Discrete Models of Fabric – Effect of Yarn-Yarn Interactions

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1. SUMMARY

A meso/macro discrete model of fabric has been developed, based on the yarn-yarn interactions occurring at the crossing points of the interwoven yarns. The fabric yarns shape is modeled by a Fourier series development, and the yarns are discretized into a set of elastic bars associated to stretching springs, connected at frictionless hinges by rotational springs. The motion of each node is described by a vertical displacement and a discrete rotation. Considering a single yarn within the woven structure, the reaction force exerted by the transverse yarns at the contact points, is expressed, and the work of the reaction forces is established. Simulations of a traction curve of a fabric in the warp direction are performed, that evidence the effect of the yarn-yarn interactions.

2. INTRODUCTION

The widespread use of woven structures in various domains (clothes, vascular prosthesis, armour, mechanical parts) has triggered many research activities related to the analysis of their deformation and shape forming capacities. Discrete structural models of fabric have been developed recently, see /1, 2, 3/ and the references therein, whereby the yarns are idealized as a set of extensible bars connected at nodes, endowed with a rotational rigidity. Few works in the literature have been devoted to the analysis of the contact or interactions between the yarns, notwithstanding 3D finite element analysis within a context of continuum contact mechanics (see e.g. /4/). The goal of the present approach is to incorporate the effect of the yarn interactions in a manner compatible with a discrete modeling, at a mesoscopic scale of description, without considering the detailed 3D analysis inherent in a microscopic view of the yarns' contact.

3. DISCRETE MODEL DESCRIPTION ACCOUNTING FOR THE YARN INTERACTIONS

We consider here the planar motion of a single yarn (the warp) subjected to a traction force at its extremities and to the punctual contact reactions exerted by the transverse yarns (the weft), Fig. 1, assuming the contact to be perfect (no sliding). Although it is somewhat artificial to isolate mentally the yarn from the trellis, this approach will give an initial insight into the coupling effect between both sets of yarns, at a mesoscopic scale of description. The discretized yarn consists of a set of punctual nodes connected by extensional rigidities $C_{ei} = EA/\Delta$; each node is given a rigidity in flexion $C_{bi} = EI/\Delta$ (Δ is the distance between two consecutive nodes), Fig. 1. The kinematics of the yarn is described by the vertical displacements w_i and the rotations ψ_i (the rotation axis being orthogonal to the plane of the figure) of the nodes. The contact forces exerted by each transverse yarn (marked with a cross in Fig. 1) are first expressed, from the Timoshenko beam theory /5/.

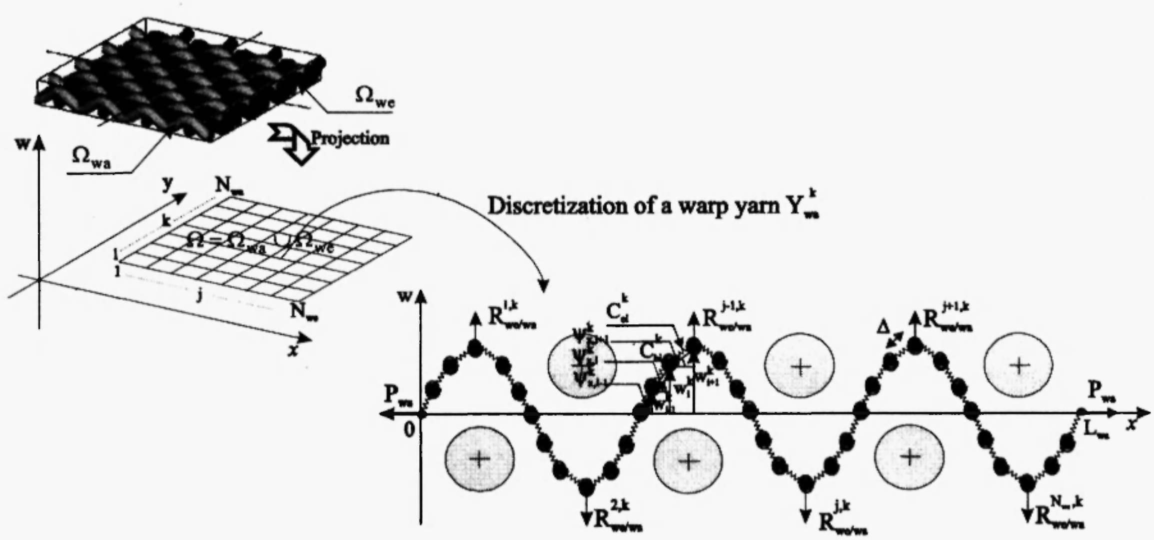

Fig. 1: Discrete model of the yarn isolated from the trellis

Determination of the weft and the warp force interactions (reaction forces)

We shall first express the reaction force, occurring at the interlacing points, in terms of the mechanical and geometrical parameters characteristic of the yarn. At equilibrium, the deformed shapes of the fabric yarns are assumed to be periodic and expressed as the following Fourier series:

$$w_{we}^{j}(y)=\sum_{n=1}^{N_{wa}} a_{n,j}^{we}\sin\left((j-1)\pi+n\frac{\pi y}{L_{we}}\right)\text{, for a weft yarn having index } j \quad (1)$$

$$w_{wa}^{k}(x)=\sum_{n=1}^{N_{we}} a_{n,k}^{wa}\sin\left((k-1)\pi+n\frac{\pi x}{L_{wa}}\right)\text{, for a warp yarn having index } k \quad (2)$$

In order to establish the expression of the reaction force exerted at the yarn-yarn contact points, we first consider the mechanical behavior of the weft yarns Ω_{we}: the sub-mechanical system Ω_{we} is there considered as an external system. We next express the coefficients of the previous series vs. the reaction force $R_{wa/we}^{k,j}$, exerted by the warp on the weft. By using Timoshenko's beam theory, in the case of an elastic beam subjected to a lateral force F exerted at a point having the abscissa c (Fig. 2), the equilibrium shape of the elastic beam, supposed to be periodic, is given by:

$$w(x)=\frac{2FL^3}{\pi^4 EI}\sum_{k=1}^{N}\frac{1}{k^4}\sin\left(\frac{k\pi c}{L}\right)\sin\left(\frac{k\pi x}{L}\right) \quad (3)$$

with L the projected beam length and EI the beam bending rigidity.

Fig. 2: Timoshenko's beam model

Using the superposition principle (the force F in (3) playing the role of the reaction force $R_{wa/we}^{k,j}$), the equilibrium shape associated to a weft yarn, viewed as an elastic beam subjected to periodic lateral forces (Fig. 3), is directly deduced from (3) by

$$w_{we}^{j}(y)=\frac{2R_{wa/we}L_{we}^{3}}{\pi^4 EI_{we}}\sum_{n=1}^{N_{wa}}\sum_{m=1}^{N_{wa}/2}\frac{i}{n^4}\left(\sin\left(\frac{n\pi}{N_{wa}}\left(2m-\frac{3}{2}\right)\right)-\sin\left(\left(2m-\frac{1}{2}\right)\frac{n\pi}{N_{wa}}\right)\right)\sin\left((j-1)\pi+\frac{n\pi y}{L_{we}}\right) \quad (4)$$

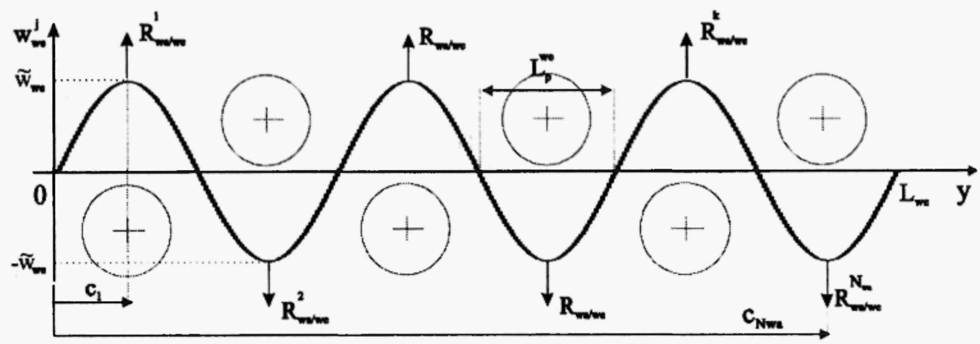

Fig. 3: Distributed contact forces exerted on a weft yarn having index j

The coefficients of the previous series involve the reaction force and the mechanical and geometrical parameters of the weft. We note $\tilde{w}_{we} = A_{we}$ the amplitude of the weft within the woven structure, at the abscissas of the contact points (Fig. 3), viz

$$\tilde{w}_{we} = \left| w^j_{we}(c_j) \right| \tag{5}$$

At the interlacing points, the double sum in Equation 4 simplifies as

$$\left| \sum\sum \right| = \frac{1}{N^3_{wa}} \quad \text{for } y = c_k \quad \forall k \in [1, N_{wa}] \tag{6}$$

We then deduce, from equations (4) to (6), the expression of the reaction forces exerted by the transverse yarns on the weft yarns : $\forall \; y = c_k$, we have

$$|w_{we}(y)| = \tilde{w}_{we} = \frac{2R_{wa/we}}{\pi^4 EI_{we}} \frac{L^3_{we}}{N^3_{wa}} \Leftrightarrow R_{wa/we} = \frac{\pi^4}{2} \frac{EI_{we}}{\left(L^{we}_p\right)^3} \tilde{w}_{we} \tag{7}$$

A result identical to that of Timoshenko /4/ is obtained, viz

$$F = \frac{\pi^4}{2} \frac{EI}{L^3} \tilde{w} \tag{8}$$

4. IMPACT OF THE YARN-YARN INTERACTIONS ON THE TRACTION BEHAVIOUR

In this section, we assess and illustrate the effect of the weft and warp interaction within a woven structure loaded by a traction force acting in the warp direction only. For this purpose, we consider and compare the cases of a warp within a woven structure with a yarn being initially deformed (extracted from the woven structure), the yarns in both cases sharing identical mechanical and geometrical properties. In both cases, the yarn shapes are assumed to be periodic, their equilibrium shapes being expressed as a Fourier series development limited to the N_{we} first harmonics, as

$$w_{wa}(x) = \sum_{n=1}^{N_{we}} a_n^{wa} \sin\left(n\pi \frac{x}{L_{wa}}\right) \tag{9}$$

with L_{wa} the projected length of the yarn on the x-axis and N_{we} the number of half-periods of undulation which corresponds, in the case of a warp yarn within the woven structure, to the number of the transverse yarns contacts. Under the traction loads P_{wa}, the equilibrium state – with or without lateral contacts – is obtained as the minimum of the total potential energy of the deformed yarns. In the sequel, the expression of the total potential energy is established (in each of previous cases) as a function of the kinematical yarn parameters, namely the displacements w_i and the rotations ψ_i of the yarn's nodes, for an inextensible yarn, focusing here on the analysis of the process of loss of undulation.

The assumed continuity of the displacement of the yarns at the contact points leads to the following relationship between the vertical displacements of the summits of the undulations of the yarns (Fig. 4), neglecting the compressibility of both yarns:

$$\delta_{wa} = \delta_{we} \;\Rightarrow\; w_{s-we} = w_{so-we} + w_{s-wa} - w_{so-wa} \tag{10}$$

This relationship can be interpreted as a transfer of undulation between the two mechanical systems Ω_{ch} and Ω_{tr} (set of warp and weft yarns respectively). In other words, when the undulation decreases in one direction, it increases in the other direction (transverse yarn).

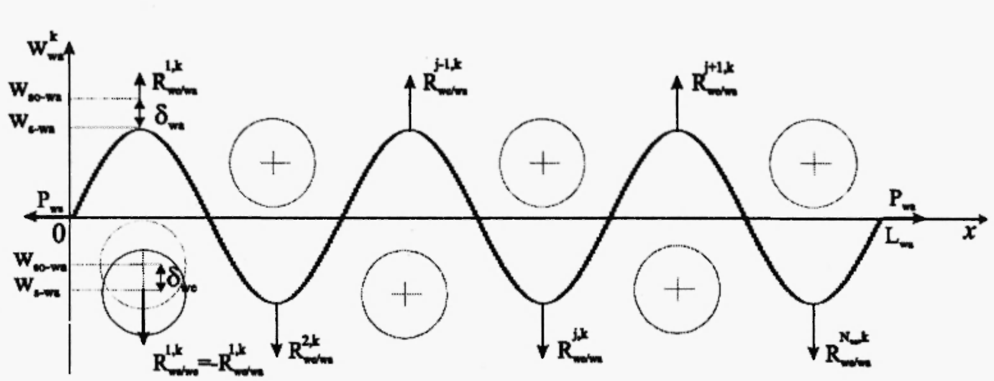

Fig. 4: Motion of the undulated warp

From the relations (7) and (10), we deduce the expression of the reaction force at any contact point :

$$R^j_{wa/we} = \frac{\pi^4}{2}\frac{(EI)_{we}}{\left(L_p^{we}\right)^3} w^j_{s-we} = \frac{\pi^4}{2}\frac{(EI)_{we}}{\left(L_p^{we}\right)^3}\left[(w_{so-we} - w_{so-wa}) + w^j_{s-wa}\right] \tag{11}$$

Using the action-reaction principle, the reaction force exerted by the weft on the warp at the crossing points having index j (which coincide with the summits of the undulations, see Fig. 4) is then given by

$$R^j_{we/wa} = -R^j_{wa/we} = -\frac{\pi^4}{2}\frac{(EI)_{we}}{\left(L_p^{we}\right)^3}\left[(w_{so-we} - w_{so-wa}) + w^j_{s-wa}\right] \tag{12}$$

The work of the reaction force exerted by the weft at a crossing node (index j) is given by

$$W_{R^j_{we/wa}} = \int_{w^j_{so-wa}}^{w^j_{s-wa}} R_{we/wa}\,dw = \int_{w^j_{so-wa}}^{w^j_{s-wa}} -\frac{\pi^4}{2}\frac{(EI)_{we}}{\left(L_p^{we}\right)^3}\left[(w^j_{so-we} - w^j_{so-wa}) + w\right]dw$$

$$= -\frac{\pi^4}{2}\frac{(EI)_{we}}{\left(L_p^{we}\right)^3}\left[\left((w^j_{so-we} - w^j_{so-wa})w^j_{s-wa} + \frac{1}{2}{w^j_{s-wa}}^2\right) - \left((w^j_{so-we} - w^j_{so-wa})w^j_{so-wa} + \frac{1}{2}{w^j_{so-wa}}^2\right)\right] \tag{13}$$

Furthermore, the total work of the reaction forces exerted at the warp yarn contact points is

$$W_{contact} = \sum_{j=1}^{N_{we}} W_{R^j_{we/wa}} \tag{14}$$

which is added to the total work of the external forces (gravity forces, traction loads), as

$$W_{ext} = W_{tr} + W_{gr} + W_{contact} \tag{15}$$

with W_{tr} the work of the traction loads P_{wa}, and W_{gr} the work of the gravity forces. The explicit expression of the external work W_{ext} is then

$$W_{ext} = -\sum_{j=1}^{N_{we}} \frac{\pi^4}{2}\frac{(EI)_{we}}{\left(L_p^{we}\right)^3}\left[\left((w^j_{so-we} - w^j_{so-wa})w^j_{s-wa} + \frac{1}{2}{w^j_{s-wa}}^2\right) - \left((w^j_{so-we} - w^j_{so-wa})w^j_{so-wa} + \frac{1}{2}{w^j_{so-wa}}^2\right)\right]$$

$$+P_{wa}\left(\sum_{i=1}^{N_d}\frac{\Delta}{2}\left(\psi_i^2 - \psi_{oi}^2\right) + u_{N_d+1}\right) - \sum_{i=1}^{N_d-1} m_i g\left(w_i - w_i^o\right) \tag{16}$$

N_d is the number of discrete elements and $(u_i)_{i\in[1,N_d+1]}$ denote the nodal extensional displacements ; the index o refers to the initial value corresponding to a kinematical variable in its initial state.

The total potential energy V_1 (yarn-yarn interactions) is then obtained as the difference between the internal deformation energy U (due to the flexion and the extension of the yarn), given by

$$U = U_F + U_{ex} = \sum_{i=1}^{N_d-1} \frac{1}{2} C_{bi} \left(\psi_{x,i+1} - \psi_{x,i}\right)^2 + \sum_{i=1}^{N_d} \frac{1}{2} C_{ei} \left(u_{i+1} - u_i\right)^2 \tag{17}$$

and the work of the external forces W_{ext}, expression (16), thus leading to

$$V_1 = \sum_{i=1}^{N_d-1} \frac{1}{2} C_{bi} \left(\psi_{x,i+1} - \psi_{x,i}\right)^2 + \sum_{i=1}^{N_d} \frac{1}{2} C_{ei} \left(u_{i+1} - u_i\right)^2 - P\left(\sum_{i=1}^{N_d} \frac{\Delta}{2}\left(\psi_{x,oi}^2 - \psi_{x,i}^2\right) + u_{N_d+1}\right) + \sum_{i=1}^{N_d-1} m_i g\left(w_i - w_i^o\right)$$
$$+ \sum_{j=1}^{N_{we}} \frac{\pi^4}{2} \frac{(EI)_{we}}{\left(L_p^{we}\right)^3}\left[\left((w_{so-we}^j - w_{so-wa}^j)w_{s-wa}^j + \frac{1}{2} {w_{s-wa}^j}^2\right) - \left((w_{so-we}^j - w_{so-wa}^j)w_{so-wa}^j + \frac{1}{2} {w_{so-wa}^j}^2\right)\right] \tag{18}$$

The simplified case without yarn-yarn interactions corresponds to a total potential energy V_2 given by

$$V_2 = \sum_{i=1}^{N_d-1} \frac{1}{2} C_{bi} \left(\psi_{x,i+1} - \psi_{x,i}\right)^2 + \sum_{i=1}^{N_d} \frac{1}{2} C_{ei} \left(u_{i+1} - u_i\right)^2 - P\left(\sum_{i=1}^{N_d} \frac{\Delta}{2}\left(\psi_{x,oi}^2 - \psi_{x,i}^2\right) + u_{N_d+1}\right) + \sum_{i=1}^{N_d-1} m_i g\left(w_i - w_i^o\right) \tag{19}$$

The index j, giving the successive labeling of the crossing points, is further replaced in the two previous expressions by the global discretization index i, such that

$$\begin{cases} w_{s-wa}^j = w_p \\ \sin\left(\psi_{x,i}\right) = \dfrac{w_i - w_{i-1}}{\Delta} \end{cases}, \text{ with } p = \frac{(2j-1)N_d}{2N_{we}}. \tag{20}$$

The discrete nodal displacements $(w_i)_i$ are obtained from the discretization of the continuous position

$$w_{wa}(x) = \sum_{n=1}^{N_{we}} a_n^{wa} \sin\left(n\pi \frac{x}{L_{wa}}\right),$$

according to

$$\forall\, i \in [1, N_d\text{-}1] \quad w_i = w_{wa}(x_i) = \sum_{n=1}^{N_{we}} a_n^{wa} \sin\left(\frac{n\pi}{L} x_i\right) \quad \text{with } x_i = \frac{iL}{N_d} \tag{21}$$

Using the relation (21), the total potential energies V_1 and V_2 given by (18) and (19) can be expressed as a function of the coefficients $\left(a_n^{wa}\right)_{n\in[1,N_{we}]}$ of the Fourier series, and of the nodal extensions $\left(u_2,u_3,...,u_{N_d+1}\right)$ of the discrete warp yarn (accounting for the condition $u_1=0$), thus

$$V_{1,2}=V\left(a_1^{wa},...a_2^{wa},...a_{N_{we}}^{wa},u_2,...u_i,...u_{N_d+1}\right) \tag{22}$$

The yarn equilibrium shape is given as the minimum of the total potential energy with respect to the set of arguments $\left(a_1^{wa},...a_2^{wa},...a_{N_{we}}^{wa},u_2,...u_i,...u_{N_d+1}\right)$. Thus, the equilibrium state of the yarns (with and without yarn-yarn interactions) is formally given by the solution of the set of algebraic equations:

$$\frac{\partial V_{1,2}}{\partial a_1^{wa}}=...=\frac{\partial V_{1,2}}{\partial a_i^{wa}}=...=\frac{\partial V_{1,2}}{\partial a_{N_{we}}^{wa}}=0 \;;\; \frac{\partial V_{1,2}}{\partial u_2}=...=\frac{\partial V_{1,2}}{\partial u_i}=...=\frac{\partial V_{1,2}}{\partial u_{N_d+1}}=0 \tag{23}$$

Carbon fibers reinforced fabric under uniaxial tension

When considering carbon fiber reinforced fabric used in the aerospace industry, the following input parameters are used /6/: the mechanical properties of the warp and weft yarns are taken respectively as

$$EI_{wa}=1.47e^{-7}N.m \;;\; EI_{we}=1.47e^{-7}N.m \;;\; EA_{wa}=13.72\ N\,.$$

The rigidities in flexion / extension of the springs are then evaluated as $C_b=\frac{EI_{wa}}{\Delta}$ and $C_e=\frac{EA_{wa}}{\Delta}$ respectively. The geometrical parameters of the discretization are taken as:

$$L_0=0.1m \;;\; w_{so-wa}=0.5mm \;;\; w_{so-we}=0.5mm \;;\; N_{we}=16 \;;\; N_d=224$$

The yarn is subjected to an increasing traction load at its extremities, and one represents the traction load vs. the yarn end-displacement (Fig. 5); the calculations have been performed with the software Maple©. The mean curve of the yarn is restricted to the (x,y) plane (Fig. 4), and both extremities of the yarn keep aligned with the direction of traction. The simulation without yarn interactions gives a reference comparison case to assess the importance of the yarn-yarn interaction.

The extension of the yarn is here defined as the displacement of the end node of the undulated beam. The simulation leads to the J-shape measured unidirectional traction curve, and it is thought that the model gives the essence of the behaviour of a piece of fabric, although restricting here to the behaviour of a single yarn. The consideration of the yarn-yarn interactions leads to a stiffer response of the yarn (Fig. 5) : during the traction, the transverse yarns resists to the yarn-yarn undulation transfer by increasing the reaction force. This explains why, without yarn-yarn interactions, the loss of undulation's is more rapid than in the case of yarn-yarn interactions. The extension of the present work for a whole trellis, accounting for the yarn compressibility, is under way.

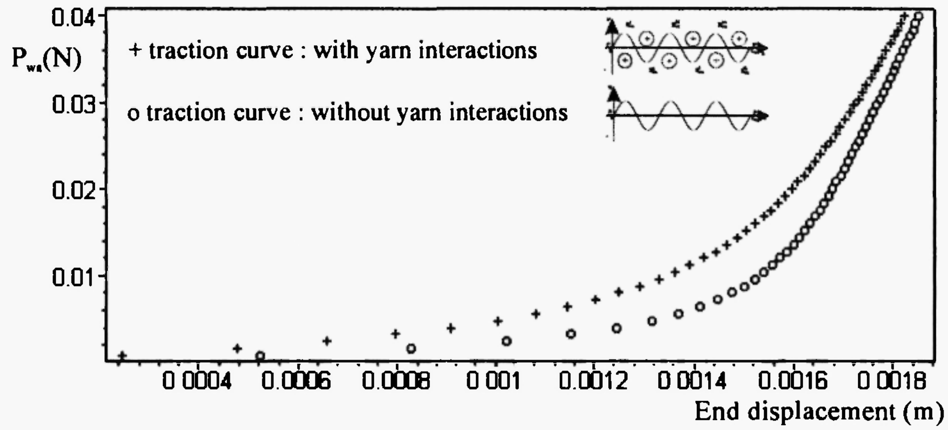

Fig. 5: Unidirectional traction curve of the warp yarn. Effect of yarn-yarn interactions

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