Development of Control Charts Using Fuzzy Sets

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ABSTRACT

Quality characteristics are made up of different types of sets and can be classified as variables from the instrument measurements based on infinite natural sets, and attributes for nonconforming descriptions based on the binary or natural sets. However, there is also a type of quality characteristics – linguistic – which is used in this paper for the subjective estimations of welding defects of Liquefied Petroleum Gas (LPG) cylinders. The goal of this paper is to show how a fuzzy set theory can be applied to well known powerful techniques of the quality control, using linguistic data for the welding defects.

Keywords: fuzzy set theory, Pareto analysis, cause and effect diagram, control chart, design of experiment, welding defects.

1. INTRODUCTION

Complex quality evaluations of materials, products, processes and services are carried out by the use of quality characteristics. These characteristics are established according to the customer's requirements and have specification limits and target values. The formalisation of quality characteristics makes it possible to apply powerful tools of quality improvement such as Pareto analysis, design of experiments, control charts and capability studies.

With the success of automatic control and of expert systems an endorsement of fuzzy concept in the technology can be witnessed. Lofti A Zadeh /1/ presented a substantive departure from the conventional quality technique for system analysis. This approach has three main distinguishing features :

a) Use of so called linguistic variables in place of or in addition to the numerical variables.
b) Characterisation of the simple relations between variables by the fuzzy conditional statements.
c) Characterisation of complex relation by fuzzy algorithms.

By relying on the use of the linguistic variables and fuzzy algorithms this approach is an approximate and yet effective means of describing the behaviour of the systems which are too complex or are too ill defined for precise mathematical analysis.

Jyh-Hong Wong and Tzvi Raz /2/ studied two approaches to construct control chart for linguistic data. The authors claim that the linguistic data can provide more information than the binary classification used in the control charts of attributes.

E.A. Glushkovsky and R.A. Florescu /3/ successfully formalised linguistic quality characteristics on the basis of the theory of fuzzy sets. They suggested a step which ensured the successful application of the fuzzy set theory as a powerful quality control tool. M. Laviolette *et al.* /4/ discussed the application of fuzzy set in engineering.

Considering the aforesaid state of art literature the authors in the present paper put forward a successful formalisation for the 'Quality Control of welded components of LPG cylinders' using fuzzy logic. Such an approach for welded components cannot be found in the existing literature.

2. FUZZY ANALYSIS AND COMPUTATION

It may be noted that welded components may have many types of defects. Investigations on the defects of welded components reveal that the welding defects like uneven and intermittent gap during welding, welding patches, lump and improper welding occur very frequently. The methods used in this investigation to detect the said defects are visual tests, dye test, magnetic particle inspection and ultrasonic testing. The results were interpreted with fuzzy logic in order to compute the average loss, plot the Pareto chart for quality loss and to draw 'd' bar control chart for linguistic quality characteristics of the welded Liquefied Petroleum Gas cylinder made of medium carbon steel. In order to formalise linguistic evaluations based on fuzzy sets, a universal set U is defined and membership functions μA, μB, μC, ------$:U \rightarrow (0,1)$ are determined corresponding to different fuzzy terms: A, B, C ----- in combination with:

(a) modifiers

$$\text{<very>}A = A^2 = \sum \mu A(u)^2/u, u \in U \quad (1)$$

$$\text{<not>}\ A = \neg A = \sum [1 - \mu A\ (u)]\ /u \quad (2)$$

(b) union

$$A\text{<or>}B = A \cup B == \sum \max[\mu A(u), \mu B(u)]\ /u \quad (3)$$

(c) intersection

$$A\text{<and>}B = A \cap B = \sum \min[\mu A\ (u), \mu B(u)]\ /u \quad (4)$$

This makes it possible to define linguistic quality characteristics quite flexibly.

3. PARETO ANALYSIS

The initial step of any quality improvement is the identification of the problem.

The main tool applied in this case is Pareto Analysis. The transformation of quality characteristics into cost lost is most widely used today. For variables it is carried out with the help of loss functions. The loss function concept can be applied to the linguistic data formalised by the membership functions. In this paper, the average loss is determined as:

$$\text{Average loss} = \sum_{u \varepsilon U} \left[k * (u - u_{TARG})^2\right] * \mu(u) \tag{5}$$

Where $\mu(u)$ is the average membership functions which is a peculiar analogue to histogram, u_{TARG} is the target value and k is the proportional (scale) coefficient. Loss calculations and the average loss for five different component types (L1 – L5) are given in Table I, where k=1, and u_{TARG} =5. Fig. 1(a), 1(b) and 1(c) depicts the relationship between the degree of membership and quality.

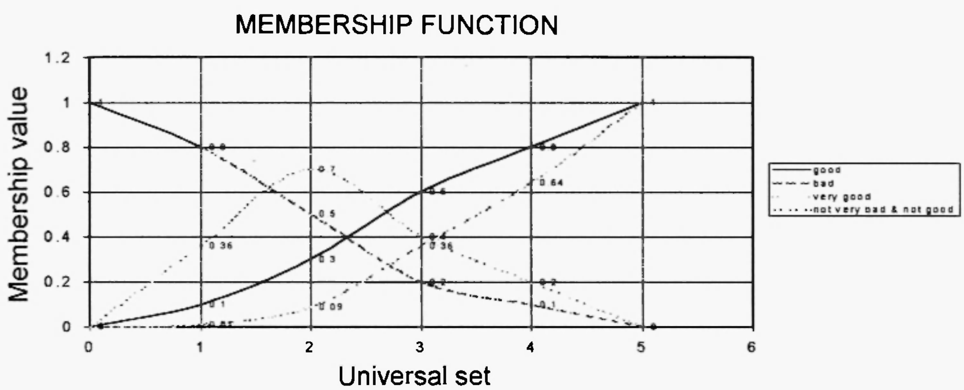

Fig.1 (a)

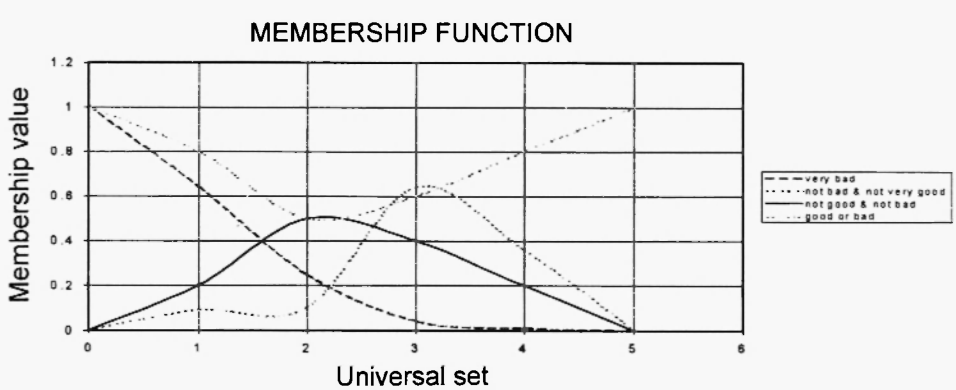

Fig. 1(b)

Fig. 1 (continued)

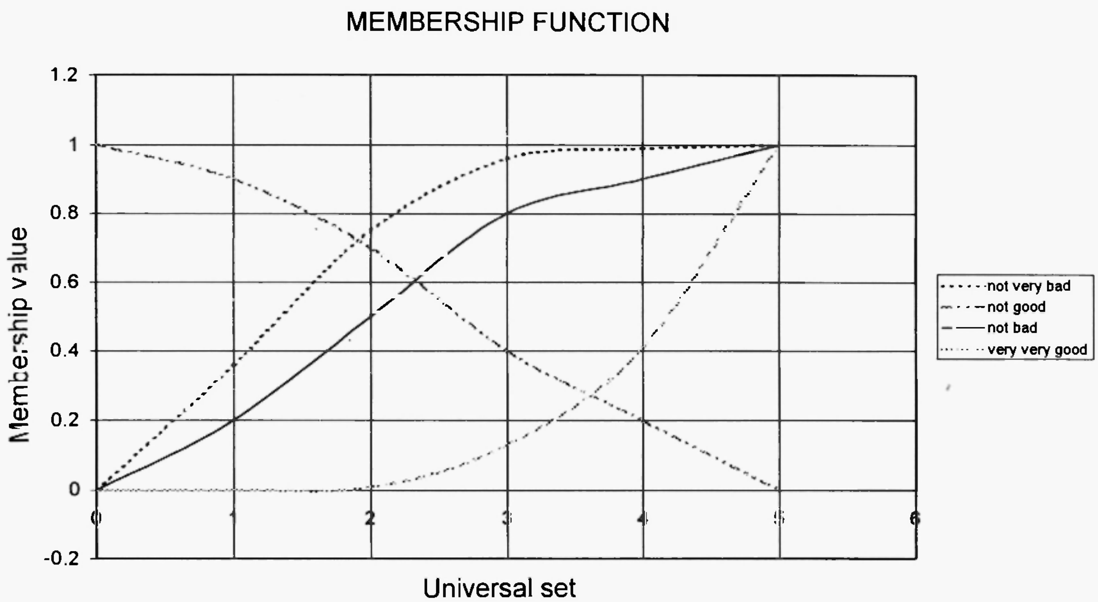

Figure 1(c)

Table 1

Loss Calculation

COMPONENT TYPE	OBSERVED RESULTS	AVERAGE MEMBERSHIP FUNCTION	AVERAGE LOSS
L1	10% VERYGOOD 84.45% GOOD 3.2% NOT BAD AND NOT VERY GOOD 2.35% BAD	0.0235/0+0.10713/1+0.27442/2+ 0.63188/3+0.75347/4+0.9445/5	8.05235
L2	97.3% GOOD 0.5 % BAD 2.2% NOT BAD AND NOT VERY GOOD	0.005/0+0.10328/1+0.29462/2+ 0.598888/3+0.78682/4+0.973/5	7.6114
L3	98.01% GOOD 0.19% BAD 1.8% NOT BAD AND NOT VERY GOOD	0.0019/0+0.10115/1+0.29516/2+ 0.59996/3+0.79075/4+0.9810/5	7.51293
L4	97.89% GOOD 0.11% BAD 2.0% NOT BAD AND NOT VERY GOOD	0.0011/0+0.10057/1+0.29442/2+ 0.60036/3+0.79043/4+0.9789/5	7.47827
L5	98.23% GOOD 0.57% BAD 1.2% NOT BAD AND NOT VERY GOOD	0.0057/0+0.10387/1+0.29766/2+ 0.5982/3+0.79073/4+0.9823/5	7.66689

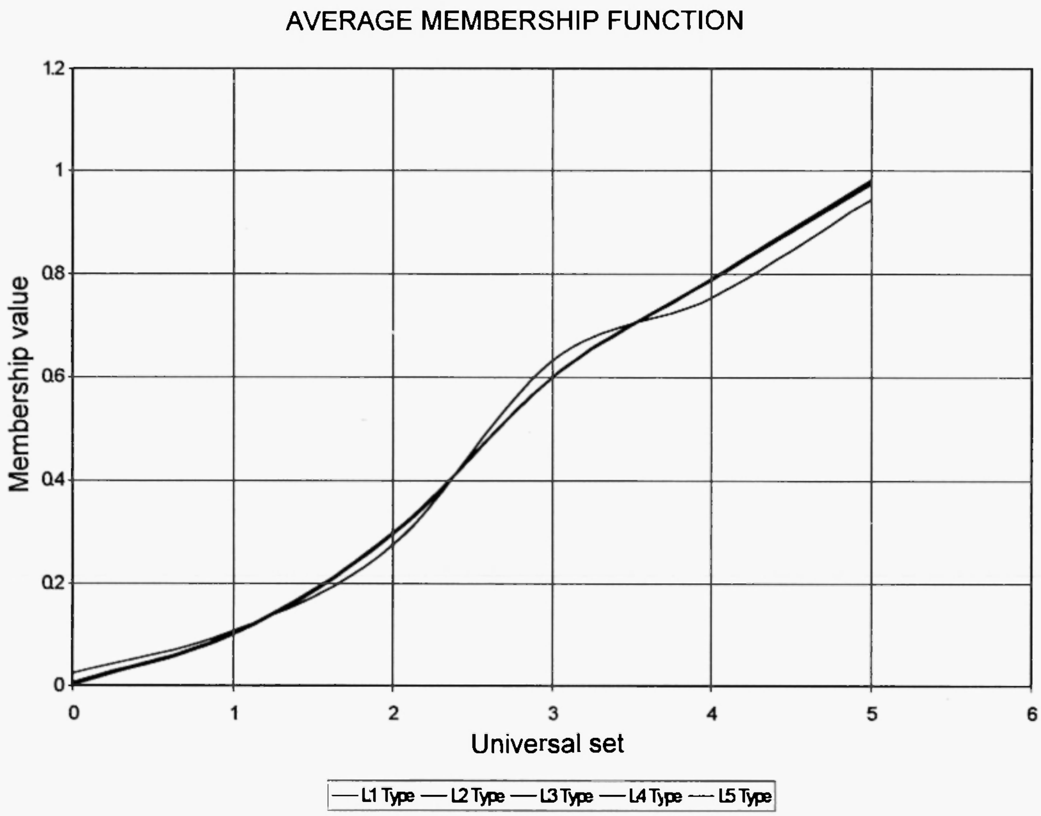

Figure 2

Fig. 2 represents computed average membership values with respect to universal sets. There are various ways to get reasonable membership functions but the probabilities cannot be confused with the degree of membership in a fuzzy set. Membership functions are not probability distribution. In the preliminary stage of the quality control of the welded LPG cylinder is often imprecise.

Method of operation of the fuzzy numbers (when the fuzzy set is of a certain restricted type it is called fuzzy number) are extended addition and extended multiplication in order to distinguish them from operations on real numbers. Fig.3 depicts the Pareto chart for quality loss. The Pareto chart clearly indicates that the average losses of the component type L1 are the highest due to the presence of subsurface defects.

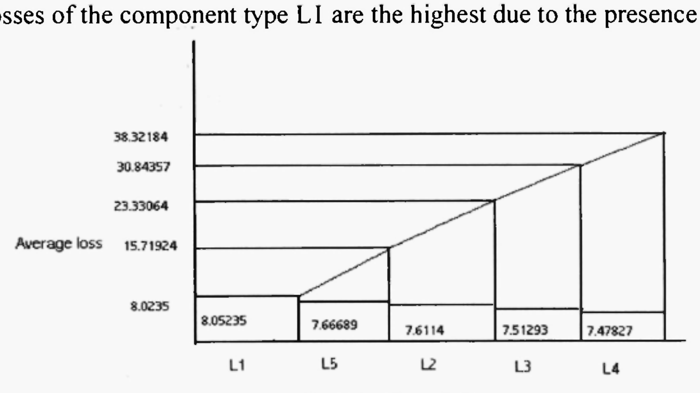

Figure 3

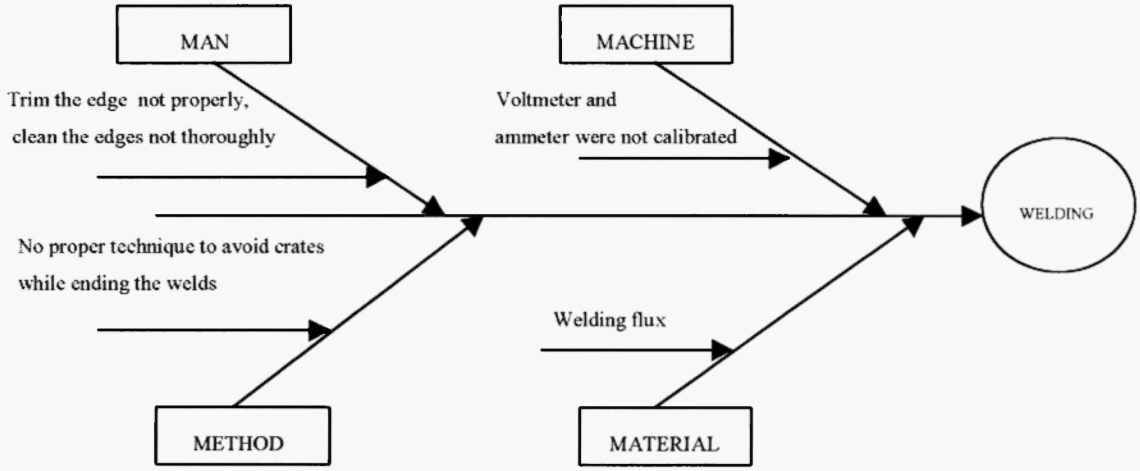

Figure 4

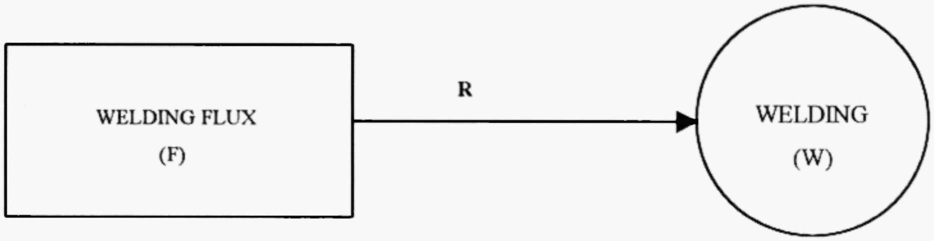

Figure 5

4. CAUSE AND EFFECT DIAGRAM

The advantage of the cause-and-effect diagram approach is the presentation both of a list of causes and of their structure. The application of fuzzy set theory can accelerate the problem analysis at a preliminary stage, based on expert knowledge. In this case the expert knowledge is presented by fuzzy relationships between causes and quality characteristics. A fragment of the cause-and-effect diagram for the welding characteristic is given in Figure 4.

Let us consider the example of a single fuzzy relationship between cause (Welding Flux F) and effect (Welding W) as shown in Figure 5.

The following rule can be used:

IF the welding flux is F = adequate THEN the welding is W1 = good

ELSE the welding is W2 = very bad

Then the fuzzy relationship is the matrix i.e. $R = (F \times W1) \cup (\neg F \times W2)$

W1 = good = 0/0 + 0.1/1 + 0.3/2 + 0.6/3 + 0.8/4 + 1/5

W2= very bad = 1/0 + 0.64/1 + 0.25/2 + 0.04/3 + 0.01/4 + 0/5

Cause = welding flux = F = adequate = $\Sigma\ \mu$ (u)/u = 0/0 + 0.2/3 + 0.4/6 + 0.8/9 + 1.0/12

$u \in U = \{0, 3, 6, 9, 12\}$

$\neg$F = not adequate = $\Sigma[\ 1 - \mu$ (u)]/u = 1/0 + 0.8/3 + 0.6/6 0.2/9 + 0/12

IF F = adequate THEN W1 = good

IF $\neg$F = not adequate THEN W2 = very bad

very adequate = 0/0 + 0.04/3 +0.16/6 + 0.64/9 + 1/12

F1 = not very adequate = 1/0 + 0.96/3 + 0.84/6 + 0.36/9 + 0/12

$$\text{FXW1} = \begin{vmatrix} 0.0 & 0.1 & 0.3 & 0.6 & 0.8 & 1.0 \\ 0.2 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.8 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \\ 1.0 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \end{vmatrix}$$

$$\neg\text{FXW2} = \begin{vmatrix} 0 & 0.01 & 0.04 & 0.25 & 0.64 & 1.0 \\ 0 & 0.01 & 0.04 & 0.25 & 0.64 & 0.8 \\ 0 & 0.01 & 0.04 & 0.25 & 0.6 & 0.6 \\ 0 & 0.01 & 0.04 & 0.20 & 0.20 & 0.20 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$R = (F \text{ X } W1) \cup (\neg F \text{ X } W2)$

$$\text{or } R = \begin{vmatrix} 0 & 0.1 & 0.3 & 0.6 & 0.8 & 1.0 \\ 0.2 & 0.1 & 0.2 & 0.25 & 0.64 & 0.8 \\ 0.4 & 0.1 & 0.2 & 0.25 & 0.6 & 0.6 \\ 0.8 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \\ 1.0 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \end{vmatrix}$$

$W = |\,F1\,|\ o\ |\,R\,|$

$$\text{or } W = |\ 1.0\ 0.96\ 0.84\ 0.36\ 0\ |\ o \begin{vmatrix} 0 & 0.1 & 0.3 & 0.6 & 0.8 & 1.0 \\ 0.2 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.8 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \\ 1.0 & 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \end{vmatrix}$$

$\therefore\ W = |\ 0.4\ 0.1\ 0.3\ 0.6\ 0.8\ 1\ |$

The result indicates that the welding is sufficiently good.

By simulating different linguistic descriptions of causes it is possible to analyse their influences upon the quality characteristic.

5. DESIGN OF EXPERIMENTS (DOE)

Design of experiments (DOE) is applied for more detailed analysis of:

a) the influence of different factors on the significant quality characteristics
b) determination of the optional factor levels.
c) the determination of the specification limits.

In design of experiments quality characteristics is the response parameter. The important feature of DOE with the linguistic description of quality characteristics is the formulation by the membership functions with further defuzzification. For defuzzification of the linguistic data the distance (d) between the center of gravity of the fuzzy sets and the target values has been used. The variable "d " is used to construct control charts using linguistic data for the welding defects, where, d

$$D = \frac{\Sigma aabs(U-U_{TARG})\,\mu(u)}{\Sigma\,\mu(u)}$$

The further processing of experimental results is carried out according to traditional schedule for distance variable 'd'.

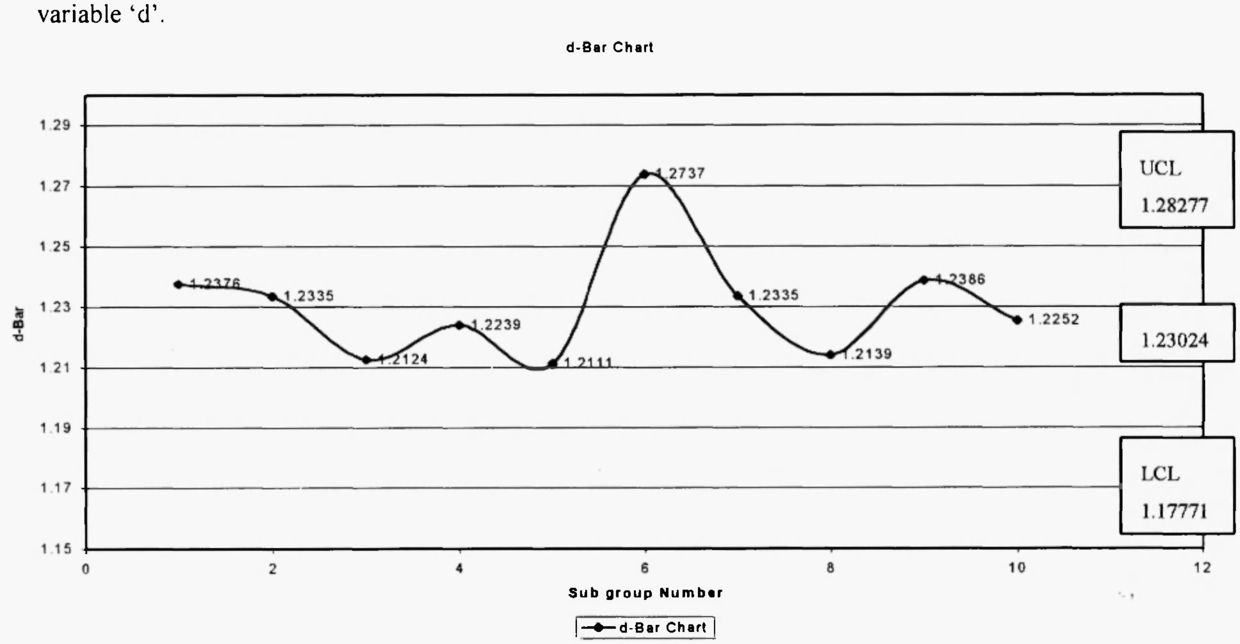

Figure 6

6. CONTROL CHARTS

The variable 'd' can also be applied to control chart construction in case of linguistics data. In spite of sufficient flexibility and the resolution of linguistic descriptions of the quality characteristics, the distribution of the variable 'd' is discrete. This is a consequence of the fact that both the set of term dictionary and the set of modifiers are finite. According to the central limit theorem the average 'd-BAR' for the subgroup with large size 'n' can be presented as a normally distributed variable. This makes it possible to use a conventional control chart construction technique for variable 'd'. The d bar control chart for the linguistic quality characteristics for the welded components of LPG cylinder as shown in Fig.6 is drawn with the help of data from Table-II.

Table II

Data sheet

Linguistic estimate	d	Observed results for welding of LPG cylinder									
		1	2	3	4	5	6	7	8	9	10
Very good	0.79524	10	5	0	0	2	0	0	0	2	0
Good	1.17857	82	90	98	97	95	95	98	96	94	96
Not bad and not very good	1.81818	6	3	1	2	2	2	0	2	2	3
Bad	3.9231	1	2	1	0	1	3	2	1	2	1
Very bad	4.3299	1	0	0	1	0	0	0	1	0	0
d - bar		1.2376	1.2335	1.2124	1.2229	1.2111	1.2737	1.2335	1.2139	1.2386	1.2252

7. CONCLUSION

On the basis of fuzzy set theory, linguistic quality characteristics can be formalised by providing 3 general steps.

1) Universal set choosing (scale of measuring device).
2) Definitional and adequate formalisation of terms (measuring device resolution).
3) Relevant linguistic description of the observation (adequate measurement).

These steps ensure the successful application of the fuzzy set theory to the powerful quality control tools considered in this paper, using linguistic characteristics.

Linguistic descriptions of the welding defects of the components do not require expensive instrument support. However, the inspection must be subjective and visual. In the design of experiments, the quality characteristics can be the response parameter. The variable 'd' that is the distance between the center of gravity of the fuzzy sets and target values can be applied also to the control chart construction in the case of the linguistic data.

8. ACKNOWLEDGEMENT

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