A Method for Inversion of Tunnel Closure Measurements – Application in the Tempi Railway Tunnel

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1. SUMMARY

A simple hierarchical methodology is proposed herein for back-analysis of *in situ* closure measurements in tunnels that is based on *in situ* convergence measurements, the equivalent plane strain concept, analytical and numerical modeling of continuous elastic and elasto-plastic rocks and dimensional analysis. An example of the application of the proposed methodology is given for the Tempi high-speed railway tunnel in Greece. It is demonstrated that both the *in situ* lateral-to-vertical stress ratio, rock mass deformation modulus and cohesion are indirectly inferred from the proposed data-inversion analysis, which in turn may be used for future design of tunnels in similar geotechnical conditions. It is also shown that deformation modulus of the rock mass exhibits size effect and stress-dependency, hence the *in situ* stress ratio depends on the deformation modulus.

2. INTRODUCTION

Rock mass deformability and cohesion are important input parameters for the estimation of the appropriate tunnel excavation sequence and shape, as well as support pressure, since they affect the rock mass displacements and strength. Moreover, of all quantities that the undergound excavation engineer is required to estimate or to measure while undertaking any major underground excavation stability problem, the *in situ* or pre-excavation stress field in a rock mass is one of the most difficult. The vertical stress can be approximated, to an acceptable level of accuracy, by the product of the depth below surface and the unit weight of the rock mass. The latter, with a good level of accuracy, may be taken as equal to 0.027 MN/m that in turn gives an average stress gradient of 0.027 MPa/m /1/. On the other hand, according to the well-known age-old concept, the horizontal stress merely occurs as a result of Poisson's restraint; thus, the vertical and horizontal stresses σ_v and σ_h are given by

$$\sigma_v = \gamma H, \qquad \sigma_h = \frac{\nu}{1-\nu}\gamma H \tag{1}$$

in which H is the height of the overlying rock, γ is the unit rock weight and ν is the Poisson's ratio of the rock mass. The second of the above expressions was derived assuming: (a) that the rock mass is an ideal, homogeneous, linear - elastic and isotropic half-space with horizontal surface; (b) that the rock mass is under gravity alone with vanishing horizontal displacements; (c) that the loading history has no influence on how *in situ* stresses build-up, and (d) that horizontal and vertical stresses vanish at Earth's surface. However, this relationship has been proven by numerous well-documented measurements /1/ to be invalid. These measurements indicate that the horizontal-to-vertical stress ratio $k = \sigma_h / \sigma_v$ not only varies with depth but is likely to be much greater than the value predicted by (1) nearer to the surface.

On the other hand, monitoring has now been widely proven to be efficient in the construction of underground excavation, both for the safety and economical aspects. The easiest and most reliable parameter recorded in the field is certainly the convergence of the tunnel walls. Hence, monitoring of wall displacements in conjunction with the "equivalent plane strain analysis concept" – that is based on the rock-support interaction analysis /2/ – has become an integral part of the design of underground openings. However, it is surprising that the collected data from these measurements are rarely used for the estimation of important soil or rock mass properties and/or pre-excavation stress conditions through appropriate inversion procedures.

The aim of this work is the development of a hierarchical methodology for the determination of the elasticity and cohesion of rock mass, as well as the *in situ* ratio of horizontal-to-vertical stress based on back-analysis of tunnel convergence measurements and dimensional analysis. In this first attempt, the rock mass is assumed to obey the simple Mohr-Coulomb elastoplastic model – though it may be not the most appropriate for all the geological formations intersected by the tunnel – and the peak internal friction angle and Poisson's ratio of the rock mass are assumed to be known. An example of the application of the proposed methodology is given for the case study of the Tempi high-speed railway tunnel in Greece. It utilizes analytical and numerical modeling for back-analysis of *in situ* closure measurements in tunnels. The deformation modulus, *in situ* lateral-to-vertical stress ratio and rock mass cohesion are indirectly inferred from the proposed analysis, which in turn may be used for future design of tunnels in similar geotechnical conditions.

3. INVERSE ANALYSIS METHODOLOGY

Geological setting and technical characteristics of theTempi railway tunnel

The Tempi tunnel is located at the North-West part of the Tempi valley (Sterea Hellas), and is a part of the new double-truck, high-speed railway line of Athens – Thessaloniki. The main tunnel has a final internal diameter of 6.1 m and a length of 4 km.

The rock masses through which the tunnel passes are either of sedimentary origin, consisting of Cretaceous crystalline limestones which are interrupted by irregular intrusions of fyllitic schists, or semi-metamorphic, consisting of marbles and mica schists (Fig. 1).

The rock mass has been classified according to Bieniawski's RMR system and Barton's Q – system. As may be seen in Table 1, the rock mass is characterized from medium (RMR class III) to poor (RMR classes IV - V) with dominating the medium quality (class III, RMR=41- 60).

Table 1

Rock mass classes intersected by Tempi tunnel between stations 8+163 and 11+088.

RMR Class No	Q Class No.	Rock type	Length [m]	Percentage of tunnel length [%]
III	IIIa	Crystalline limestone	938.6	42.17
III	IIIb	Crystalline limestone	474.9	21.34
IV	IVa	Crystalline limestone – fyllites – mica schists	101.7	4.57
IV	IVb	Crystalline limestone – fyllites – mica schists	160.7	7.22
V	Va	Fyllites – mica schists – heavily jointed limestones	549.7	24.70

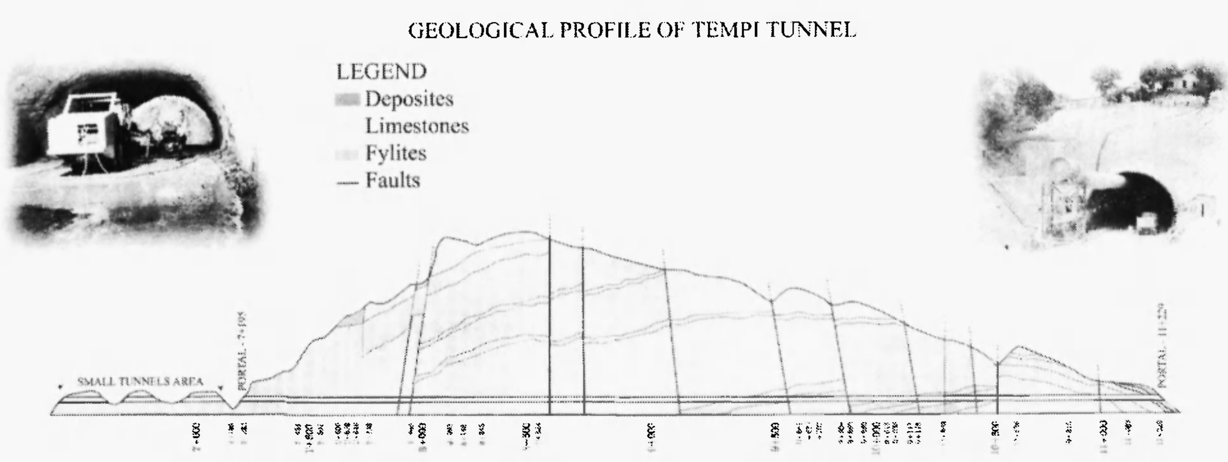


Fig. 1: Geological profile of Tempi railway tunnel.

The excavation of the tunnel has been carried out in two successive stages following the NATM philosophy (i.e. the top semi-circular head in the 1^{st} stage and the bottom bench in the 2^{nd} stage) by using conventional drilling and blasting rounds. Based on the geomechanical classification of the rock mass by RMR and Q systems, there have been designed and applied four typical tunnel cross-sections and corresponding temporary support systems involving rock bolting and shotcreting.

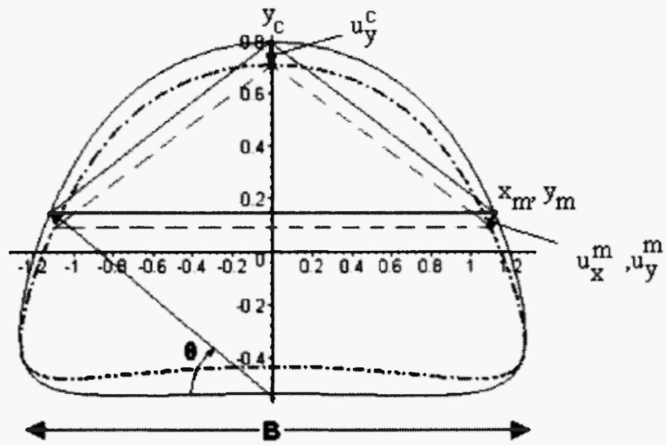
Interpretation of tunnel convergence measurements

Monitoring of excavations has now been widely proven to be efficient in the construction of underground excavations, both for the safety and the economical aspects. The easiest and most reliable parameter recorded in the field is certainly the convergence of tunnel walls (Fig. 2). The aim of this measurement is to determine the variation in the distance between two opposite points of the tunnel wall. The measuring stations must be installed as near to the face as possible at a distance x_0 at time t_0. Provided that the tunnel wall deformations are small, the estimation of the maximum convergence of tunnel wall may be accomplished by best-fitting the in situ measurements through the formula /2/

$$C(x) = C_{\infty x}\left(1 - e^{-\frac{x}{X}}\right) \tag{2}$$

in which C(x) is the convergence at a distance x from the tunnel face calculated as the relative displacement between two points i.e. $C(x) = \Delta L_i \ (i = a, b, c)$ where L is the distance between these points in Fig. 2, $C_{\infty x}$ is the maximum convergence, and X denotes a best-fit constant. Since at time t = 0, the distance of the instrumented section to the face is x_0, then the convergence actually measured is $C(x) - C(x_0)$ and this fact should be also taken into account in the estimation of $C_{\infty x}$. This correction is illustrated in Fig. 3 for the measurement of the horizontal convergence C_h at the station 10+150 of the Tempi tunnel. These convergence measurements at a number of stations along the tunnel have been performed by the contractor during tunnel construction by employing a conventional tape-meter and their accuracy was of the order of 10^{-5} of the measured length.

For the specific case of Fig. 3 it may be observed that the convergence at the tunnel face is approximately 22% of the maximum tunnel convergence and that the tunnel displacements reach the far field constant value at a distance behind the face of approximately 10 times the diameter of the tunnel.



$$d = \sqrt{x_m^2 + (y_m - y_c)^2},$$

$$d' = \sqrt{\left(x_m - u_x^m\right)^2 + \left[\left(y_m - u_y^m\right) - \left(y_c - u_y^c\right)\right]^2},$$

$$C_h = 2 \cdot u_x^m, \quad C_p = d - d'$$

Fig. 2: Undisturbed and deformed cross-section of the Tempi tunnel in the first stage of excavation and positions of convergence measurements (B denotes tunnel span).

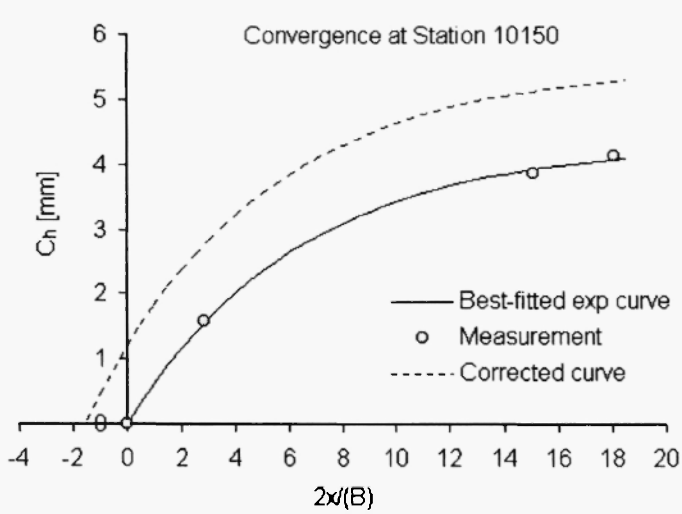


Fig. 3: Tunnel wall convergence as a function of the distance from the face (x=0) at the station 10+150 of Tempi tunnel.

Rock mass model and dimensional analysis

In order to simplify the inversion analysis, the rock mass is modeled as an isotropic, linear elastic - perfectly plastic, Mohr–Coulomb material with zero dilatancy angle (i.e. it is assumed that the rock mass follows the non-associated flow rule and that the effect of plastic volumetric strains on tunnel deformations is negligible). The approach to keep the rock mass model as simple as possible is in accordance with the inherent variability in space and high degree of uncertainty of the values of its properties even for the same geological formation. Further, for the specific case of Tempi tunnel that is excavated in a medium to good quality rock mass, the effect of the initial temporary support measures (i.e. shotcrete and rock bolts) are considered to have a second order effect on the measured tunnel displacements and thus they are not taken into account in the numerical model.

Based on the above considerations it is assumed that the horizontal convergence C_h (Fig. 2) of the initial unsupported semi-circular tunnel is given by the following relation

$$C_h = f_0\left(C_P, B, \sigma_v, \sigma_s, k, E, \nu, c, \varphi, \theta\right) \tag{4}$$

In these relations C_h and C_p denote the horizontal convergence and the oblique convergence of the tunnel section, respectively, as it is illustrated in Fig. 2, E denotes the deformation modulus of the geological material and σ_s is the tunnel support pressure that is related to the tunnel wall displacement through the ground reaction curve /2/. The peak mobilized cohesion and internal friction angle of the rock mass are denoted by the symbols c and φ, respectively. The symbol θ denotes the angle subtended between the line formed by connecting an end of the horizontal tape and the centre of the tunnel floor (Fig. 2). In the case study of Tempi tunnel this angle is $\theta = 10°$ for cross-sections in rock mass belonging to Class III and $\theta = 30°$ for rock classes IV and V according to RMR classification (e.g. Table 1). Moreover the span of the unsupported Tempi railway tunnel is B=13.4 m.

Based on Buckingham's π-theorem and the well-known inverse relation of displacements with elasticity modulus /3/, we may reduce the above relation (4) into a non-dimensional form as follows

$$\frac{E}{\sigma_v}\frac{C_h}{B} = f_1\left(\frac{E}{\sigma_v}\frac{C_p}{B}, \frac{c}{\sigma_v}, \frac{\sigma_s}{\sigma_v}, k, \nu, \varphi, \theta\right) \tag{5}$$

A further simplification of the above function of dimensionless variables may be obtained from the following transformation of normal stresses (Fig. 4)

$$\hat{\sigma}_v = \sigma_v + \frac{c}{\tan\varphi}, \quad \hat{\sigma}_s = \sigma_s + \frac{c}{\tan\varphi} \tag{6}$$

Hence, due to (6) formula (5) takes the form

$$\frac{E}{\hat{\sigma}_v}\frac{C_h}{B} = f_2\left(\frac{\hat{\sigma}_s}{\hat{\sigma}_v}, k, \nu, \varphi, \theta\right), \quad \frac{E}{\hat{\sigma}_v}\frac{C_p}{B} = f_3\left(\frac{\hat{\sigma}_s}{\hat{\sigma}_v}, k, \nu, \varphi, \theta\right) \tag{7}$$

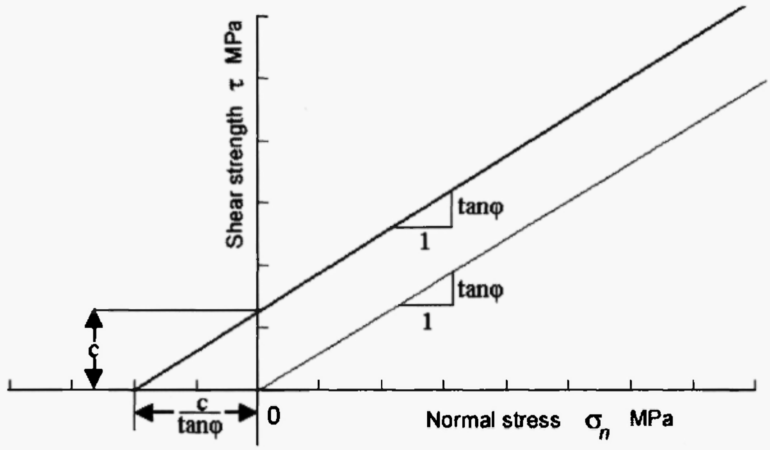


Fig. 4: Transformation of normal stresses.

Next, based on the above dimensional analysis, tunnel wall displacement charts were constructed by virtue of numerical analyses with the aid of the 2D explicit finite difference code with dynamic relaxation FLAC^{2D} of ITASCA /4/. Such charts are displayed in Fig. 5.

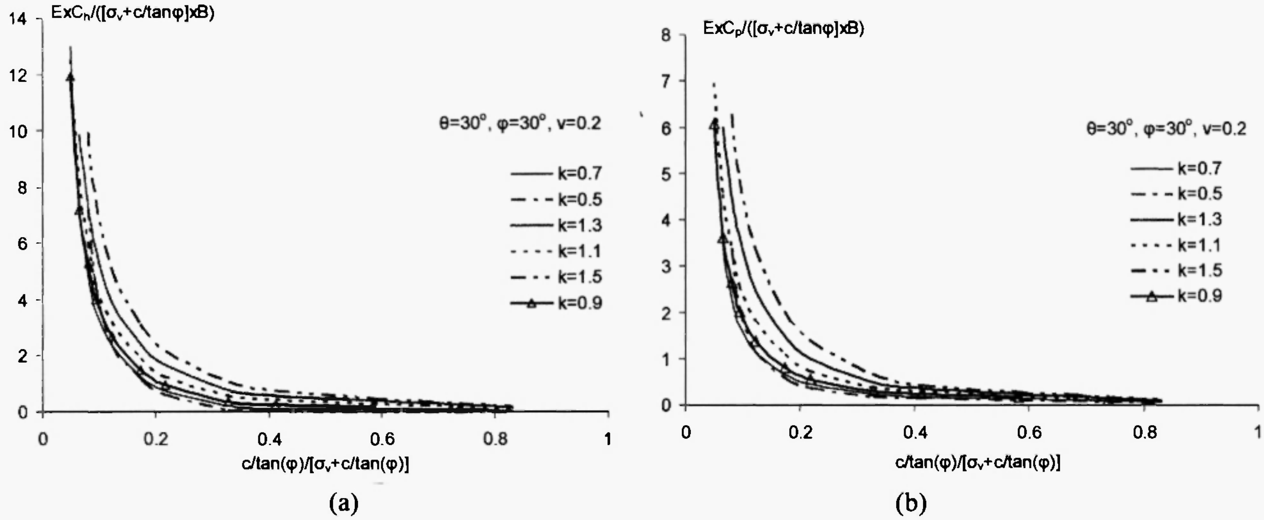


Fig. 5: Relation of normalized tunnel wall displacements for various values of the in situ stress ratio k for zero value of the support pressure σ_S.

The advantages of the above dimensionless expressions (7) for the tunnel wall displacements are obvious:

1. For all numerical elastoplastic computations one may use a single deformation modulus and tunnel span since their effect on tunnel wall displacements is explicit.
2. Back-analysis procedure is significantly simplified since various curves of tunnel wall displacements versus rock mass cohesion may be easily produced from a single curve of Fig. 5.

4. BACK-ANALYSIS RESULTS

The proposed back-analysis approach is as follows:

In a first approximation the tunnel convergence measurements at various distances from the tunnel face (e.g. Fig. 3) are back-analyzed by employing a linear elastic analytical solution for the semi-circular tunnel that takes into account the support pressure /5/. The elastic solution is valid close to the tunnel face but at large distances from it may be not true if the soil-rock mass exhibits damage and plasticity. Typical results of the above method pertaining to seven stations of the Tempi tunnel are illustrated in Fig. 6a. It is worth noting that the k-E experimental relations are linear which agrees with the results of /6/ even if a different approach was followed in that paper. This linear relation may be explained by the hypothesis that the deformation modulus is linearly-dependent on the *in situ* compressive stress, i.e. $E = \alpha + \beta\sigma$, where α is the initial deformation modulus and β is a positive proportionality constant. Another remarkable finding is the fact that the deformation modulus exhibits a size-effect, namely as the volume of the rock mass around the tunnel or the distance x from the tunnel face increases, the deformation modulus decreases (Fig. 6b).

In a second stage the values of E and k that have been found from the elastic tunnel model and correspond to a large distance from the face (i.e. far-field solution) are introduced into the elastoplastic solution in the form of nomograms (Fig. 5) and the unknown peak cohesion is estimated in such a way that the relative errors of the predicted and measured far-field tunnel wall displacements are minimum. For example, by employing the far-field values of k=1.1and E=7 GPa for the rock mass of station 10+150 (Fig. 6) it was found from the corresponding curve of Fig. 5 that c=2.2 MPa for the assumed values of $\varphi = 30^{o}$ and $\nu = 0.2$.

Hence, by virtue of the proposed 2-step hierarchical approach (elastic solution close to the tunnel face and elastoplastic solution for the far-field measurements) and some necessary assumptions regarding Poisson's ratio and peak friction angle, the *in situ* stress ratio, the deformation modulus and peak cohesion of a geological formation transected by the tunnel may be estimated by measuring the tunnel convergence at various distances from the tunnel face.

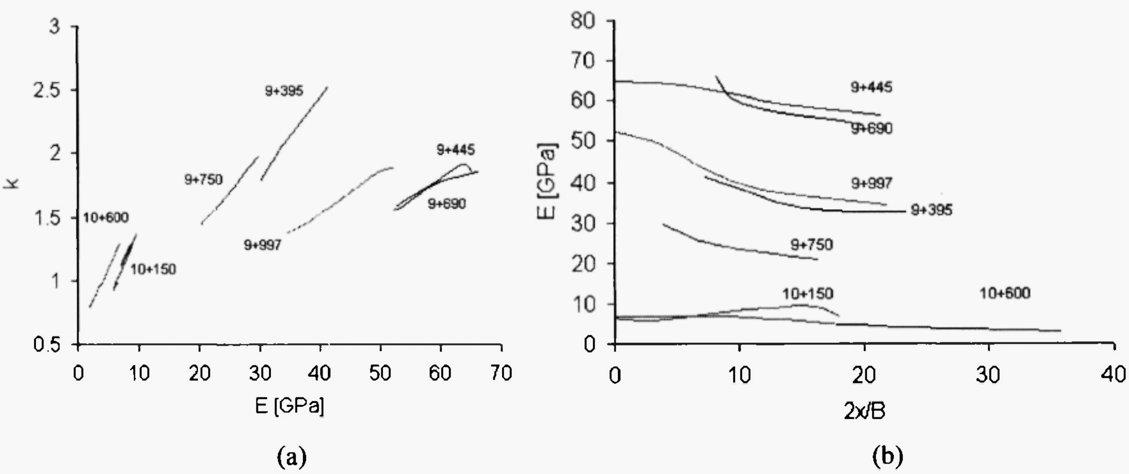


Fig. 6: (a) Dependence of the horizontal stress to vertical stress ratio on the deformation modulus (stress dependency) and (b) dependence of the deformation modulus on the relative distance from the face (size effect) of the rock mass intersected by the Tempi tunnel. These results have been found by employing the elastic tunnel solution /5/ for $\nu = 0.2$.

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