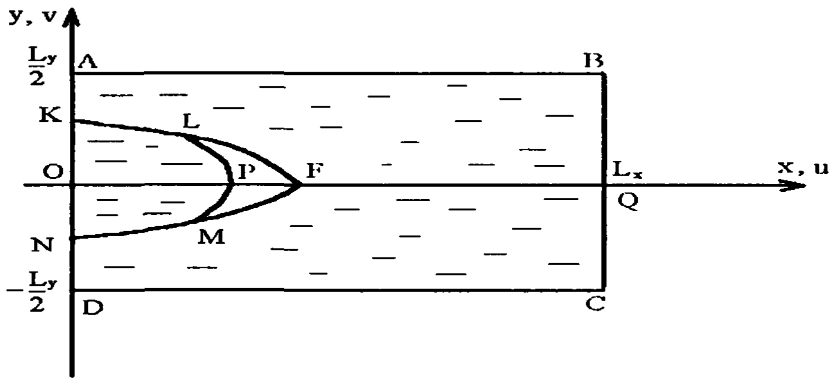

Fig. 1: Scheme for a two-dimensional fracturing problem

$L_x \times L_y$, and the Descartes coordinates system (x, y); the ordinates axis is directed upwards along the well, the abscises axis is horizontal (Figure 1). The parameters do not depend on the third co-ordinate z orthogonal to the xy-plane. The region KFN (fig. 1) is a hollow originated after perforation of the layer, e.g. by means of a cumulative explosive device. The line AD is the external boundary of the well area.

At the first instance $t = 0$ the fracturing liquid starts to be pumped into KFN from the well stem. The Fig. 1 shows the region KLPMN filled with the liquid pumped by some moment of time $t = t_1$. The region LFMP is a hollow situated between the pumped liquid and the broken layer.

3. MODELS OF MECHANICS OF CONTINUAL FRACTURE

Mathematically the problem is formulated as follows. The equations of mass, momentum and energy balance in the layer region ABCDNMFIKA take the form:

$$\frac{\dot{\rho}}{\rho} = -\dot{\varepsilon}_{xx} - \dot{\varepsilon}_{yy},\ \rho\dot{u} = \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y},\ \rho\dot{v} = \frac{\partial\sigma_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y}, \tag{1}$$

$$\rho c_\sigma \dot{T} + \alpha_v \dot{\sigma} T = S_{xx}\dot{\varepsilon}^p_{xx} + S_{yy}\dot{\varepsilon}^p_{yy} + 2S_{xy}\dot{\varepsilon}^p_{xy} + S_{zz}\dot{\varepsilon}^p_{zz} + \Lambda\dot{\omega}^2 \tag{2}$$

The equations describing damageable (porous) thermoelastoplastic media are the following:

$$\dot{\sigma}' = K_0\left(\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} - \alpha_v\dot{T} - \frac{\Lambda}{3}\dot{\omega}\frac{\partial\dot{\omega}}{\partial\sigma}\right), \tag{3}$$

$$\left(S'_{ij}\right)^\nabla + \lambda S'_{ij} = 2\mu_o\dot{e}_{ij},\ e_{ij} = \varepsilon_{ij} - \tfrac{1}{3}\varepsilon_{kk}\delta_{ij},\ S_{zz} = -\left(S_{xx}+S_{yy}\right),\ S'_{ij}S'_{ij} \le (2/3)\cdot Y^2{}_o(\sigma),$$

$$Y_0(\sigma) = \alpha\sigma + \beta,\ S'_{ij} = S_{ij}/(1-\omega),\ \sigma' = \sigma/(1-\omega)$$

Here, the superscript ∇ denotes Yaomann's temporal derivative applied to tensor components /8/.

The kinetic equation describing the evolution of the damage parameter ω is the following:

$$\frac{\dot{\omega}}{\omega} = \frac{\sigma - \sigma^+}{4\eta_0}H(\sigma - \sigma^+) + \frac{\sigma - \sigma^-}{4\eta_0}H(\sigma^- - \sigma), \tag{4}$$

$$\sigma^+ = -(2/3)\cdot Y_0\log\omega - p_0\left(\omega_o/\omega\right)^k,\ \sigma^- = (2/3)\cdot Y_0\log\omega - p_0\left(\omega_o/\omega\right)^k$$

Here, Y_0 is the plasticity limit under material stretching, ω_0 is the initial porosity, p_0 is the initial pressure in pores ("rock layer pressure"), k is the adiabatic exponent of the media filling the pores, η_0 is the dynamic viscosity of material.

The criterion of the destruction beginning (that is appearance of new free surfaces within the material) uses the principle of critical value of specific dissipated energy, which has for considered problem following form /2/:

$$D=\int_0^{t_*}\frac{1}{\rho}\left(S'_{ij}\dot{\varepsilon}^p_{ij}+\Lambda\dot{\omega}^2\right)dt=D_* \tag{5}$$

Thus, the model (2) - (4) is similar to the model of porous thermoviscoelastoplastic medium which was worked out for mathematical describing of dynamic deformation and fracture of solid fuel under impact loading /1/.

To determine the direction of a crack, we use the following procedure.

We seek the direction of extreme normal and tangential stresses in the cell, where the criterion of limited specific dissipation energy (5) took place: $D \geq D_*$. The stress vector for the given square with normal unit vector $\vec{n}(n_x, n_y)$ $(n_x^2 + n_y^2 = 1)$

$$\vec{\sigma}_n(\sigma_{nx}, \sigma_{ny}) \ \ (\sigma_{nx} = \sigma_{xx}n_x + \sigma_{xy}n_y,\ \sigma_{ny} = \sigma_{xy}n_x + \sigma_{yy}n_y)$$

is split into normal σ_n and tangential σ_τ components. It is obvious that

$$\sigma_n = \vec{\sigma}_n\vec{n} = \sigma_{xx}n_x^2 + 2\sigma_{xy}n_xn_y + \sigma_{yy}n_y^2,\ \sigma_\tau = \sqrt{\vec{\sigma}_n\vec{\sigma}_n - \sigma_n^2}.$$

Denoting $n_x = \cos\theta$, $n_y = \sin\theta$, we can write σ_n and σ_τ as following:

$$\sigma_n = \sigma_{xx}\cos^2\theta + \sigma_{xy}\sin 2\theta + \sigma_{yy}\sin^2\theta,\ \sigma_\tau = \left|0.5\cdot(\sigma_{xx} - \sigma_{xy})\cdot\sin 2\theta + \sigma_{xy}\cos 2\theta\right|.$$

Then, the components of normal direction $\vec{n}(n_x, n_y)$ are found, where maximums of σ_n and σ_τ take place, and the corresponding maximal values are obtained: $\sigma_n^{\max}$ and $\sigma_\tau^{\max}$. After that, the Davidenkov – Friedman criterion is applied /8/:

1. If $\sigma_n^{\max}/\sigma_B > \sigma_n^{\max}/\tau_B$, then the " separation" damage took place, and the direction of the separation fracture is determined by the normal direction to the square where $\sigma_n^{\max}$ took place.
2. If $\sigma_n^{\max}/\sigma_B < \sigma_n^{\max}/\tau_B$, then the "shear" damage took place and the direction of the shear fracture is determined by the normal direction to the square where $\sigma_\tau^{\max}$ took place.

Reconstruction of a grid is described in /8/.

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The initial conditions at moment $t = 0$ are: $u = v = 0$, $\rho = \rho_0$, $S_{ij} = 0$, $\sigma = -p_0$, $T = T_0$.

The boundary conditions are the following.

At the boundaries AB and CD: $\partial\sigma_{ij}/\partial y = 0$, $\partial\sigma/\partial y = 0$, $\partial T/\partial y = 0$.

At the boundary BC: $\partial\sigma_{ij}/\partial x = 0$, $\partial\sigma/\partial x = 0$, $\partial T/\partial x = 0$.

At the boundary AK and ND: $u > o$, $\rho\dot{v} = \partial\sigma_{xy}/\partial x + \partial\sigma_{yy}/\partial y$, $T = T_0$.

To simplify the test problem, it is assumed, that the pumped liquid front LPM is flat and moves at a given speed $V = V(t)$; the pressure at the boundaries KL and MN is also assumed to be $P = P(t)$. The simplest case is $P = P_0 = \text{const}$, $V = V(t)$, where the velocity has a linear decrease.

4. METHODS FOR DETERMINATION OF MATERIAL CONSTANTS OF DAMAGEABLE MEDIA

Models for damageable media contain some "nonstandard" constants, connected with damage parameters and subjected to determination. For example, the models with damage parameters /6/ contain seven such constants. To determine these constants under dynamical loading we use a method, based on comparing of the results of physical and numerical experiments of the problem of flat collision of two plates with spallation destruction in a plate-target /9/. Note, that nowadays, experiments with spallation destruction are the most informative and detailed for constructing dynamic constitutive equations for materials under high parameters.

For determining these "nonstandard" constants under quasidynamical deforming we use the method, based on numerical and physical modeling of processes of quasidynamical twisting and tension of thin-walled tubular samples with destruction and with following mathematical data handing /5/.

For the model of porous medium /1/ intended for describing behavior of solid fuels and explosive under impact loading, we used the problem of compression of a spherical microscopical pore filled with gas medium /10/.

5. RESULTS OF CALCULATIONS

To solve the problem of hydraulic fracture development, we suggest the finite difference scheme with Lagrangean approach, the method used by M. L. Wilkins /8/.

The numerical modelling of fracturing could be applied in several ways. Using the continuous approach (which involves the model of damaged media with pores presented above), we calculate its stressed and deformed state assuming development microdamages and accumulation. All this results reducing the durability of rock material.

In many practical cases, which scope the hydraulic problem, it is necessary to monitor the details of origination of damage processes and monitor the development of explicit motion of breaches in continuity, their interaction and joining. There are two approaches for this: either explicit distinguishing the boundaries of continuity violation, or using discrete particles instead of broken material. Each particle has its mass, finite size, momentum. The particles interact with each other and with the continuous region in accordance with a given rule /8/. It is worth to notice that the second approach is not an alternative to the first one but rather its development.

The most effective method of explicit distinguishing the boundaries of continuity violation is based on local reconstruction of the Lagrangean grid in the vicinity of the fracture origination /8/. If a fracture originates in a given cell of the grid, then two banks of the fracture are built instead of it. Afterwards the cell is excluded from calculations. Its mass, momentum and other characteristics are shared among the neighbouring cells.

The boundary conditions of stress (free surface or contact surface) are set on the borders of the fracture (depending on situation) /8/. If the borders of the fracture diverge, then the stress free conditions are set on, if they collapse, then the algorithm describing the contact surface is applied similar to one described in /8/.

The results of numerical modelling of the layer using hydraulic fracturing were obtained for the following governing parameters: $L_x = 2.75\,m$, $L_y = 2.145\,m$; $L_0 = 0.55\,m$, $d = 0.055\,m$ – the initial length and height of the perforated area; $\rho_0 = 2000\,kg/m^3$; $\mu_0 = 16.5\,GPa$, $E = 42.9\ GPa$, $\nu = 0.3$, $\eta_0 = 100\,Pa\cdot s$, $\Lambda = 1500\,Pa\cdot s$, $\alpha = -0.09$, $\beta = 0.04\,GPa$, $k = 1.4$; $D_* = 333.4\,J/kg$; $p_0 = 30\,MPa$, $p_f = 90\,MPa$; $\rho_f = 1000\,kg/m^3$, $v_0 = 10\,m/s$ - density and initial velocity of the pumped fluid. With time, this velocity decreases.

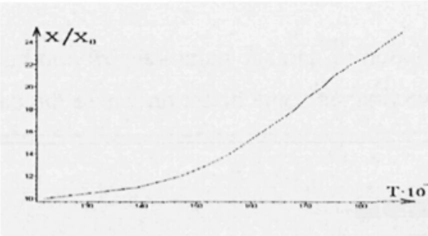


Fig. 2: Dependence of the hydraulic fracture length X / X_0 on time t

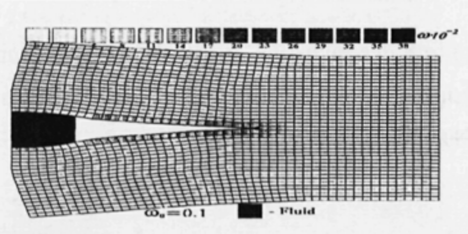
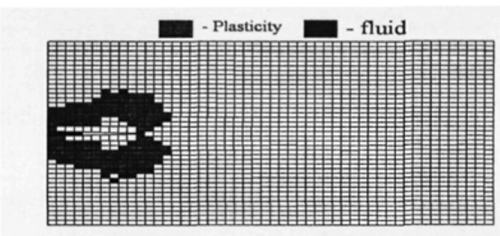
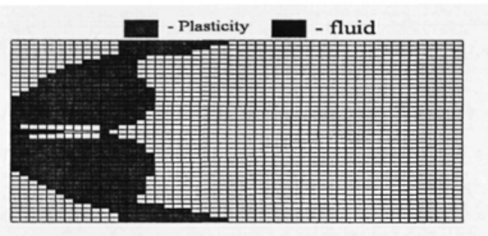


Fig. 3: Deformed mesh and distribution of damage parameter ω (porosity)

Figures 4, 5 show the hydraulic fracture propagation and the region of plasticity (horizontal case) at the different moment of time. Near the tip of the crack the region of plasticity looks like the "rabbit's ears".



Figs. 4: Regions with active plastic deformations



Figs. 5: Regions with active plastic deformations

Figures 6, 7, 8 show the hydraulic fracture propagation and distribution of damage parameter ω in the following cases. The Figures 6, 7 describe the hydraulic fracture propagation and distribution of damage parameter ω with the initial damage parameter $\omega = 0.1$. Besides, in this case we first set some direction where the damage parameter $\omega = 0.4$.

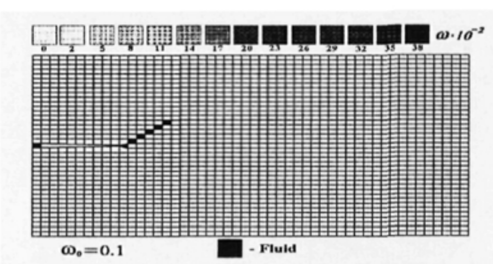


Fig. 6: The beginning configuration and the beginning distribution of damage parameter ω (porosity)

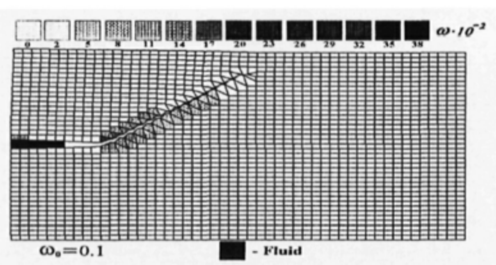


Fig. 7: Deformed mesh and distribution of damage parameter ω (porosity)

Figure 8 describes the hydraulic fracture propagation and distribution of damage parameter ω too but with the initial damage parameter $\omega = 0.05$. Besides, in this case we first set some direction where the damage parameter $\omega = 0.4$.

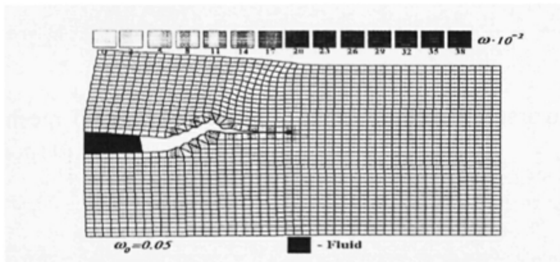


Fig. 8: Deformed mesh and distribution of damage parameter ω (porosity)

6. CONCLUSIONS

The following conclusions could be derived from those results:

1. For the expanding fracture, there are vivid regions of high plastic deformations near its pit (so-called "rabbit's ears"). Significant growth of the damage parameter ω takes place within those regions.
2. The level of the critical dissipation D_* influences very much on the fracture expansion. Therefore, it is significant to make special experiments to determine this parameter value for various materials, which could be present in an oil or gas host rock layer.

The following aspects of hydraulic fracturing problem are being studied:

1. Dynamical behavior of the tip of a hydraulic fracture in an elastoplastic material.
2. The influence of the pre-existing fracture network on hydraulic fracture propagation.
3. Investigation of curved hydraulic fracture propagation.

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REFERENCES

1. A.B. Kiselev and M.V. Yumashev, A mathematical model of deformation and fracture of a solid fuel under impact loading, *J. Appl. Mech. Tech. Phys.*, Vol. **33**, (1992) 126-134.
2. A.B. Kiselev and M.V. Yumashev, Deforming and fracture under impact loading. The model of thermoelastoplastic medium, J. Appl. Mech. Tech. Phys., Vol. 31, (1990) 116-123.
3. A.B. Kiselev and M.V. Yumashev, Numerical investigation of dynamic processes of deforming and microfracture of damageable thermoelastoplastic medium, Moscow Univ. Mech. Bull., Vol. 49, (1994) 14-24.
4. A.B. Kiselev, Mathematical modelling of dynamical deforming and microfracture of damageable thermoelastoviscoplastic medium, in M. Predeleanu, P. Gilormini, eds., Studies in Applied Mechanics 45: Advanced Methods in Materials Processing Defects, Elsevier, Amsterdam, (1997) 43-50.
5. A.B. Kiselev et al., Deforming and fracture of metals. The model of damageable thermoelastoviscoplastic medium, J. of Materials Processing Technology, Vol. 80-81, (1998) 585-590.
6. A.B. Kiselev, Mathematical modelling of dynamical deformation and combined microfracture of a thermoelastoviscoplastic medium, Moscow Univ. Mech. Bull., Vol. 53, (1998) 32-40.
7. V.I. Kondaurov and L.V. Nikitin, Theoretical Foundations of Rheology of Geomaterials (in Russian), Nauka, Moscow, (1990).
8. V.M. Fomin et al., High-speed interaction of bodies (in Russian), Sibirian Branch of RAS, Novosibirsk, Russia (1999).
9. A.B. Kiselev and A.A. Lukyanov, Mathematical modeling of dynamic processes of irreversible deforming, micro- and macrofracture of solids and structures, Int. J. of Forming Processes, Vol. 5, No. 2-3-4, (2002) 351-362.
10. A.B. Kiselev and M.V. Yumashev, Numerical investigation of micropore shock compression in a thermoelasto-viscoplastic material, Moscow Univ. Mech. Bull., Vol. 47, (1992) 31-36.