A Framework for the Analysis of Damaged Composite Materials Using the Homogenization Method

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ABSTRACT

The purpose of this work is to outline a systematic strategy for the analysis of damaged composite materials using the homogenization method. It is noted that this work does not provide specific models for the analysis of damaged composites. However, it provides guidelines with schematic diagrams based on a sound systematic strategy envisioned by the authors. A framework is built using special generalized equations that can be used by researchers to derive specific formulations and models based on their needs. It should be emphasized that the models derived must fit within the framework outlined in this work. Three types of homogenization methods are proposed to analyze damaged composite materials – overall homogenization, local homogenization, and mixed homogenization.

1. INTRODUCTION

The homogenization method has been used extensively in the literature to analyze undamaged composite materials /1-3/. Originally, the homogenization method was formulated to solve equations with differential operators with periodically and rapidly oscillating coefficients /4,5/. It was applied later to composite materials because of their periodic structure. However, the method does not seem to have been applied to damaged composite materials yet – this is exactly the subject of this work.

Continuum damage mechanics has been used extensively in the past to analyze both damaged metals and composite materials /6-10/. The formulation of damage mechanics is usually based on the concept of effective stress originally introduced by Kachanov /11/. In this work, it is proposed to use a new general method for damage mechanics based on homogenization in which the concept of effective stress would be utilized only as a special case.

The homogenization method can be used theoretically with modifications to analyze damaged materials with voids, cracks, microvoids, and microcracks. However, because the distribution of these defects is usually random with a nonperiodic structure, the homogenization method as it is available today cannot be used directly to solve these problems. The homogenization method needs to be modified in order to use it to solve problems with nonperiodic structures like damaged and fissured materials. In this work, it is assumed that such a method can be devised and will be available for use. It is noted that formulating the exact details of this modified homogenization method is a mathematical problem that is beyond the scope of this work.

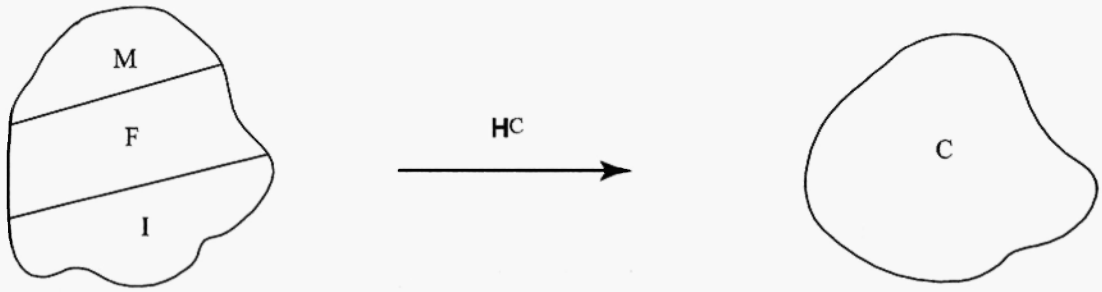
The concept of effective stress as used in continuum damage mechanics can be viewed as a special model or case of the modified homogenization method. Since a modified homogenization method has not yet been developed for randomly damaged media, damage mechanics can be used in its place in this work but only as a special case.

The homogenization method can be used in different ways to attack the problem of damaged composite materials. First, an overall homogenization method is outlined where the damage in the composite material is homogenized as one medium. This is followed by a local homogenization method where each of the composite constituents (matrix, fibers, and interface) is homogenized separately. Finally, a mixed homogenization method is outlined in which only one or two of the composite constituents is homogenized separately. It turns out that mixed homogenization can be performed in several ways – a total of six mixed homogenization approaches are proposed.

2. HOMOGENIZATION OF UNDAMAGED COMPOSITES

The method of homogenization of undamaged composites has been studied extensively in the literature /12-14/. The method is used to determine what are called effective material parameters for the composite. However, in this work the word "homogenized" material parameters will be used instead. The word "effective" material parameters will be reserved for parameters associated with the effective configuration of undamaged material (see the next section for more details).

The process of homogenization of undamaged composite media is illustrated in Figure 1. The homogenizing transformation H^C is used to represent the homogenization of the composite material where the superscript C is used to denote the composite material. The composite material is assumed to be composed of three constituents – matrix, fibers, and interface, denoted by the superscripts M, F, and I respectively. It should be noted that the composite material is exactly the union of M, F, and I, that is, $C_0 = M \cup F \cup I$, where C_0 denotes the undamaged composite configuration. In Figure 1-a, the undamaged composite configuration is shown where all three constituents are assumed to be undamaged. The homogenized composite configuration, denoted by C, is shown in Figure 1-b.



(a) Undamaged Composite Configuration

(b) Homogenized Composite Configuration

Fig. 1: Homogenization of Undamaged Composites

Next, the following generalized framework relation can be obviously written:

$$C = H^{C}(M,F,I) \tag{1}$$

where H^C represents a general homogenization procedure from the constituents M, F, and I to C. The nature of H^C and the exact details are available in the literature on homogenization of undamaged periodic structures /1-3/. The interested researchers may pick the method of homogenization of their choosing to insert in this part of the formulation. It is again emphasized that no specific model will be utilized in this work – only a general framework and guidelines will be provided. It should be emphasized that the function H^C is always a function of three arguments in this work.

In equation (1), homogenized mechanical quantities may be used for C like the effective elastic modulus, while constituent related quantities may be used for M, F, and I such as the matrix elastic modulus, fiber elastic modulus, and interface elastic modulus. Other quantities related to the constituents may also be used such as the matrix stress, matrix strain, fiber stress, fiber strain, interface stress, and interface strain.

3. HOMOGENIZATION OF DAMAGED METALS

The homogenization method can theoretically be modified to be applied to damaged metals with inclusions like voids, cracks, microvoids, and microcracks. However, the authors are not aware of any available works in the literature that have used homogenization for damaged metals. It is assumed in this work that such a modified procedure is or will be available to the researcher. In case that such a procedure is not developed mathematically, then the concept of effective stress within the framework of continuum damage mechanics can be used instead, but only as a special case. Actually, damage mechanics as applied through the effective stress concept may be regarded as some form of homogenization of the damaged metal. In this case, the effective undamaged configuration will be identical to the homogenized configuration. It should be mentioned that Attouch and Murat /15/ performed a study on the homogenization of fissured elastic materials. For the damage mechanics literature, see references /16-30/.

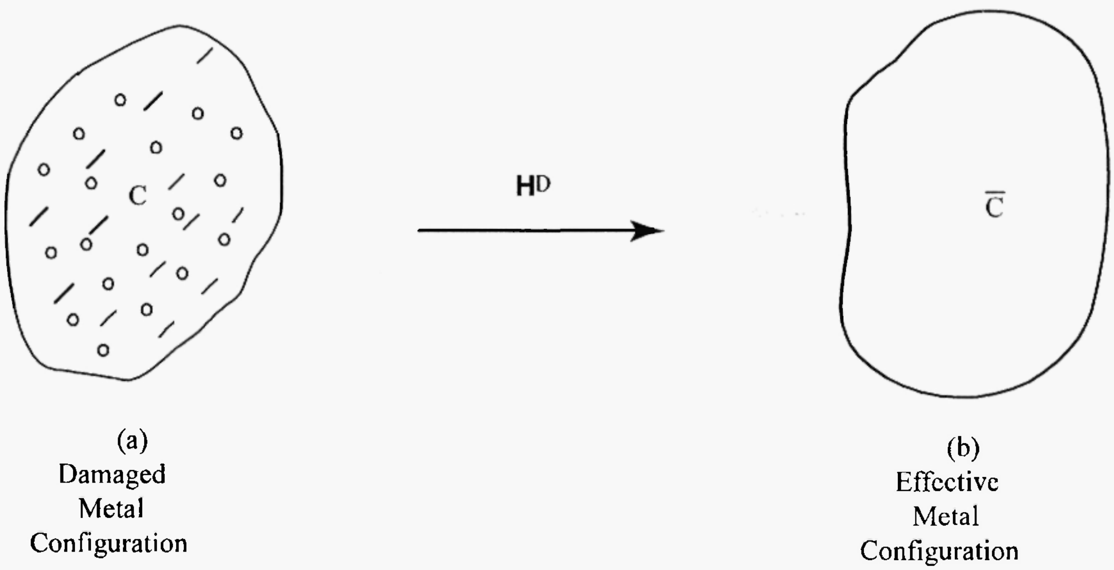


Fig. 2: Homogenization of Damaged Metals

The process of homogenization of damaged metals is illustrated in Figure 2. The homogenizing transformation H^D is used to represent the homogenization of the damaged metal where the superscript D is used to denote the damaged metal configuration. The damaged metal is assumed to include voids, cracks and other types of inhomogenities. In Figure 2-a, the damaged metal configuration, denoted by C, is shown. The homogenized metal configuration, denoted by $\bar{C}$, is shown in Figure 2-b.

Next, the following generalized framework relation can obviously be written:

$$\bar{C} = H^D(C) \tag{2}$$

where H^D represents a general homogenization procedure from the damage configuration C to the effective undamaged configuration $\bar{C}$. The nature of H^D and the exact details may not be available in the literature. However, a continuum damage mechanics model may be used here where the fourth-order damage effect tensor M can be used for H^D in equation (2) – see reference /6/. It is again emphasized that no specific model will be utilized in this work – only a general framework and guidelines will be provided. It is also emphasized that the function H^D is always a function of one argument in this work.

In equation (2), mechanical quantities related to the damaged metal may be used for C like the elastic modulus, while effective quantities may be used for $\bar{C}$ like the effective elastic modulus. Other quantities related to the damaged and effective configurations may be used also like the stress tensor and strain tensor for C, and the effective stress tensor and the effective strain tensor for $\bar{C}$. For the general case of anisotropic damage, tensors will be used for the stresses and strains – therefore, the transformation H^D will be a tensor-valued function.

4. HOMOGENIZATION OF DAMAGED COMPOSITES

The application of the homogenization method to damaged composite materials has not been systematically investigated in the literature. Voyiadjis and Deliktas /31/ and Voyiadjis and Park /32/ investigated the coupling of damage and inelastic deformation in metal matrix composites but without using the homogenization method. It is the purpose of this work to provide a general framework in which damaged composites can be analyzed by homogenization through the use of a consistent and systematic procedure. The authors have concluded that this process may be achieved through several approaches that may be followed. Actually, in this case, a form of double homogenization may be needed – the first homogenization for the damaged configuration followed by a second homogenization for the composite constituents. The order of the two homogenizations may also be reversed thus giving rise to another model that should theoretically be equivalent to the first model. In this section, this order of homogenizations is investigated in detail where three methods are outlined in the following subsections.

4.1 Overall Homogenization Method

Analysis of damaged composite materials using an overall approach has been studied before in the literature /33/ but not using homogenization. The overall homogenization method is the simplest method proposed to analyze damaged composite materials. It utilizes a double homogenization technique as illustrated in Figure 3. The term "double homogenization" is used to mean that two homogenizing transformations are used in sequence in the model. Figure 3-a shows the damaged composite configuration C_0, which is composed of the three constituents M, F, and I, denoting the matrix, fibers, and interface respectively. Figure 3-b shows the damaged homogenized composite configuration C, while Figure 3-c shows the effective homogenized composite configuration $\bar{C}$.

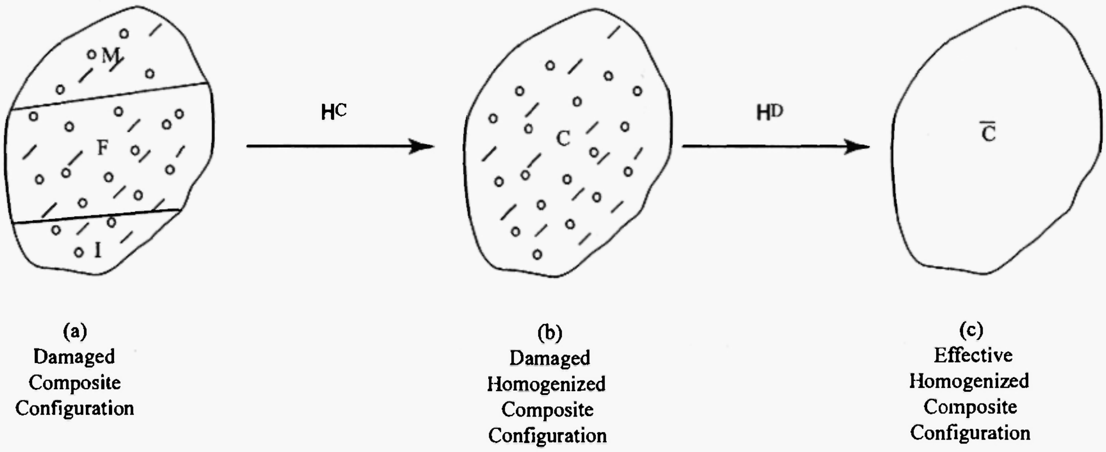


Fig. 3: Overall Homogenization Method

The overall homogenization of the damaged composite configuration is performed in two steps in sequence as follows:

1. The homogenizing transformation H^C is used to obtain the damaged homogenized composite configuration C. This step can be represented using the following framework relation.

$$C = H^C(M, F, I) \tag{3}$$

It is noted that equation (3) looks identical to equation (1) except for the fact that both configurations in equation (1) are undamaged (see Figure 1) while both configurations in equation (3) are damaged (see Figure 3).

2. The homogenizing transformation H^D is then used to obtain the effective homogenized composite configuration $\bar{C}$ from C. This step can be represented using the following framework relation.

$$\bar{C} = H^D(C) \tag{4}$$

It is noted that equation (4) is exactly identical to equation (2).

Substituting equation (3) into equation (4), we obtain the general transformation for the overall homogenization method as follows.

$$\bar{C} = H^D(H^C(M, F, I)) \tag{5}$$

In equation (5), effective homogenized mechanical quantities may be used for $\bar{C}$ like the effective elastic modulus and the effective elasto-plastic modulus, while constituent related quantities may be used for M, F, and I like the matrix elastic or elasto-plastic modulus, fiber elastic or elasto-plastic modulus, and interface elastic or elasto-plastic modulus. Other quantities related to the constituents may be used also like the matrix stress, matrix strain, fiber stress, fiber strain, interface stress, and interface strain. In this method, the complete composite material is homogenized as one medium without distinguishing between the matrix, fibers, and interface. This is especially true of the second step outlined above.

4.2 Local Homogenization Method

Analysis of damaged composite materials using a local approach has been studied before in the literature /34/ but not using homogenization. The local homogenization method is the second simplest method proposed to analyze damaged composite materials. It utilizes a quadruple homogenization technique as illustrated in Figure 4. The term "quadruple homogenization" is used to mean that four homogenizing transformations are used in the model – three in parallel and one in sequence. Figure 4-a shows the damaged composite configuration C_0, which is composed of the three constituents M, F, and I, denoting the matrix, fibers, and interface respectively. Figure 4-b shows the effective composite configuration $\bar{C}_0$, which is composed of the three constituents $\bar{M}$, $\bar{F}$, and $\bar{I}$, denoting the effective matrix, fiber, and interface configurations, respectively. Finally, Figure 4-c shows the effective homogenized composite configuration $\bar{C}$.

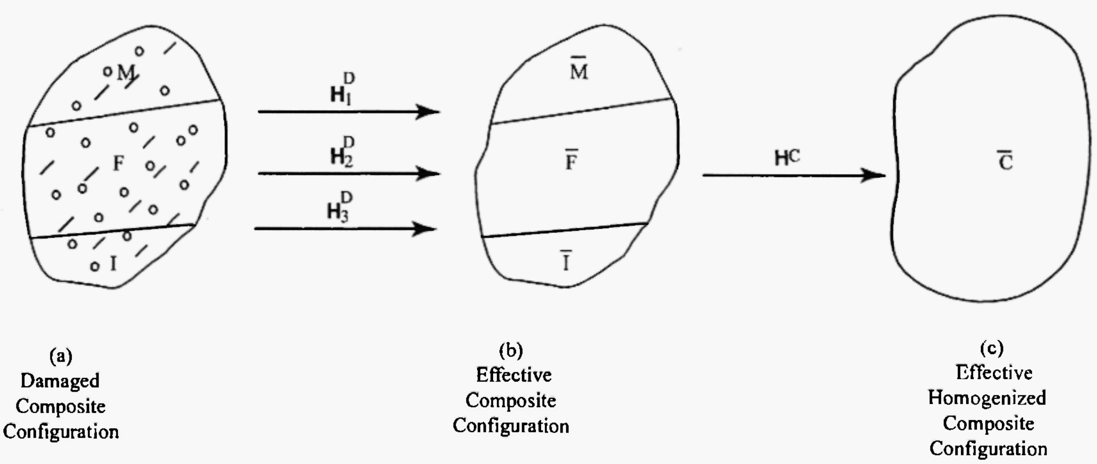


Fig. 4: Local Homogenization Method

The local homogenization of the damaged composite configuration is performed in two steps in sequence as follows:

1. The three homogenizing transformations H_1^D, H_2^D, and H_3^D are used in parallel to obtain the effective composite configuration $\bar{C}_0$. This step can be represented using the following three framework relations.

$$\bar{M} = H_1^D(M)$$
$$\bar{F} = H_2^D(F) \qquad (6)$$
$$\bar{I} = H_3^D(I)$$

   It is noted that equations (6) are in fact identical to equation (2) except for the fact that in this step, each constituent configuration is homogenized separately. This is the reason why we need three homogenizing transformations working in parallel – one for each of the three constituents.

2. The homogenizing transformation H^C is then used to obtain the effective homogenized composite configuration $\bar{C}$ from $\bar{C}_0$. This step can be represented using the following framework relation.

$$\bar{C} = H^C(\bar{M}, \bar{F}, \bar{I}) \qquad (7)$$

   It is noted that equation (7) is similar to equation (1) although effective quantities are used in equation (7).

Substituting equations (6) into equation (7), we obtain the general transformation for the local homogenization method as follows:

$$\bar{C} = H^C(H_1^D(M), H_2^D(F), H_3^D(I)) \qquad (8)$$

In equation (8), effective homogenized mechanical quantities may be used for $\bar{C}$ like the effective elastic modulus and the effective elasto-plastic modulus, while constituent related quantities may be used for *M*, *F*, and *I* like the matrix elastic or elasto-plastic modulus, fiber elastic or elasto-plastic modulus, and interface elastic or elasto-plastic modulus. Other quantities related to the constituents may also be used like the matrix stress, matrix strain, fiber stress, fiber strain, interface stress, and interface strain. This type of homogenization is important particularly in composites where damage may be primarily isolated at the interface between the metal matrix and the ceramic fibers. In that case identity tensors may be used to represent the damage in the metal matrix and the ceramic fiber.

In comparing equations (5) and (8) for the overall and local homogenization methods, respectively, caution must be employed so as not to expect similar results. Theoretically, we should expect equations (5) and (8) to yield identical results. However, the complexities of the models to be employed may prevent us from proving the equivalence of these two methods analytically – although the final numerical results should be similar.

4.3 Mixed Homogenization Methods

One may choose to initially homogenize one or two constituents only using one or two of the three homogenizing transformations H_1^D, H_2^D, and H_3^D, and follow this first step with reaching an effective homogenized composite configuration, leaving the remaining constituent(s) to be homogenized in a last third step. These three steps constitute what is called a mixed homogenization method. This process can be performed in six different ways, thus leading effectively to six different mixed homogenization methods. These mixed methods are illustrated schematically in Figures (5) to (10). One may wish to utilize a mixed method of homogenization if one wants to isolate the effect of damage in one of the constituents or in two constituents operating in parallel. In the remaining part of this section, details of the six methods are outlined.

The first mixed homogenization method is illustrated in Figure 5. In this method, four homogenizing transformations are used in three steps. The matrix and fibers are homogenized in parallel in the first step, while the interface is homogenized separately in the third step. Figure 5-a shows the damaged composite configuration while Figure 5-b shows the effective composite configuration with interfacial damage. Figure 5-c shows the effective homogenized composite configuration with interfacial damage while Figure 5-d shows the effective homogenized composite configuration.

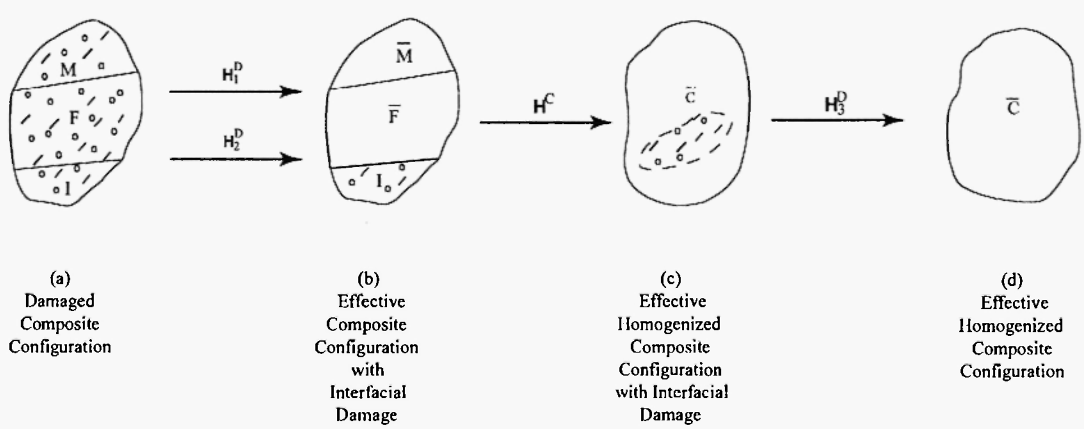


Fig. 5: First Mixed Homogenization Method

The first mixed homogenization of the damaged composite configuration is performed in three steps in sequence as follows:

1. The two homogenizing transformations H_1^D and H_2^D are used in parallel to obtain the effective composite configuration with interfacial damage. This step can be represented using the following two framework relations.

$$\bar{M} = H_1^D(M)$$
$$\bar{F} = H_2^D(F) \quad (9)$$

2. The homogenizing transformation H^C is then used to obtain the effective homogenized composite configuration with interfacial damage $\tilde{C}$. This step can be represented using the following framework relation.

$$\tilde{C} = H^C(\bar{M},\bar{F},I) \quad (10)$$

3. The homogenizing transformation H_3^D is finally used to obtain the effective homogenized composite configuration $\bar{C}$ from $\tilde{C}$. This step can be represented using the following framework relation.

$$\bar{C} = H_3^D(\tilde{C}) \quad (11)$$

Substituting equations (9) into equation (10), and then substituting the resulting equation into equation (11), we obtain the general transformation for the first mixed homogenization method as follows.

$$\bar{C} = H_3^D(H^C(H_1^D(M),H_2^D(F),I)) \quad (12)$$

It is noted that in this method the interfacial damage is isolated to be dealt with in the last step.

The second mixed homogenization method is illustrated in Figure 6. In this method, three homogenizing transformations are used in three steps. The interface is homogenized in the first step, while the matrix and fibers are homogenized in parallel in the third step. Figure 6-a shows the damaged composite configuration while Figure 6-b shows the effective composite configuration with matrix and fiber damage. Figure 6-c shows the effective homogenized composite configuration with matrix and fiber damage while Figure 6-d shows the effective homogenized composite configuration.

The second mixed homogenization of the damaged composite configuration is performed in three steps in sequence as follows:

1. The homogenizing transformation H_3^D is used alone to obtain the effective composite configuration with matrix and fiber damage. This step can be represented using the following framework relation.

$$\bar{I} = H_3^D(I) \quad (13)$$

2. The homogenizing transformation H^C is then used to obtain the effective homogenized composite configuration with matrix and fiber damage $\tilde{C}$. This step can be represented using the following framework relation.

$$\tilde{C} = H^C(M, F, \bar{I}) \tag{14}$$

3. The homogenizing transformation $H_{1,2}^D$ is finally used to obtain the effective homogenized composite configuration $\bar{C}$ from $\tilde{C}$. This step can be represented using the following framework relation.

$$\bar{C} = H_{1,2}^D(\tilde{C}) \tag{15}$$

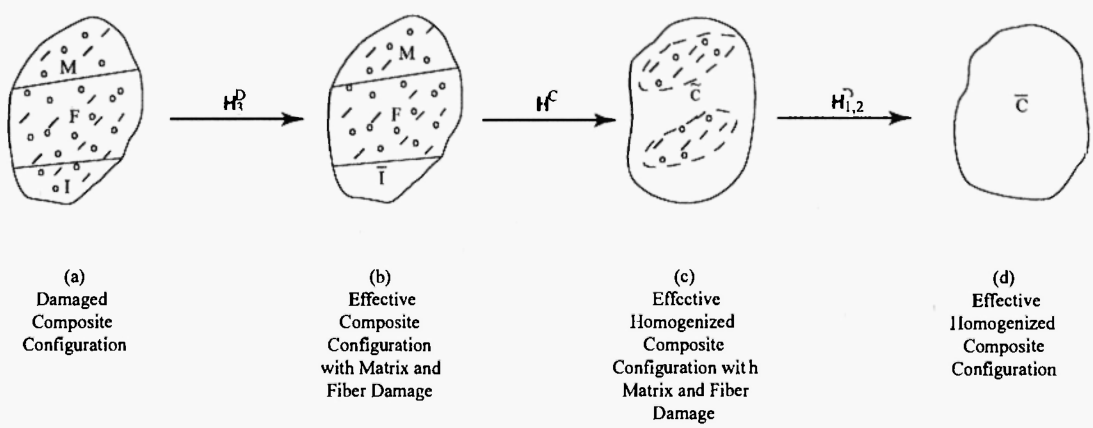


Fig. 6: Second Mixed Homogenization Method

Substituting equation (13) into equation (14), and then substituting the resulting equation into equation (15), we obtain the general transformation for the second mixed homogenization method as follows.

$$\bar{C} = H_{1,2}^D(H^C(M, F, H_3^D(I))) \tag{16}$$

It is noted that in this method there is a coupling between the damage in the matrix and the damage in the fibers, while the interfacial damage is isolated and dealt with in the first step.

The third mixed homogenization method is illustrated in Figure 7. In this method, four homogenizing transformations are used in three steps. The interface and fibers are homogenized in parallel in the first step, while the matrix is homogenized separately in the third step. Figure 7-a shows the damaged composite configuration while Figure 7-b shows the effective composite configuration with matrix damage. Figure 7-c shows the effective homogenized composite configuration with matrix damage while Figure 7-d shows the effective homogenized composite configuration.

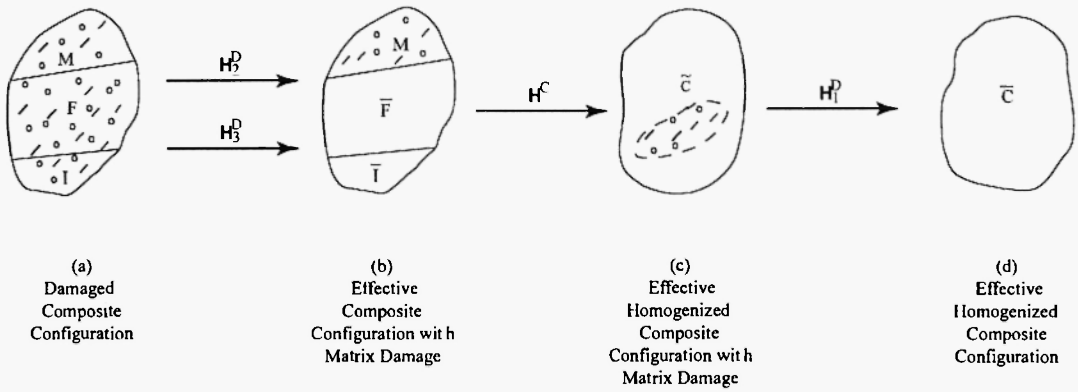


Fig. 7: Third Mixed Homogenization Method

The third mixed homogenization of the damaged composite configuration is performed in three steps in sequence as follows:

1. The two homogenizing transformations H_2^D and H_3^D are used in parallel to obtain the effective composite configuration with matrix damage. This step can be represented using the following two framework relations.

$$\bar{F} = H_2^D(F)$$
$$\bar{I} = H_3^D(I) \tag{17}$$

2. The homogenizing transformation H^C is then used to obtain the effective homogenized composite configuration with matrix damage $\tilde{C}$. This step can be represented using the following framework relation.

$$\tilde{C} = H^C(M, \bar{F}, \bar{I}) \tag{18}$$

3. The homogenizing transformation H_1^D is finally used to obtain the effective homogenized composite configuration $\bar{C}$ from $\tilde{C}$. This step can be represented using the following framework relation.

$$\bar{C} = H_1^D(\tilde{C}) \tag{19}$$

Substituting equations (17) into equation (18), and then substituting the resulting equation into equation (19), we obtain the general transformation for the third mixed homogenization method as follows.

$$\bar{C} = H_1^D(H^C(M, H_2^D(F), H_3^D(I))) \quad (20)$$

It is noted that in this method the matrix damage is isolated to be dealt with in the last step.

The fourth mixed homogenization method is illustrated in Figure 8. In this method, three homogenizing transformations are used in three steps. The matrix is homogenized in the first step, while the interface and fibers are homogenized in parallel in the third step. Figure 8-a shows the damaged composite configuration while Figure 8-b shows the effective composite configuration with interfacial and fiber damage. Figure 8-c shows the effective homogenized composite configuration with interfacial and fiber damage while Figure 8-d shows the effective homogenized composite configuration.

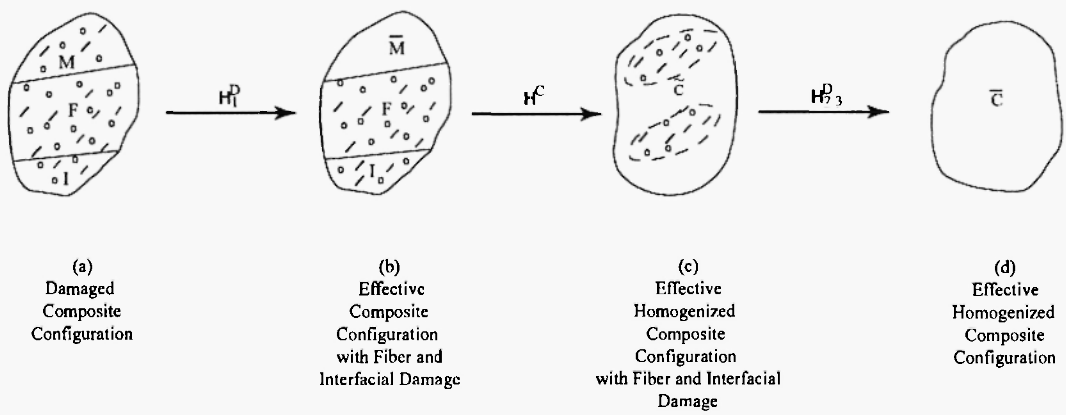


Fig. 8: Fourth Mixed Homogenization Method

The fourth mixed homogenization of the damaged composite configuration is performed in three steps in sequence as follows:

1. The homogenizing transformation H_1^D is used alone to obtain the effective composite configuration with interfacial and fiber damage. This step can be represented using the following framework relation.

$$\bar{M} = H_1^D(M) \quad (21)$$

2. The homogenizing transformation H^C is then used to obtain the effective homogenized composite configuration with interfacial and fiber damage $\tilde{C}$. This step can be represented using the following framework relation.

$$\tilde{C} = H^C(\bar{M}, F, I) \tag{22}$$

3. The homogenizing transformation $H_{2,3}^D$ is finally used to obtain the effective homogenized composite configuration $\bar{C}$ from $\tilde{C}$. This step can be represented using the following framework relation.

$$\bar{C} = H_{2,3}^D(\tilde{C}) \tag{23}$$

Substituting equation (21) into equation (22), and then substituting the resulting equation into equation (23), we obtain the general transformation for the fourth mixed homogenization method as follows.

$$\bar{C} = H_{2,3}^D(H^C(H_1^D(M), F, I)) \tag{24}$$

It is noted that in this method there is a coupling between the damage in the interface and the damage in the fibers, while the matrix damage is isolated and dealt with in the first step.

The fifth mixed homogenization method is illustrated in Figure 9. In this method, four homogenizing transformations are used in three steps. The interface and matrix are homogenized in parallel in the first step, while the fibers are homogenized separately in the third step. Figure 9-a shows the damaged composite configuration while Figure 9-b shows the effective composite configuration with fiber damage. Figure 9-c shows the effective homogenized composite configuration with fiber damage while Figure 9-d shows the effective homogenized composite configuration.

The fifth mixed homogenization of the damaged composite configuration is performed in three steps in sequence as follows:

1. The two homogenizing transformations H_1^D and H_3^D are used in parallel to obtain the effective composite configuration with fiber damage. This step can be represented using the following two framework relations.

$$\begin{aligned} \bar{M} &= H_1^D(M) \\ \bar{I} &= H_3^D(I) \end{aligned} \tag{25}$$

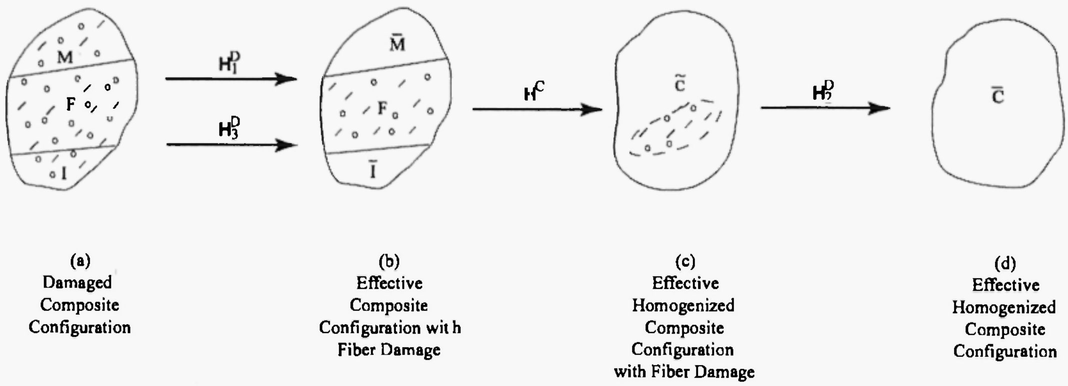


Fig. 9: Fifth Mixed Homogenization Method

2. The homogenizing transformation H^C is then used to obtain the effective homogenized composite configuration with fiber damage $\tilde{C}$. This step can be represented using the following framework relation.

$$\tilde{C} = H^C(\bar{M}, F, \bar{I}) \tag{26}$$

3. The homogenizing transformation H_2^D is finally used to obtain the effective homogenized composite configuration $\bar{C}$ from $\tilde{C}$. This step can be represented using the following framework relation.

$$\bar{C} = H_2^D(\tilde{C}) \tag{27}$$

Substituting equations (25) into equation (26), and then substituting the resulting equation into equation (27), we obtain the general transformation for the fifth mixed homogenization method as follows.

$$\bar{C} = H_2^D(H^C(H_1^D(M), F, H_3^D(I))) \tag{28}$$

It is noted that in this method the fiber damage is isolated to be dealt with in the last step.

The sixth mixed homogenization method is illustrated in Figure 10. In this method, three homogenizing transformations are used in three steps. The fibers are homogenized in the first step, while the interface and matrix are homogenized in parallel in the third step. Figure 10-a shows the damaged composite configuration while Figure 10-b shows the effective composite configuration with interfacial and matrix damage. Figure 10-c shows the effective homogenized composite configuration with interfacial and matrix damage while Figure 10-d shows the effective homogenized composite configuration.

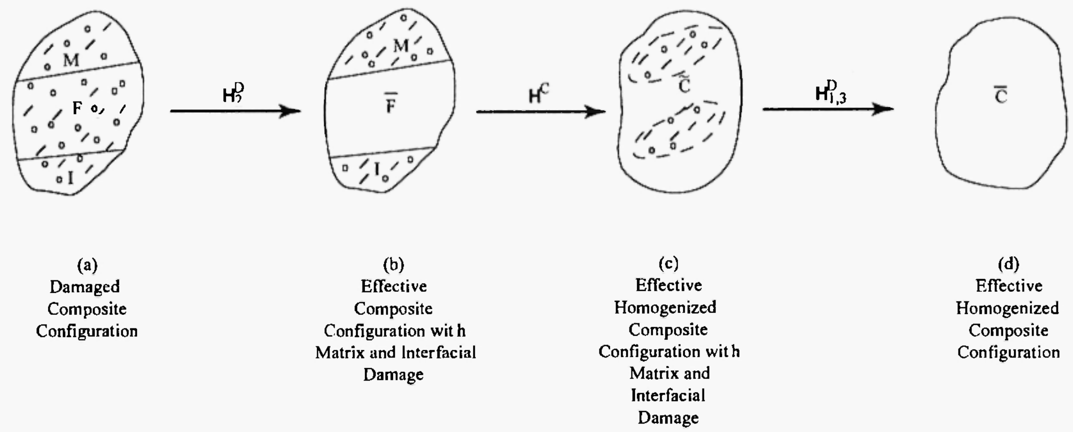


Fig. 10: Sixth Mixed Homogenization Method

The sixth mixed homogenization of the damaged composite configuration is performed in three steps in sequence as follows:

1. The homogenizing transformation H_2^D is used alone to obtain the effective composite configuration with interfacial and matrix damage. This step can be represented using the following framework relation.

$$\bar{F} = H_2^D(F) \tag{29}$$

2. The homogenizing transformation H^C is then used to obtain the effective homogenized composite configuration with interfacial and matrix damage $\tilde{C}$. This step can be represented using the following framework relation.

$$\tilde{C} = H^C(M, \bar{F}, I) \tag{30}$$

3. The homogenizing transformation $H_{1,3}^D$ is finally used to obtain the effective homogenized composite configuration $\bar{C}$ from $\tilde{C}$. This step can be represented using the following framework relation.

$$\bar{C} = H_{1,3}^D(\tilde{C}) \tag{31}$$

Substituting equation (29) into equation (30), and then substituting the resulting equation into equation (31), we obtain the general transformation for the sixth mixed homogenization method as follows.

$$\bar{C} = H_{1,3}^{D}(H^{C}(M, H_{2}^{D}(F), I)) \tag{32}$$

It is noted that in this method there is a coupling between the damage in the interface and the damage in the matrix, while the fiber damage is isolated and dealt with in the first step.

In comparing equations (12), (16), (20), (24), (28), and (32) for the six mixed homogenization methods, caution must be employed so as not to expect similar results. Theoretically, we should expect these six equations to yield identical results. However, the complexities of the models to be employed may prevent us from proving the equivalence of these six methods analytically – although the final numerical results should be similar.

5. CONCLUSIONS

The authors have presented a systematic strategy within a consistent framework for the analysis of damaged composite materials using the homogenization method. A total of eight homogenization methods are proposed including an overall homogenization method, a local homogenization method, and six mixed homogenization methods. For each method, framework equations are derived for the general characteristics of the method. The authors provide no specific models to be used within these methods – it is left for the interested researcher to fill in the models of his or her choice. Ample flexibility is provided especially within the mixed methods so that a specific damage mechanism related to one or two of the composite constituents can be isolated and investigated separately. Also, coupling between two damage mechanisms is provided for three of the six mixed methods. It is hoped that this work will prove to be valuable for active researchers engaged in modeling of damage in composite materials as it provides guidelines and strategy for this very important topic.

6. REFERENCES

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