Thermoelastic damping in materials with a complex coefficient of thermal expansion

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Abstract

The effect of a complex coefficient of thermal expansion upon the thermoelastic relaxation mechanism is analyzed. A phase angle in the thermal expansion has the effect of generating a very broad band of mechanical damping, in addition to the peak usually observed. Phase angles in the thermal expansion have been observed in several polymers, and they may be generated in composite materials in which one or more phases is viscoelastic. However the resulting enhancement of the overall damping of the composite is modest, unless new materials can be found with very high relaxation strength or with a large intrinsic phase angle in the expansion.

1. INTRODUCTION

Thermoelastic relaxation is a coupled-field type viscoelastic mechanism. Relaxation proceeds to a non-zero asymptotic value of stiffness, therefore it is regarded as anelastic. Thermoelastic relaxation, initially explored by Zener [1-3], occurs in all materials which exhibit thermal expansion. It is present whenever there is inhomogeneity of temperature, since the consequent flow of heat gives rise to a dissipation of energy. Temperature inhomogeneity can arise due to an inhomogeneity of dilatational stress. Stress inhomogeneity occurs in some types of vibration, such as bending vibration of reeds. Inhomogeneity of stress also is present if the material has cavities, discrete phases, or anisotropic crystallites with random orientation. There is also a homogeneous thermoelastic relaxation governed by heat flow between the specimen

and the environment. The maximum tan δ due to thermoelastic damping depends on the relaxation strength Δ, defined as the change in stiffness during relaxation divided by the stiffness at long time t, or in the formulation of creep compliance J(t), as follows.

$$\Delta = \frac{J(\infty) - J(0)}{J(0)} . \tag{1}$$

For thermoelastic relaxation, the relaxation strength is

$$\Delta = \frac{\alpha^2 T}{C_v J_S}, \tag{2}$$

with α as the thermal expansion coefficient, $J_S = J(0)$ as the adiabatic compliance, T as the absolute temperature and C_v as the heat capacity per unit volume. When the relaxation strength is small as it is for this mechanism, the maximum tan δ for a Debye peak is $\frac{1}{2}\Delta$. Values of $\frac{1}{2}\Delta$ for some common materials are 0.0012 for Fe, 0.0024 for Al, 0.0003 for SiC, and 0.0089 for Zn. Thermoelastic relaxation is most important in metals and ceramics in which it may comprise most of the total relaxation. Experimental verification of the theory was reported by Zener and co-workers [3,4].

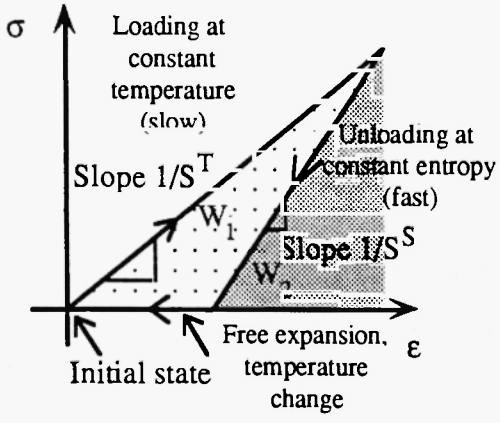


Fig. 1 Cyclic history of stress σ vs. strain ε showing energy dissipated due to thermoelastic damping. Conversion of mechanical energy into thermal energy via the thermal expansion and piezocaloric effects. Energy densities W are shown as shaded areas. Material is loaded slowly at constant temperature, then unloaded rapidly at constant entropy (adiabatically), which causes a temperature change, finally allowed to thermally equilibrate at zero stress.

Dissipation of mechanical energy in a cyclic load history with heat flow is illustrated in Fig. 1. The history consists of three portions. First the solid is loaded slowly at constant

temperature (isothermally). It is then unloaded adiabatically, too rapidly for heat flow to occur, with the slope of the stress strain curve as the adiabatic modulus which differs from the isothermal modulus. The solid is then held at constant stress, and it exchanges heat with the environment. Thermal expansion occurs, so the strain changes. Mechanical energy is dissipated in this cycle, since there is a non-zero area enclosed by the load history. The loss tangent as a measure of damping refers to sinusoidal loading which gives an elliptical stress-strain diagram, governed by the same general principles.

Thermoelastic damping has received renewed attention in view of the fact that it is operative in stiff materials which are of use structurally. Since the figure of merit for many aspects of structural damping is E tan δ, the thermoelastic mechanism is of interest even though the maximum damping values from it are relatively small. Thermoelastic damping in *composite* materials arises due to the inhomogeneity of the thermal and mechanical properties of such materials, leading to heat flow between constituents, hence mechanical energy dissipation. The damping depends on the specific phase geometry as well as the constituents involved. The reason is that damping depends on inhomogeneity of dilatational stress, and on the nature of the boundary value problem under consideration. Composites of the following structure have been analyzed for thermoelastic damping: one dimensional inclusions by Milligan and Kinra [5], laminates by Bishop and Kinra [6], laminates with perfect and imperfect thermal interfaces by Bishop and Kinra [7], and composites with particulate inclusions by Bishop and Kinra [8].

In this article we consider the effect upon damping of phase angles in the thermal expansion, in view of the fact that similar phase angles in piezoelectric moduli can substantially affect the mechanical damping due to piezoelectric coupling as demonstrated by Lakes [9].

II. MECHANICAL LOSS DUE TO THERMAL CURRENTS

A. Homogeneous case, with complex expansion coefficient

In this section we obtain the mechanical damping due to thermoelastic coupling. The

analysis parallels that of Zener [3], with the exception that the thermal expansion coefficient is assumed to be a complex quantity,

$$\alpha^* = \alpha'(1 + i \tan \delta_\alpha), \tag{3}$$

with δ_α as a phase angle, $\tan \delta_\alpha = \mathrm{Im}\{\alpha^*\}/\mathrm{Re}\{\alpha^*\}$ and α' as $\mathrm{Re}\{\alpha^*\}$.

The relation between strain ε, stress σ, and temperature T is, in one dimension:

$$\varepsilon = J_T\sigma + \alpha^*\Delta T, \tag{4}$$

with J_T as the isothermal compliance, so

$$\Delta T = \frac{1}{\alpha^*}(\varepsilon - J_T\sigma). \tag{5}$$

Thermal diffusion is governed by the following, with τ as the thermal diffusion time. This is homogeneous thermal diffusion, from the specimen to its environment. It is assumed that the exchange of heat with the environment is much faster than the thermal diffusion within the specimen.

$$\frac{d\Delta T}{dt}\Big|_{\text{diffusion}} = -\frac{\Delta T}{\tau}. \tag{6}$$

In the frequency domain,

$$i\omega\Delta T = -\frac{\Delta T}{\tau}. \tag{7}$$

Under adiabatic conditions, an increase in length of a specimen of material results in a decrease in temperature, for $\alpha > 0$.

$$\frac{d\Delta T}{dt}\Big|_{\text{adiabatic}} = -\gamma\frac{d\varepsilon}{dt}, \tag{8}$$

with

$$\gamma = \frac{\partial T}{\partial \varepsilon}\Big|_{\text{adiabatic}}.$$

There may also be a phase angle in γ:

$$\gamma^* = \gamma'(1 + i \tan \delta_\gamma),$$

The change in temperature in response to strain is a manifestation of the piezocaloric effect,

which is complementary to thermal expansion. In the frequency domain,

$$i\omega\Delta T = - \gamma^* i\omega\varepsilon, \tag{9}$$

Combining Eq. 6 and 8, since there are two independent sources for rate of temperature change,

$$\frac{d\Delta T}{dt} = - \frac{\Delta T}{\tau} - \gamma^* \frac{d\varepsilon}{dt}. \tag{10}$$

In the frequency domain,

$$i\omega\Delta T = - \frac{\Delta T}{\tau} - \gamma^* i\omega\varepsilon, \tag{11}$$

Eliminating ΔT with Eq. 5,

$$i\omega \frac{1}{\alpha^*}(\varepsilon - J_T\sigma) = - \frac{1}{\alpha^*\tau}(\varepsilon - J_T\sigma) - i\omega\gamma^*\varepsilon. \tag{12}$$

Calculating the ratio of stress to strain, the product $\gamma^*\alpha^*$ appears. It may be written as follows in terms of the phase angles. The prime denotes the real part.

$$\gamma^*\alpha^* = \gamma'(1 + i \tan \delta_\gamma)\alpha'(1 + i \tan \delta_\alpha) = \gamma'\alpha'\{1 + i(\tan \delta_\gamma + \tan \delta_\alpha) - \tan \delta_\gamma \tan \delta_\alpha\}.$$

For small phase angles,

$$\gamma^*\alpha^* \approx \gamma'\alpha'\{1 + i(\tan \delta_\gamma + \tan \delta_\alpha)\} \equiv \gamma'\alpha'\{1 + i \tan \delta_{\alpha\gamma}\}.$$

$$E^* = \frac{1}{J_T} \frac{1 + i\omega\tau\alpha'\gamma'(1 + i \tan \delta_{\alpha\gamma}) + \omega^2\tau^2\alpha'\gamma'(1 + i \tan \delta_{\alpha\gamma}) + \omega^2\tau^2}{1 + \omega^2\tau^2}. \tag{13}$$

But

$$\tan \delta_E = \frac{\text{Im}\{E^*\}}{\text{Re}\{E^*\}}, \tag{14}$$

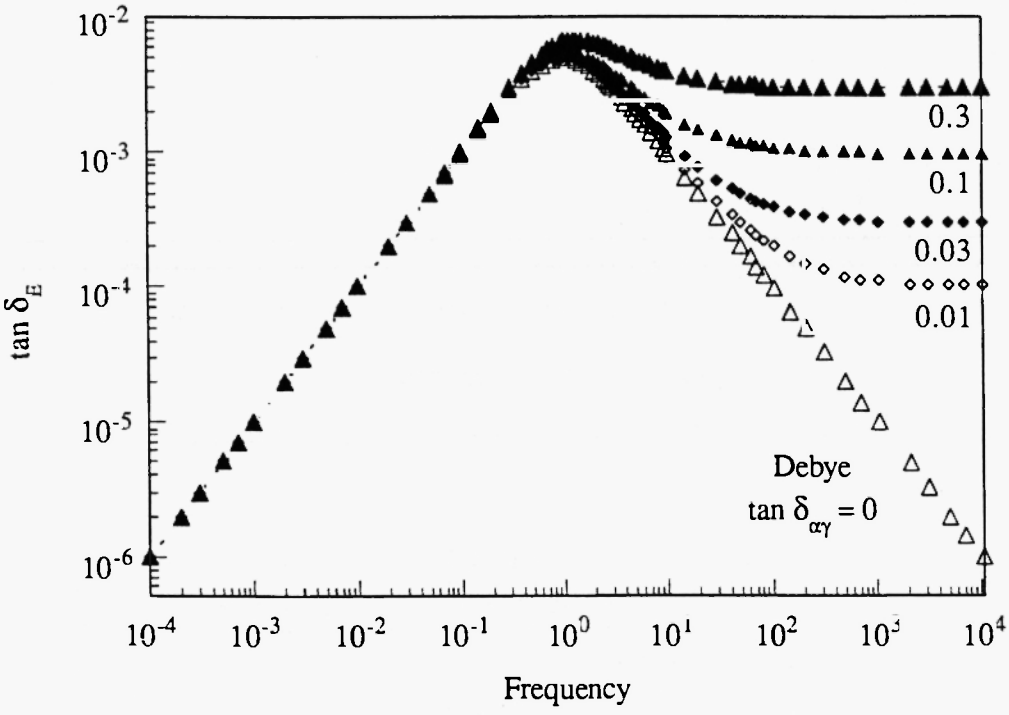
so the mechanical damping due to thermoelastic effects is

$$\tan \delta_E = \frac{\omega\tau\gamma\alpha + \omega^2\tau^2\alpha'\gamma' \tan \delta_{\alpha\gamma}}{1 + \omega^2\tau^2(1 + \alpha'\gamma') - \omega\tau\alpha'\gamma' \tan \delta_{\alpha\gamma}}. \tag{15}$$

This damping is shown for various values of $\tan \delta_{\alpha\gamma}$, in Fig. 2. Observe that in the absence of a phase angle in thermal expansion ($\delta_\alpha = 0$), the damping due to thermoelasticity in this example follows a Debye peak. Following Zener [3], the relaxation strength for the Debye peak is

$$\Delta = \alpha\gamma = \frac{\alpha^2 T}{C_v J_S},$$

with the second form obtained via thermodynamic arguments based on the fact that an increment of free energy is a perfect differential. The Debye form is obtained for the case of macroscopic diffusion of heat. In cases of transverse vibration of reeds or cylinders, tan δ exhibits a series of peaks of progressively smaller magnitude.



2 Theoretical mechanical damping tan δ_E for thermoelastic damping with and without a phase angle δ_α in the thermal expansion coefficient; effect of different values of tan δ_γ + tan δ_α, labeled as tan $\delta_{\alpha\gamma}$

When $\delta_\alpha \neq 0$, the high frequency damping for $\omega >> \tau^{-1}$, is as follows.

$$\tan \delta_E \approx \gamma'\alpha' \tan \delta_{\alpha\gamma} \qquad (16)$$

This damping is constant, independent of frequency corresponding to the flat region to the right in the curves in Fig. 2. In a continuum, this damping extends to arbitrarily high frequency, but

if the material has microstructure, an upper limit on frequency is to be expected as discussed in section III.

The effect of a positive phase angle in thermal expansion increases the mechanical damping. This behavior is in contrast to a positive phase angle in the piezoelectric 'd' tensor which reduces the mechanical damping [9]. Indeed, prior theoretical treatments which neglected piezoelectric phase angles overestimated observed mechanical damping in piezoelectric ceramics by more than a factor of two.

B. Restrictions on the coefficients

In a passive material, the total damping tan δ_E must be nonnegative. However the thermal expansion coefficient α can be positive or negative. Damping in the Debye peak is always positive regardless of the sign of α since it is governed by α^2. As for the phase δ_α, Eq. 16 shows that a negative phase combined with a positive expansion coefficient, gives rise to a negative contribution to the mechanical loss in the high frequency region. Consequently negative values of δ_α are excluded in passive materials unless viscoelastic mechanisms other than thermoelasticity are operative. As demonstrated in section III, negative values of δ_α can be generated in composite materials, but only by having at least one phase which is already viscoelastic.

C. Loss due to thermal diffusion among inhomogeneities

Damping which results from thermal diffusion between the grains in a polycrystalline metal was studied by Randall, *et al.* [4]. Experiments showed that this damping, though small in magnitude, can account for virtually all the damping in the kHz region in some metals. Damping due to thermoelastic effects associated with inclusions in a one dimension was presented by Milligan and Kinra [5]. Predicted damping for one dimensional inclusions is proportional to $(\alpha_1/\rho_1 c_1 - \alpha_2/\rho_2 c_2)^2$ with α as a real thermal expansion, ρ as density, c as specific heat per unit mass and the subscripts representing the phase. The peak is 1.2 to 1.9

decades wide at half maximum depending on the ratio of thermal conductivities compared with 1.1 decades for a Debye peak. The more realistic and more complicated case of spherical inclusions was examined by Bishop and Kinra [8]). The damping peak in this case is broader than a Debye peak: about 1.7 decades wide at half maximum. Peak tan δ_E values for one-dimensional inclusions are, for SiC in Mg, 0.0091, for SiC in Al, 0.0049; for spherical inclusions, for SiC in Mg, 0.0067, for SiC in Al, 0.0045.

Examination of the effect of a phase angle in α for such cases is considerably more difficult than in the case considered above. One cannot apply the correspondence principle to a result for the loss tangent. The loss is a real quantity, and substitution of a complex α in its expression does not yield a physically meaningful quantity. Nevertheless, to the extent that the behavior is approximated by a Debye peak, the effect of a complex thermal expansion coefficient is expected to be similar to that obtained above.

III. Materials with Complex Coefficients of Thermal Expansion

A. Unidirectional composite

Complex thermal expansions are known to occur in polymeric materials [10,11]. We demonstrate here that they can be obtained in a controlled fashion in composite materials, provided that at least one phase is viscoelastic. Consider the longitudinal thermal expansion coefficient α_L for a unidirectional fibrous composite [12,13].

$$\alpha_L = \frac{1}{E_L}(\alpha_1 E_1 V_1 + \alpha_2 E_2 V_2), \tag{17}$$

in which α_1 is the thermal expansion coefficient of the fibers, E_1 is Young's modulus of the fibers, and V_1 is the volume fraction of the fibers; α_2, E_2, V_2 are corresponding values for the matrix, with $V_1+V_2 = 1$. E_L is given by the Voigt relation or rule of mixtures,

$$E_L = E_1 V_1 + E_2 V_2, \tag{18}$$

Applying the dynamic elastic-viscoelastic correspondence principle, and assuming that each component is viscoelastic but has a real thermal expansion coefficient,

$$\alpha_L^* = \frac{1}{E_L^*} (\alpha_1 E_1^* V_1 + \alpha_2 E_2^* V_2). \quad (19)$$

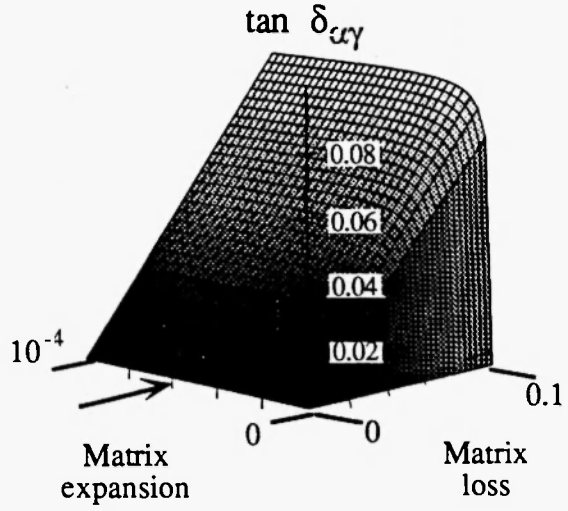
We remark that this form can be readily obtained from the dynamic stress-strain-temperature relation without any appeal to the correspondence principle.

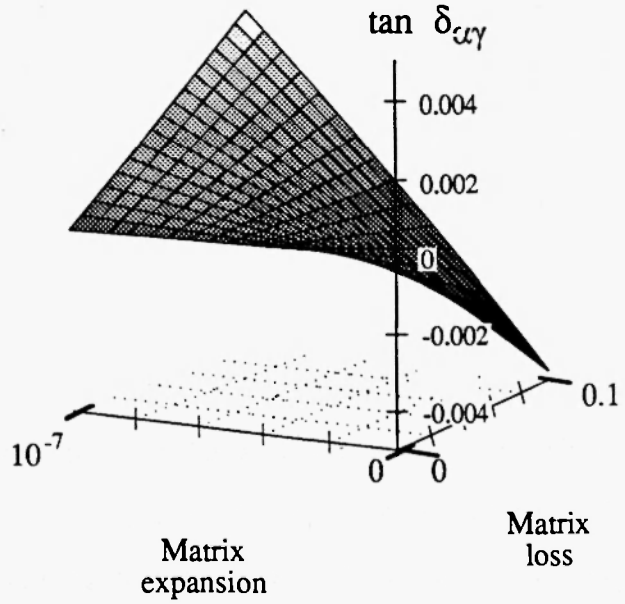
Combining, and incorporating $V_2 = 1 - V_1$,

$$\alpha_L^* = \frac{\alpha_1 + \alpha_2 \frac{E_2^*}{E_1^*} \frac{1-V_1}{V_1}}{1 + \frac{E_2^*}{E_1^*} \frac{1-V_1}{V_1}}. \quad (20)$$

Materials with complex thermal expansions can therefore be prepared as composite materials in which at least one constituent is viscoelastic. Fig. 3 shows the tangent of the phase δ_α of the thermal expansion for such a composite. Tan δ_α can be made on the order of the mechanical damping tan $\delta_{E.2}$ of the matrix phase, for reasonably large volume fractions of fibers. The magnitude of the thermal expansion in this example is rather small, since the expansion of the fibers is assumed to be small. The phase δ_α can be negative in composites if the expansion in the stiff phase is greater than that in the more compliant phase, as shown in the lower part of Fig. 3.

The above calculation of the complex thermal expansion is based on a continuum view of the composite. Such a view is warrantable provided the frequency is sufficiently below the characteristic frequency of thermal diffusion between the fiber and matrix phases of the composite.





3 Theoretical tangent of the phase angle δ_α in the thermal expansion of a unidirectional fibrous composite in terms of the mechanical damping tan δ_E of the matrix phase, and thermal expansion α_m of the matrix. It is assumed that the fibers are purely elastic and are 45 times stiffer than the matrix and that they have a longitudinal expansion $\alpha_f = 4.5 \times 10^{-8}$ /°C. These figures are representative of graphite fibers in an epoxy matrix. The volume fraction of fibers is assumed to be 0.5.

Top diagram: range of α_m 0 to 10^{-4} /°C.

Arrow: a typical epoxy, $\alpha_m = 6 \times 10^{-5}$ /°C.

Bottom diagram: range of α_m 0 to 10^{-7} /°C, showing the possibility of a negative phase if the matrix expansion is very small.

B. Isotropic composite

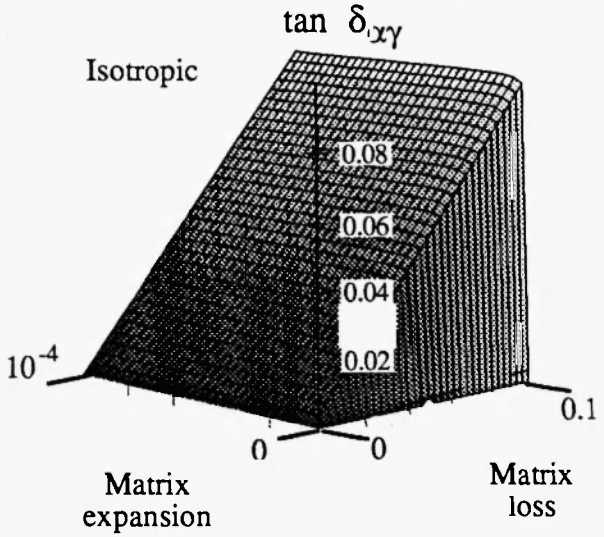
The thermal expansion for a macroscopically isotropic composite is given as follows [14]:

$$\alpha = \alpha_2 + \frac{\alpha_1 - \alpha_2}{K_1^{-1} - K_2^{-1}} \{\frac{1}{K} - \frac{1}{K_2}\}, \qquad (21)$$

in which K_1 and K_2 are the bulk moduli of the two phases and K is the bulk modulus of the composite as a whole. Let K be given as the Hashin-Shtrickman upper bound, assuming a Poisson's ratio of 0.3 for both phases, so $K_1 = 2.166\ G_1$ with G_1 as the shear modulus of the first phase. The Hashin-Shtrickman upper bound is given by:

$$G = G_1 + \frac{V_2}{\frac{1}{G_2 - G_1} + \frac{6(K_1+2G_1)V_1}{5(3K_1+4G_1)G_1}}. \qquad (22)$$

Again, consider the stiffness of the matrix (phase 2) to be a complex quantity. The phase in the thermal expansion is shown in Fig. 4, for constituent material properties as in Fig. 3. As in the case of the unidirectional composite, tan δ_α can be made on the order of the mechanical damping tan $\delta_{E.2}$ of the matrix phase, for reasonably large volume fractions of inclusions.



4 Theoretical tangent of the phase angle δ_α in the thermal expansion of an isotropic composite in terms of the mechanical damping tan δ_E of the matrix phase, and thermal expansion α_m of the matrix. Hashin-Shtrickman upper bound model. Assumed constituent properties are the same as those for Fig. 3. Poisson's ratio is assumed to be 0.3 for both phases.

C. Prospects for high-loss materials. Structural hierarchy.

If a composite material is used to generate a phase δ_α for use in homogeneous thermoelastic damping, there are two length scales, that of the composite structure and that of the specimen or structural member itself. This is an example of structural hierarchy, in which there are multiple nested length scales. Hierarchical materials are known to offer the potential for superior stiffness, strength and toughness [15]. Complex composite structures need not arise by manufacturing. For example spherulitic structures in polymers, and eutectic structures in metal alloys are heterogeneous. In the context of viscoelastic composites, the high frequency plateau for the example of homogeneous relaxation extends over frequencies at which a continuum view applies to any composite structures used. At sufficiently high frequency, thermal flow between constituents gives rise to an additional damping peak as considered for laminates [6] and for particulate composites [8]. One may envisage hierarchical composites in which each lamina, fiber, or particle has its own internal structure. In such a case a high frequency plateau is expected above the damping peak associated with the largest structural elements.

Thermoelastic damping due to the phase δ_α in a unidirectional composite, was calculated in the plateau region governed by Eq. 16, for several combinations of constituent materials. This damping was compared with the composite damping due to the assumed viscoelasticity of the matrix phase, following a correspondence principle analysis of the Voigt relation, Eq. 18, for elastic materials:

$$E_c^* = E_1^* V_1 + E_2^* V_2.$$

Taking the ratio of real and imaginary parts, the mechanical loss tangent of the composite due to the viscoelasticity of the constituent phases is given by:

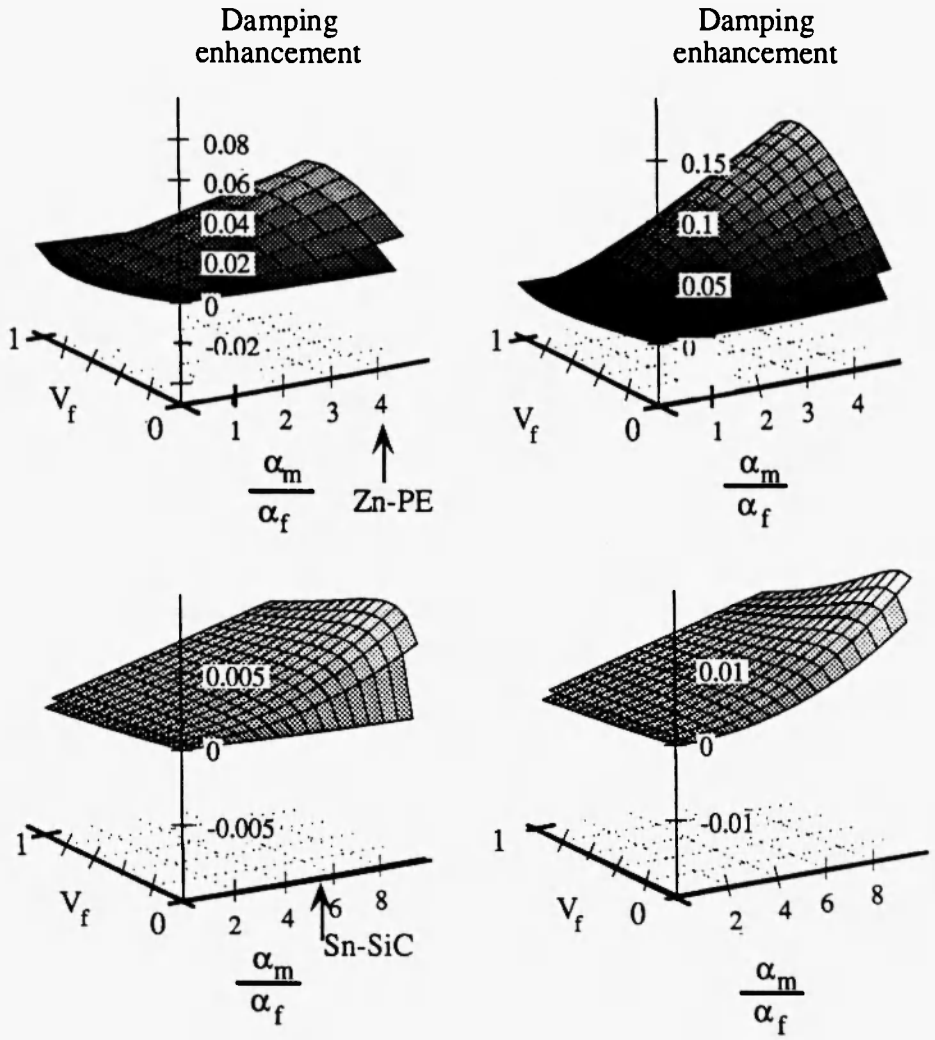
$$\tan \delta_c = \frac{V_1 \tan \delta_1 + V_2 \frac{E_2'}{E_1'} \tan \delta_2}{V_1 + \frac{E_2'}{E_1'} V_2}. \qquad (23)$$

The thermoelastic enhancement of the damping, defined as the ratio ξ of the plateau damping of Eq. 16 to the damping due to the viscoelasticity of the matrix phase, Eq. 23, was calculated.

$$\xi = \frac{\Delta \tan \delta_{\alpha\gamma}}{\tan \bar{\delta}_c} . \qquad (24)$$

This enhancement increases with E_1/E_2, with V_1, with α_2/α_1, and with the relaxation strength. For composites made of commonly known materials, the enhancement is small, less than 20% for all cases considered. Several examples, calculated using thermal data reported by Milligan and Kinra [5] are given in Fig. 5. Observe that if the matrix itself has a phase angle in its thermal expansion (to be distinguished from the phase angle of the composite) then there is additional mechanical damping.

Applications of thermoelastic damping all suffer from the drawback that the relaxation strength, hence the maximum tan δ, is relatively small for known materials considered thus far by investigators of thermoelastic damping. It is possible to obtain very high values of thermal expansion in certain cellular composites [16,17]. Such materials are compliant in view of the bending deformation which occurs in them, hence they will also have a small relaxation strength. Moreover such compliant materials cannot be expected to exhibit a large figure of merit E tan δ. As for a broader view of materials, Ashby [18] has compiled mechanical and thermal properties of many engineering materials. Ashby plots α vs E and shows contours of the product αE which governs thermal stress in a constrained system. In the context of thermoelastic relaxation, the figure of merit is the relaxation strength, which contains α^2E. By drawing contours of α^2E, we observe that the figure of merit for metals such as magnesium, zinc, aluminum, steel and copper is similar to that of polymers such as nylon and polystyrene. Ceramics and porous materials are less promising in this regard. The specific heat of a wide range of engineering materials, expressed as ρC, does not deviate much from 3×10^6 J/m^{3}K, so it seems unlikely to enhance the relaxation strength by control of that variable.



5 Theoretical thermoelastic damping enhancement at high frequency for composites. Matrix damping is assumed to be 0.1 for all cases. Matrix α_m shown by arrow.

Top left, zinc-polyethylene; damping of zinc assumed to be 10^{-4} based on pinned dislocations. Top right: tan $\delta_\alpha = 0.1$ for the polymer. Bottom left, tin-silicon carbide. Damping of silicon carbide assumed to be zero. Bottom right, tan $\delta_\alpha = 0.1$ for the tin.

Future development of high-damping structural materials based on thermoelasticity will be facilitated by a search for new materials, possibly alloys, which exhibit a large thermoelastic

relaxation strength. If such materials are to be used in composites they need not be as strong as common engineering materials. This study demonstrates that thermoelastic phase angles can be beneficial. If needed, they can be generated in composite materials.

IV. CONCLUSIONS

A phase angle in the thermal expansion coefficient α gives rise to mechanical damping over a broad range of frequency. This damping is in addition to the Debye peak expected for homogeneous thermoelastic relaxation. A phase angle in α is known to exist in certain polymers. A phase angle in α can be obtained in a controlled fashion in composite materials, provided that at least one phase is viscoelastic. However the resulting enhancement of the overall damping of the composite is modest, unless new materials with very high relaxation strength can be found.

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