DOES THE GURSON-TVERGAARD MATERIAL MODEL SATISFY THE SECOND LAW OF THERMODYNAMICS?

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Abstract

The Gurson-Tvergaard material model was studied from the basic laws of thermomechanics point of view. The model was successfully used for prediction of crack growth in fracture mechanics specimens of various materials used in nuclear reactor pressure vessels. Theoretical evaluation of the model showed that when the hydrostatic part of the stress is compression there are states which do not satisfy the second law of thermodynamics. The evolution equation for void volume fraction f applied by Tvergaard and Needleman is in contradiction of the continuity equation.

1. Background

In the case of realistic nuclear reactor pressure vessel materials, only a very limited amount of material is usually available for testing. With the help of micromechanical material models, fracture mechanical parameters can be estimated by testing small tensile specimens only. An important advantage of micromechanical material models over classical fracture mechanics is that, at least in principle, the parameters do not depend on specimen size or geometry. This makes micromechanical material modelling at present a subject of worldwide intensive research.

The present work belongs to the RAKE project (Structural Analyses for Nuclear Power Plant Components). The main objective of the RAKE project is to create, evaluate and apply effective and reliable structural analysis methods for the safety and availability assessment of nuclear power plant applications. In particular, they are applied on pressure vessels and piping. There are three target research areas:

- to develop fracture assessment tools,
- to assess component behaviour under realistic loading cases, and
- to verify the methods using large scale experiments.

The evaluation and utilisation of the Gurson-Tvergaard material model forms part of the first item on the list.

The RAKE project belongs to the Finnish research programme on the ‘Structural Integrity of Nuclear Power Plants’ (RATU2). The RATU2 programme is set for the period 1995-1998 and its total budget is FIM 40-45 million ($7.7-8.7 million US). The RATU2 research programme has been funded mainly by the Ministry of Trade and Industry (KTM), the Finnish Centre for Radiation and Nuclear Safety (STUK), Imatran Voima (IVO), Teollisuuden Voima Oy (TVO) and the Technical Research Centre of Finland (VTT).

2. Description of the Gurson-Tvergaard model

Ductile fracture occurs in metals as nucleation, growth and coalescence of microvoids. Microvoids are caused by inclusions and carbides breaking or debonding from the matrix material. Among micromechanical material models for ductile fracture, the Gurson-Tvergaard-Needleman model is the most widely used. Gurson (1977) derived the original yield condition for porous materials by analysing a spherical void in a rigid-plastic spherical solid. It was later modified by Tvergaard (1981) to improve the agreement with numerical analyses. The modified yield condition is

$$\Phi = \frac{3\sigma_{ij}\sigma_{ij}}{2(\sigma^{M})^2} + 2q^1 f \cosh\left(\frac{q^2\sigma_{kk}}{2\sigma^{M}}\right) - \left[1 + \left(q^3 f\right)^2\right] = 0, \qquad (1)$$

where σ_{ij} is the Cauchy stress tensor, σ^M is the current flow stress of the matrix material, f the void volume fraction and q^1, q^2, q^3 are parameters introduced by Tvergaard. In the case of zero porosity Eq. (1) reduces to the von Mises yield condition. Further modifications were undertaken by Tvergaard and Needleman (1984) to consider the final material failure by void coalescence, which starts when a critical porosity f_c is reached. They replaced f by a "damage" function f*

$$f^* = \begin{cases} f \ for f \le f_c \\ f_c + K(f - f_c) for f > f_c with K = \dfrac{f_u^* - f_c}{f_f - f_c}, \end{cases} \qquad (2)$$

and introduced a model to consider void nucleation. The ultimate porosity value, at which the stress carrying capacity vanishes according to (1) is $f_u^* = 1/q$ and final porosity f_f is the value at which stress carrying capacity has been observed to occur in reality.

3. Application of the model in practical analyses

The main problems in applying the modified Gurson model are the large number of parameters and the difficulties in determining their values experimentally. According to a methodology developed at Fraunhofer-Institut für Werkstoffmechanik (IWM) in Germany, the damage parameter values are obtained by numerical fitting of tensile test results and then used to predict J_R curves (Sun et al. 1988, 1992). If a parameter set succeeds in describing different tests correctly, it can be regarded as material parameter values.

Extensive numerical and experimental studies were performed jointly by IWM and VTT to assess the applicability of the methodology on several reactor pressure vessel materials: besides western ferritic steels 20 MnMoNi 5 5 (German nomination) and A533B also austenic VVER-440 reactor pressure vessel cladding was considered.

As part of an ASTM Cross-Comparison Exercise on Determination of Material Properties through the Use of Miniature Mechanical Testing Techniques with the ferritic steel ASTM

A533B, the feasibility of different specimen types for the determination of damage parameters was evaluated (Schmitt et al 1997). The specimen size should be as small as possible and the specimen geometry such that a two-dimensional (or axisymmetric) finite element model is appropriate for simulation. Dynamic tests with subsized Charpy- and precracked bend specimens (SE(B)), quasistatic tests with C(T)25 specimens as well as dynamic and quasistatic tensile tests were performed and simulated using the Gurson-Tvergaard-Needleman model (Schmitt et at 1997). These studies revealed that the most feasible specimen for the determination of damage parameter values is the SE(B) specimen and that smooth tensile specimens are suitable as well. Figure 1 compares experimental and numerical results for the SE(B) specimen. In Figure 2 the damage parameter values fitted on the SE(B) tests have been applied to predict the J_R curve.

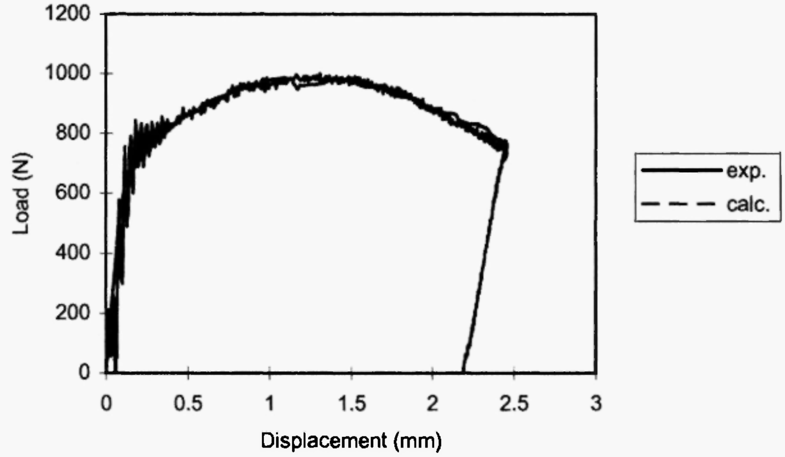

Figure 1: Subsized side-grooved SE(B)specimen: measured and simulated load vs. hammer displacement (Schmitt et al 1997).

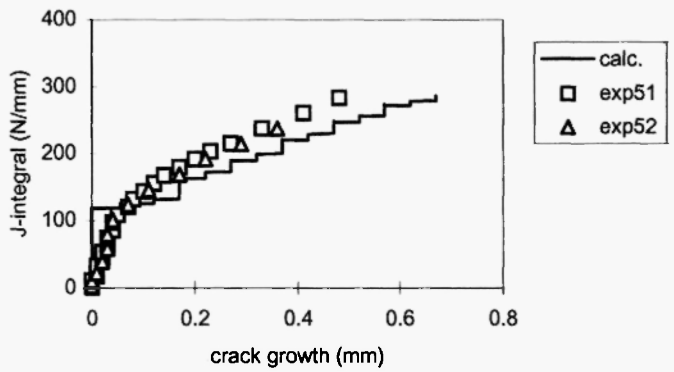

Figure 2: Side-grooved C(T) specimen: measured and simulated J_R curves (Schmitt et al 1997).

In the case of VVER-440 cladding there were problems transferring the material parameter sets between different specimen types (Talja et al 1997). This was partly due to uncertainties in the material data. For the material 20 MnMoNi 5 5 the effect of specimen orientation and size were studied considering the material scatter. Tensile and fracture mechanics specimens were tested in three orientations.

4. Physical evaluation of the Gurson-Tvergaard model

Besides computational work the foundation of the Gurson-Tvergaard material model was evaluated from the physics point of view. First, the model was studied within the framework of mechanics and then a thermomechanical derivation of the constitutive equation was carried out. This work was carried out at VTT and is published in Santaoja (1997).

Gurson used upper bound approach to derive the yield function Φ. He modelled the matrix material between the voids by a rigid-perfectly-plastic material model. In order to make the independent variable of the Cosh-function dimensionless Gurson introduced the constant σ^M in Eq. (1). Gurson may have adopted σ^M = microscopic equivalent tensile yield stress (constant), since it provides a form for the yield function where its terms take values close to unit. So in the model by Gurson there was no concept that σ^M is related to the yield stress, but it was just suitable value. In contrast to Gurson Tvergaard assumed that the matrix material shows hardening and modelled it by letting the value of σ^M vary with hardening. Tvergaard prepared an elegant formulation for the evolution equation of σ^M. If the matrix material shows hardening the upper bound approach used by Gurson is not valid. Therefore, the introduction of the variable σ^M and the parameters q^1, q^2 and q^3 is an ad hoc modification of the Gurson model. It is prepared for the examples studied by Tvergaard. It has to remember that also the original Gurson model has limitations.

The evolution equation for the void volume fraction f can be derived from the continuity equation. Already Tvergaard [1981, Eq. (5.5)] gave the result. This does not accept the replacement of f by f* shown by Eq. (2). Chu and Needleman (1980, pp. 249 and 251) argued the modification of the evolution equation for the void fraction f by referring to the second phase particles. They made a different modification than shown by Eq. (2). If someone makes changes to the continuity equation by referring to the second phase particles, he/she can modify all the other basic laws of thermomechanics by the same argumentation. It is more acceptable to keep the continuity equation and make changes to the other part of the material model.

The Gurson-Tvergaard material model combined with the damage evolution described by Eq. (2) simulates strain softening of the material. This tends to cause localisation of damage which makes the computational result mesh dependent. Although in the case when this constitutive equation is used in the simulation of crack growth, the element mesh in front of the crack tip is usually almost the same, some mesh dependency may exist. This is an important topic for prospective investigation at VTT.

Thermomechanical evaluation allows proof that the material model fulfils the requirements set by thermomechanics. The key role is the satisfaction of the second law of thermomechanics. The standard procedure of this consideration is to combine the basic laws of thermomechanics including the law of balance of momentum, the law of balance of moment of momentum, the first law of thermodynamics, the second law of thermodynamis and to dress the resulting expression as the Clausius-Duhem inequality. Thermomechanical evaluation of the Gurson model showed that when the hydrostatic part of the stress is tension, the Gurson material model always satisfies the Clausius-Duhem inequality. In compression, however, there are states which are not allowed.

5. Future Directions

There are many topics concerning the Gurson-Tvergaard material model that need further clarification before it can be used in the safety assessment of nuclear power plants. One important area for extensive work are the form of the evolution equation and the determination of the values for the parameters of the constitutive equation. A solid way to treat material anisotropy effects should be developed. In 1998 the Gurson-Tvergaard will be applied at VTT for prediction of the dependence of the J-R curves on specimen geometry and size, and the possibilities to assess mixed mode fracture will be evaluated.

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