Measurement of Structural Intensity of a Shell Based on Finite Element Method

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1. SUMMARY

The purpose of the work presented here is to propose a measurement method of the structural intensity in an arbitrary thin shell structure. Structural intensity is defined as a vector quantity representing a vibrational power flow in a structure. Since the energy source identification and the energy transmitting path detection can be possible using this method, it is a powerful way to investigate or control a vibrating structure. But various difficulties prevented this method from practical use. In this paper, we propose a new measurement theory based on FEM (Finite Element Method) and show a simulation result on a straight beam.

2. INTRODUCTION

The purpose of this paper is to develop a measurement theory of the structural intensity using FEM theory. Structural intensity technique is very useful because energy source identification and energy path finding can be achieved by this approach. Therefore, it will become a useful tool in a field such as structural design, vibration analysis and vibration testing.

The concept of structural intensity and its measurement technique is presented by Noiseux /1/ and Pavic /2/. In these papers, the structural intensity is defined as a vibrational power per unit width of cross-section of a uniform beam and plate. A more general formulation is developed by Romano *et al.* /3/ using a Poynting vector approach and they formulated structural intensity in shells.

Various measurement techniques have been tried by mainly finite difference method. But there are still many problems for practical use because of theoretical and measurement errors of the finite difference method. The complexity of structural intensity formulation and numerous points of measurement are also problems.

Our approach was to measure the structural intensity started from the spatial Fourier transform method.

Some measurement methods using this idea have been presented /4,5/. Though this method is applicable to any shells, it is difficult to deal with the complex wavenumbers (evanescent mode). Furthermore, this method is under the free field (nearfield neglected) and free vibration assumptions. This restriction limits its range of application. Especially, if the applicable field is limited within farfield which is sufficiently far from physical discontinuities, energy source identification is impossible because the evanescent wave cannot be disregarded near the source/sink or boundary. This shows inconsistency between the purpose and the means.

To resolve this problem, finite element method and Sommerfeld's condition is applied. In unbounded domain modeling, Sommerfeld's condition is applied to describe the infinite or semi-infinite region /6/. Sommerfeld's condition is a non-reflecting condition in the acoustics. Its application to the elasticity is possible /7/ and the effect of Sommerfeld's condition is identical to that of an absorbing boundary or non-reflecting boundary in FEM modeling. This condition is enforced on a boundary of some finite region and element characteristic equation is formulated. However, some parameters remain unknown. These parameters are determined by solving simultaneous equations substituting some values obtained by measurement. Only the out-of-plane displacement is needed for this procedure. The features of this method are that there is no necessity for whole structural modeling, the necessary value is easy to obtain by measurement, the analytical solution is not necessary, and it is possible to evaluate nearfield.

In this paper, structural intensity in tensor form is stated first, and measurement procedure is presented. Finally, an example case is simulated and the result is shown. Though our final goal is an arbitrary shell, the objective of this paper is to show the concept. So simulation is performed with a beam model.

3. STRUCTURAL INTENSITY

Structural intensity is defined as the vibrational power per unit width of the cross-section of a uniform beam and plate. In early studies /1,2/, only flexural motion is focused because the flexural motion is superior to others in a straight beam or flat plate. In a shell structure, because of curvature, the in-plane motion is important and expressions of structural intensity are complicated.

First, structural intensity density is defined as a three-dimensional Poynting's vector in an elastic body

$$\mathbf{i} = -\boldsymbol{\sigma} \cdot \mathbf{v} \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress tensor and $\mathbf{v}$ is the velocity vector. Structural intensity is a resultant of the vector $\mathbf{i}$ over a cross section. Therefore, it is a two-dimensional vector at the central surface of a structure. Its final expression is in tensor form

$$I = I^{\alpha}\mathbf{a}_{\alpha} = (-\dot{u}_{\beta}N^{\alpha\beta} - \dot{\theta}_{\beta}M^{\alpha\beta} - \dot{w}Q^{\alpha})\mathbf{a}_{\alpha} \tag{2}$$

where $\mathbf{a}_{\alpha}$ is base vector, (u_{β}, w) is displacement component, θ_{β} is rotation angle component and

$(N^{\alpha\beta}, M^{\alpha\beta}, Q^{\alpha})$ is membrane force, moment and shear force respectively. All components are tensor components, so if they are converted to physical components, Eq. (2) is identical to Romano's expression of intensity. To measure the structural intensity, the out-of-plane displacement and its derivative is needed in a beam or a plate, and both the out-of-plane displacement and the in-plane displacement (and its derivatives) are needed in shells.

4. MEASUREMENT THEORY

4-1. Element characteristic equation

The characteristic equation of a element, the discretized equation of motion of an element, is

$$\mathbf{K}\mathbf{q} + \mathbf{M}\ddot{\mathbf{q}} = \mathbf{g} \tag{3}$$

where **M** is the mass matrix, **K** is the stiffness matrix, **q** is the nodal displacement and **g** is the generalized loads. Generally, **g** consists of the load per unit length and the external loads at the boundary of the element and the reaction forces with adjacent elements. If there are no external loads, **g** is identical to the inter-elemental forces.

4-2. Sommerfeld's condition

At the boundary of some domains, unknown inter-elemental forces will exist. On the other hand, this boundary is in the condition of impedance matching and a non-reflective wave situation is possible. This situation can be formulated by Sommerfeld's condition. The effect of Sommerfeld's condition is identical to that of an absorbing boundary or non-reflecting boundary /6,7/.

The boundary condition is that the artificial damping reaction force is enforced. This virtual force is described as

$$\mathbf{t} = -i\omega\mathbf{c}\mathbf{u} \tag{4}$$

where **t** is the force vector, ω is the angular frequency, **c** represents the impedance matrix and **u** is the displacement vector. This is equivalent to the reaction force of the artificial damper installed in the boundary. Therefore, all the energy of the outgoing wave is absorbed by this boundary and reflection does not take place as a result.

If the boundary of some finite element is in this condition, the unknown inter-elemental forces are replaced by this virtual force. This concept is shown in Figure 1. Thus, by determining the impedance matrix **c** only nodal displacement is needed to decide the infomation inside the element.

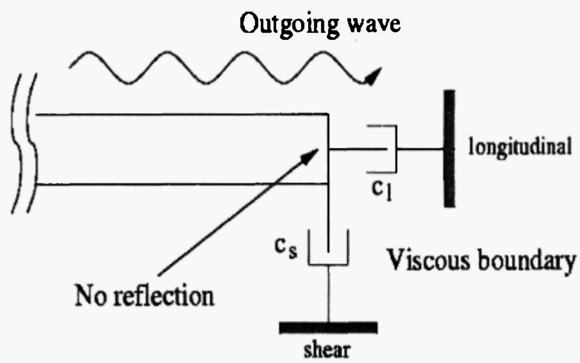

Fig. 1: Concept of Sommerfeld's condition

4-3. Impedance matrix

In a beam, for example, the shear force and the bending moment act on a cross-section. Therefore, two impedance parameters are required to express Sommerfeld's condition: namely, the shear force impedance c_F and the moment impedance c_M. Using these parameters, boundary conditions

$$F = i\,\omega\,c_F\,w \tag{5}$$

$$M = i\,\omega\,c_M\,\theta \tag{6}$$

are enforced at the end of the element. With these parameters, the external force vector

$$\mathbf{g} = \begin{Bmatrix} F_n \\ M_n \end{Bmatrix} = \begin{Bmatrix} i\omega c_F w_n \\ i\omega c_M \theta_n \end{Bmatrix} \tag{7}$$

is obtained (n is the nodal number).

The shear force impedance and the moment impedance can be assumed to have a proportional relation, presuming from their characteristic impedances. This is expressed as follows

$$c_F = \alpha\, c_M \tag{8}$$

where α means proportional parameter. Therefore the external force vector is rewritten

$$\mathbf{g} = \begin{Bmatrix} F_n \\ M_n \end{Bmatrix} = \begin{Bmatrix} i\omega \alpha c_M w_n \\ i\omega c_M \theta_n \end{Bmatrix} \tag{9}$$

With this expression, only proportional parameter α and the moment impedance c_M remain unknown. Then, it is described in matrix form

$$\mathbf{g} = i\omega \begin{bmatrix} \alpha c_M & 0 \\ 0 & c_M \end{bmatrix} \begin{Bmatrix} w_n \\ \theta_n \end{Bmatrix} = i\omega \mathbf{S}\mathbf{q} \tag{10}$$

Finally, the characteristic equation of an element in a harmonic vibration is

$$\left(\mathbf{K} - \omega^2\mathbf{M} - i\omega\mathbf{S}\right)\mathbf{q} = \mathbf{D}\mathbf{q} = \mathbf{0} \tag{11}$$

4-4. Measurement procedure

Measurement procedure is an iteration process described as follows. Initially, the proportional parameter α is assumed. Since the remaining parameter is only c_M, the coefficient matrix of the characteristic equation of an element contains only c_M as unknown. Solving the determinant equation of this coefficient matrix,

$$\text{Det } \mathbf{D}(c_M) = 0 \tag{12}$$

eigenvalues are obtained.

Next, using these assumed parameters α and c_M, the component of the matrix is determined for each c_M. For each matrix, the relationships of components of displacement vector is obtained in the form

$$\mathbf{D}(c_M)\mathbf{q} = \mathbf{0} \tag{13}$$

because all unknowns are assumed and displacement components are still variables. From these simultaneous equations, by substituting some measurement values into the nodal displacement vector **q**, the remaining unknown values can be assumed. Notice that these assumed displacements are not exact ones, so they should be confirmed. First, the rank of the coefficient matrix should be checked. Second, the proportional parameter α calculated from its definition using assumed displacement components should be compared with its initial value. If it matches, the iteration process is ended. If it does not match, this process should be tried again by changing α.

5. SIMULATION

A simple example is simulated. The objective is a steel beam with both ends fixed. As a preliminary simulation, FEM software MSC.MARC is used. Its result is treated as an alternative to an actual measurement. Namely, the out-of-plane displacement is used in the iteration as a measurement value, and the rotation angle is computed by the method presented in this paper.

5-1. Properties of the beam

The beam's Young's modulus is 2.1×10^{11} [N/m^3], density is 7850 [kg/m^3], width and depth of the beam are 0.001[m] and 0.0001[m] respectively. Both ends of the beam are fixed, so the out-of-plane displacement w and rotation angle θ are 0 at the boundary. An exciter is placed near one end with 10 [N] force and 1000 [Hz] frequency.

5-2. FEM formulations

In this case, our computation is performed on one element at the end of the beam. So it seems like a cantilever modeled by one element (see Figure.2). The other end is not free but continuous. This situation is modeled by Sommerfeld's condition.

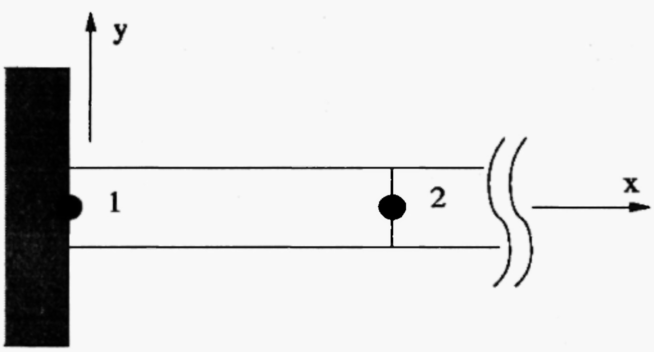

Fig. 2: Finite element modeling with a beam element at the boundary

Its characteristic equation is

$$\begin{bmatrix} K_{33} - \omega^2 M_{33} - i\omega\alpha c_M & K_{34} - \omega^2 M_{34} \\ K_{43} - \omega^2 M_{43} & K_{44} - \omega^2 M_{44} - i\omega c_M \end{bmatrix} \begin{Bmatrix} w_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{14}$$

where K_{nm} and M_{nm} are components of the stiffness and mass matrices of the beam element. Because one end is fixed, namely $w_1 = \theta_1 = 0$, the dimension of the elemental characteristic equation is (2×2).

The proportional parameter α is calculated by its definition as follows.

$$\alpha = \frac{F \cdot \theta}{M \cdot w} \tag{15}$$

The force F and the moment M are calculated from the stiffness matrix **K** and the nodal displacement **q**.

5-3. Calculation result

The length of the element is 0.015[m]. At the boundary point that x=0.015, the out-of-plane displacement is $w = -1.097 \times 10^{-5}$ [m].

With an arbitrary α, two eigenvalues (c_M) are obtained from the determinant of the coefficient matrix (Eq. (12)). For each eigenvalue, a rank check is needed and it should be 1 for the unique solution.

The theoretical value of α presumed by the simulation is used for simplicity, namely α = 5346.3. In this case, eigenvalues are: $c_M = -0.00132i$, $-0.02410i$.

The rank is 1 for each eigenvalue. So next a check for α is attempted. For the first eigenvalue, α calculated by Eq. (15) is -14904.1. This case is probably a wrong one because it is far from the initial value. In the second case, α is 5437.7. The error is merely 1.7% compared with its initial value. This value is considered to be right. Furthermore, the re-calculated rotation angle 0 is identical to the exact one (θ $= 4.845 \times 10^{-4}$).

This result is natural because simulated values are used in this calculation for simplicity. In the future, the iteration and check process should be further established.

6. CONCLUSION

This paper shows that using the impedance parameters, a vibration field can be obtained with some measurement points. With the concept of Sommerfeld's condition, a non-reflection condition can be modeled by FEM. Then, a relatively small number of measurement points can satisfy the condition equation, and it leads to the grasp of the vibration field. Once the vibration field is determined, the structural intensity vector is easy to obtain. However, details of the iteration process have not been established so far. This means that the presented process cannot be performed automatically. This problem must be overcome.

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