The Use of the Reference Stress Concept in Creep Crack Growth Studies

T.H. Hyde,
Department of Mechanical Engineering,
University Park,
Nottingham, NG7 2RD.
England.

ABSTRACT

The reference stress method used in the prediction of creep deformations and failure times of components is briefly outlined. The extension of the reference stress approach for estimating incubation times and C* values, and hence creep crack growth rates, is then described. It is concluded that estimates of incubation time and C* (and therefore creep crack growth rates), using the reference stress approach, result in reasonably accurate predictions for practical purposes.

1. INTRODUCTION

Many engineering components contain cracks and/or crack-like defects. If these components operate at elevated temperatures, creep crack growth may occur. Under steady loading conditions, the initial elastic or elastic-plastic stesses in the vicinity of a crack will begin to redistribute as creep strains are accumulated. Depending on the creep ductility of the material, one of a number of different failure mechanisms could occur. For example, a single crack may propagate due to a succession of localised crack tip events. Alternatively, general damage may occur and the presence of the initial crack becomes unimportant or failure may be the result of a large deformation mechanism.

A variety of fracture parameters have been used in an attempt to correlate and predict creep crack growth rates. Those most commonly used are the linear elastic stress intensity, K, the reference stress, σ_R, and the C* parameter. Although reasonably good correlations of some data have been obtained with each of these parameters, e.g. K(1), σ_R(2), C*(3), the C*-parameter seems to be the most generally applicable (e.g. 4-12). However, under some circumstances, the reference stress, σ_R, can be used to determine the failure time of a component (e.g. 13) and hence crack growth rate information may be unnecessary. Also, the reference stress approach can sometimes be used to estimate the C* parameter, e.g. (13-18).

In this paper, a brief description of the reference stress method, for predicting creep deformations and failure times is given. The uses of the reference stress approach, in estimating crack initiation times (on the basis of a critical crack tip opening displacement) and C* are described.

Reference stress and creep crack growth studies have been the subjects of an enormous number of papers. The author has therefore found it necessary to be selective when choosing the work to describe and the books and papers to cite. Also, only the behaviour of steadily loaded, isothermal components is considered. However, many of the methods described can be extended to non-isothermal and variable loading conditions. The list of references should therefore form a useful starting point for the reader who is interested in more complicated loading conditions, as well as steady loading situations.

1.1 Notation

A constant in creep law (equation 4)

a crack length

B component thickness

C* creep contour integral (equation 8)

D reference multiplier (equation 1)

E Youngs' modulus

I constant in HRR stress field (equation 7)

J Rice's path independent contour integral

K stress intensity factor (linear elastic)

m ratio of the collapse load of a cracked component to that of the same uncracked component

n constant in creep law (equation 4)

P load

P_L limit load

R characteristic distance

r radial distance from crack tip

s distance along contour for J- or C*- integrals

T_i traction on contour for J- or C*- integrals

t time

t_i incubation time

$\dot{u}_i$ displacement rates on contour used for C*- integral

W^* work rate term in equation 8

x, y Cartesian co-ordinates

β constant relating J to δ or C* to $\dot{\delta}$ in equations 14 and 15

Δ^c creep displacement

Δ^e elastic displacement

δ crack tip opening displacement

δ crack tip opening displacement-rate

$\bar{\delta}$ critical crack tip opening displacement

ε^c creep strain

ε^i initial strain

ε_r strain at onset of tertiary creep

ε_u failure strain

$\dot{\varepsilon}^c$ creep strain-rate

θ crack tip angular position co-ordinate

$\mu(n)$ constant ($\simeq 1.5$) in equation 21

σ stress

$\sigma_{\bar{R}}$ reference stress

σ_Y yield stress

χ ratio of collapse load to load at first yield

Γ contour used in J- or C*- integrals

2. USE OF REFERENCE STRESSES FOR PREDICTING CREEP DEFORMATIONS

The reference stress method was initially developed in order to predict the creep deformations of components, with the minimum of material data and structural analysis. Many excellent reviews of the reference stress method have appeared, e.g. (19) - (24). Knowing the reference stress, σ_R, appropriate to a particular creep deformation (displacement, twist, strain, etc.), Δ^c, at a point in a component, the creep deformation after time t, neglecting redistribution effects, is simply given by

$$\Delta^c(t) = D\,\varepsilon^c\,(\sigma_R, t) \tag{1}$$

where $\varepsilon^c\,(\sigma_{R,t})$ is the creep strain obtained from a uniaxial creep test at time t, due to stress σ_R, and D is the reference multiplier. The reference multiplier is approximately constant (independent of material properties) for a given geometry and loading, but may be a function of dimensions for some types of deformation.

For some simple components, it may be possible to obtain analytical solutions for reference stresses and reference multipliers, e.g. (25) - (27). For more complex components and loadings numerical solutions can be used to determine reference stresses and reference multipliers, e.g. (28) - (31). It is also possible to obtain approximate reference stresses and reference multipliers (32) - (34) based on limit-load and linear elastic solutions, i.e.

$$\sigma_R \simeq \frac{P}{P_L}\,\sigma_Y \tag{2}$$

and
$$D \simeq \frac{\Delta^e}{(\sigma_R/E)} \tag{3}$$

For simple components, the approximate reference stress is simply the skeletal point stress (35). In such components (e.g. beams in pure bending and thick cylinders) it is possible to accurately predict all deformations using the same reference stress; only the reference multiplier changes for different positions and types of deformation.

For more complicated components and loadings the approximate reference stresses are likely to be most accurate for the maximum strains in the components. By testing model components (36-38), accurate reference stresses and reference multipliers, for any deformation, at any point in a component can be obtained, thus avoiding the necessity of using a single, approximate reference stress.

3. USE OF REFERENCE STRESSES FOR PREDICTING CREEP RUPTURE TIMES

Following the success of the reference stress method in predicting creep deformations, attempts have been made to obtain reference stresses for predicting creep rupture times. A rupture reference stress for a component is that stress which would cause a uniaxial specimen to fail in the same time as the component. By using a material model in which creep and damage rates are governed by the existing stress state and damage, failure times were obtained for a thick cylinder (39). It was found that the failure time based on the skeletal point stress was in close agreement with the accurately determined value (39). Thus, the approximate reference stress, equation (2), gives a reasonable prediction of failure time. Other theoretical and experimental work (e.g. (33), (34), (38), (40) - (45)) indicate that the limit load reference stress provides an upper bound to the failure times for creep ductile materials. A ductile material is defined, (43, 46, 47) as a material which is able to accommodate tertiary deformation without forming a macrocrack which propagates rapidly through the structure. Adequate ductility is obtained (46) when $\varepsilon_u/\varepsilon_r \sim 10$, where ε_u is the ultimate failure strain and ε_r is the strain at the onset of tertiary creep. For creep brittle materials (i.e. $\varepsilon_u/\varepsilon_r \sim 1$ (43)), the peak stationary state stress may be used to give an accurate prediction of failure time (46). The creep strain rate for many materials can be represented reasonably accurately, by a power stress dependance (Norton creep), i.e.

$$\dot{\varepsilon}^c = A\sigma^n \qquad (4)$$

For materials which creep according to equation 4. the peak, equivalent, stationary state stress can be interpolated (48) from the elastic solution (n = 1) and the perfectly plastic solution ($n = \infty$). Therefore, the representative rupture reference stress for a component made from a creep brittle material may be obtained (41) from

$$\sigma_R \simeq \frac{P}{P_L}\,\sigma_y\,\left(1 + \frac{1}{n}\,(\chi - 1)\right) \qquad (5)$$

In equation (5), χ is the ratio of the collapse load to the load to first yield, which is a measure of the stress concentration.

For ductile materials, equation (5) overestimates the reference stress and equation (2) underestimates the reference stress. The difference in the predicted failure times using these two extreme reference stresses can be very large, e.g. (44). However, a theoretical and experimental investigation of typical structures (46, 49) has resulted in the following rupture reference stress being proposed for ductile materials, i.e.

$$\sigma_R = \frac{P}{P_L}\,\sigma_y\,\left(1 + 0.13\,(\chi - 1)\right) \qquad (6)$$

In practical situations (43, 47), for which $\chi < 2.5$, it has been suggested (43, 47) that equation (6) may be conservatively simplified by using $\sigma_R = 1.2\,\frac{P}{P_L}\,\sigma_y$.

4. USE OF REFERENCE STRESSES FOR DESCRIBING CRACK TIP BEHAVIOUR

4.1 Background

So far it has been assumed that the components do not contain cracks or crack-like defects. It has also been assumed that either the material is creep ductile or creep brittle. For creep ductile materials failure occurs due to a large deformation mechanism. Hence the failure time can be bounded by using the reference stresses given by equations (2) and (6). For a creep brittle material it is assumed that failure is governed by the peak stress (equation (5)) and that the crack propagation time is negligible compared to the initiation time. For materials which are neither creep ductile or creep brittle (as defined above), it is necessary to consider the possibility of creep crack growth due to a succession of crack tip events.

The initial stress field in the vicinity of a crack, for elastic or small-scale yielding conditioning, is described by the elastic stress intensity factor, K. For stationary cracks, stress redistribution will occur and provided continuous damage effects are negligle, the stress field in the vicinity of the crack will approach that considered by Hutchinson (50) and Rice and Rosengren (51). In the stationary state the stress field, in the vicinity of the crack tip, for a Norton power law material, equation 4, is given by (e.g. (50)-(53))

$$\sigma_{ij}(r,\theta) = \left[\frac{C^*}{I_n A r}\right]^{1/(n+1)} \tilde{\sigma}_{ij}(\theta) \qquad (7)$$

where $\tilde{\sigma}_{ij}(\theta)$ are functions of angular position, θ. The path independent C*-integral is analogous to Rices' J-integral (54), i.e.

$$C^* = \int_\Gamma \left(W^* dy - T_i \frac{\partial \dot{u}_i}{\partial x} ds\right) \qquad (8)$$

where $W^* = \int \sigma_{ij} \, d \dot{\varepsilon}^c_{ij}$.

For a growing crack it is known (13, 55) that the type of stress field in the vicinity of the crack changes abruptly at n = 3. However, it has also been shown (13) that K-controlled crack growth is only likely for low n-values and high crack growth rates. Also, the C* parameter, which characterises the stationary state stress and strain-rate fields, has wide applicability (13); particularly for low crack growth rates and high n-values. There is no theoretical basis on which crack growth rates might be expected to be directly related to reference stress. However, reference stresses can be used in estimating C* (e.g. 13-18). Hence crack growth rates can be indirectly related to reference stresses.

4.2 Estimating C* using reference stresses

For a material which creeps according to equation 4, it can be shown (e.g. 14) that

$$C^* = \frac{1}{n+1} \frac{P}{B} \frac{d\dot{\Delta}^c}{da} \tag{9}$$

for steady load situations, where $\dot{\Delta}^c$ is the load-point displacement rate.

Combining equations 1 and 9 and noting that, in general, σ_R and D are functions of crack length, a, leads (17) to the following expression for C*, i.e.,

$$C^* = \frac{n}{n+1} \frac{PD}{B} \dot{\varepsilon}^c(\sigma_R) \left\{ \frac{1}{n} \cdot \frac{dD}{da} \cdot \frac{1}{D} + \frac{d\sigma_R}{da} \cdot \frac{1}{\sigma_R} \right\} \tag{10}$$

It should be noted that the reference stress, σ_R, and reference multiplier, D, in equation 10 are those appropriate to the load point displacement.

For situations in which the variations of σ_R and D with crack length, a, have been obtained (e.g. 18) then equation 10 can be used to estimate C* values. In general $\frac{1}{n}.\frac{dD}{da}.\frac{1}{D} << \frac{d\sigma_R}{da}.\frac{1}{\sigma_R}$ and hence equation 10 reduces to

$$C^* \simeq \frac{n}{n+1} \; \frac{PD}{B} \; \dot{\varepsilon}^c(\sigma_R). \; \frac{d\sigma_R}{da}.\frac{1}{\sigma_{\bar{R}}} \qquad (11)$$

Equation 11 is similar to an approximate expression for C*, derived by Harper and Ellison (14) using a limit load approach, i.e.

$$C^* \simeq \frac{-n}{n+1} \; \frac{P\dot{\Delta}^c}{B} \; \left\{\frac{1}{m} \; \frac{dm}{da}\right\} \qquad (12)$$

where m is the ratio of the collapse load of the cracked component to that of the same uncracked component.

Equations 10 and 11 will give accurate C* estimates, but for practical situations, the variations of $\sigma_{\bar{R}}$ and D (appropriate to the load-point displacement) may not be known. However, they can be estimated by using equations 2 and 3. Also, when estimating J for structure of strain hardening material, Ainsworth (56) has shown that limit load reference stresses are reasonably accurate for both tension and bend type specimens. Therefore, since experimental results indicate an approximately linear dependence of crack growth rate on C* (rather than a strong power dependence) such estimates for $\sigma_{\bar{R}}$ and D may lead to reasonably accurate predictions.

An alternative approach to estimating C* on the basis of reference stresses may be obtained by analogy with approximate relationships between J and crack tip opening displacement, δ, used in yielding fracture mechanics. By using the Dugdale model (57), for a perfectly plastic material, it can be shown (e.g. 54) that for tensile cracks under plane stress conditions,

$$J \simeq \sigma_y \delta \tag{13}$$

Rice (54) shows how the effect of strain hardening can be included in the relationship between J and δ. However, Turner (58) points out that the effects of varying constraint, work hardening, the increase of J and lack of uniqueness in the definition of δ can be approximately included by modifying equation 13, i.e.,

$$J \doteq \beta \sigma_y \delta \tag{14}$$

where β is in the range $1 \leq \beta \leq 3$. The lower values of β apply to tensile loading and the higher values to bending (58).

The analogous relationship between C* and $\dot{\delta}$ is

$$C^* \simeq \beta \sigma_R \dot{\delta} \tag{15}$$

where $\beta \simeq 1$ for purely tensile situations and $\beta \simeq 3$ for bending situations.

By using equation 1, equation 15 becomes

$$C^* = \beta D \sigma_R \dot{\varepsilon}^c (\sigma_R) \tag{16}$$

where D and σ_R are appropriate to the crack tip opening displacement; they may be estimated using equations 2 and 3. Equations 15 and 16 have been used (59) to process data from tests of 316 stainless steel specimens with surface, thumbnail cracks, subjected to tensile loading at 600°C. Because the specimens were subjected to tensile loading, β = 1 was used. Also, the experimental results indicated that the crack faces opened up practically uniformly, except very close to the crack tips. Therefore δ was taken to be the surface opening at the centre of the thumbnail cracks. The reference stress, $\sigma_{\bar{n}}$, was simply taken as the net section stress. When using equation 15, average experimental $\dot{\delta}$ values were used and when using equation 16,a D-value was determined experimentally using an equation similar to equation 3 (i.e. D = $\delta^i / \varepsilon^i(\sigma_R)$). Using both methods, the thumbnail crack data was found to fall within the scatter-band of data obtained from compact tension tests. Therefore, equations 15 and 16 offer a very simple means of estimating C* values in practical situations.

Equation 16 is similar to an expression derived by Ainsworth (13), i.e.

$$C^* \simeq R\,\sigma_R\,\dot{\varepsilon}^c(\sigma_R) \tag{17}$$

where R is described (13) as a 'characteristic distance', which is estimated (13) from

$$R \simeq (K/\sigma_R)^2 \tag{18}$$

The characteristic distance, R, in equation 17 is equivalent to the quantity βD in equation 16. Hence, the characteristic distance, R, is related to the reference multiplier, D, appropriate to the crack tip opening displacement.

4.3 Initiation

It is known (i.e. 60) that crack growth may not start until after a significant incubation period. A critical value of crack tip opening displacement, $\hat{\delta}$, is often used (i.e. 61-63) to estimate the incubation period. Approximate solutions for crack tip opening displacements have been obtained (e.g. 61, 64).

From equation 1, the incubation time, t_i, can be estimated from

$$\varepsilon^c(\sigma_R, t_i) = \hat{\delta}/D \tag{19}$$

i.e. it is the time required to accumulate a strain of $\hat{\delta}/D$ at the reference stress, σ_R. The reference stress can be approximated by equation 3 and the reference multiplier, D, is appropriate to the crack tip opening displacement. In the absence of accurately determined D values, $D \doteq R/\beta$ can be used, where R is given by equation 18 and $\beta \doteq 1$ in tensile cases and $\beta \doteq 3$ in bending situations. Therefore equation 19 reduces to

$$\varepsilon^c(\sigma_R, t_i) \simeq \beta(\hat{\delta}/R) \tag{20}$$

By assuming a simple notch tip shape (i.e. semi-circular) and estimates for strain rates on the notch surface, Ainsworth (61, 64) has obtained expressions similar to equations 19 and 20. Ainsworth's analyses result in initiation times predicted by

$$\varepsilon^c(\sigma_R, t_i) = \mu(n) \left(\frac{\hat{\delta}}{R}\right)^{\frac{n}{n+1}} \tag{21}$$

Ainsworth (64) concludes that $\mu(n)$ is sensibly independent of n and it is suggested that $\mu \doteq 1.5$ should be taken. For high n values, equation 21 reduces to equation 20. Also, taking $\mu = 1.5$ is consistent with the likely range for β (i.e. $1 \leq \beta \leq 3$).

5. CONCLUSIONS

By using reference stresses and appropriate reference multipliers, accurate predictions can be obtained for the creep deformations of components.

For components made of creep ductile materials, rupture reference stresses can be bounded and hence failure times can be bounded. Calladines (48) interpolation method can be used to estimate peak stationary state stresses, using elastic and perfectly plastic solutions. The peak stationary state stress can then be used to predict failure times for components made from creep brittle materials.

When component failure is controlled by creep crack growth, reference stresses cannot be used directly to predict crack growth rates. The so-called C* parameter appears to be most widely applicable for predicting creep crack growth rates. However, the reference stress concept can be used to estimate C* values. Hence, reference stresses can be used indirectly to predict creep crack growth rates. Creep crack growth rates have an approximately linear dependence on C*, rather than a strong power dependence. Hence C* values estimated on the basis of reference stresses result in reasonably accurate predictions for practical purposes.

For some materials, a critical crack tip opening displacement must be achieved before creep crack growth occurs. The reference stress approach can again be used to estimate the resulting incubation time.

References

(1) SIVERNS, M.J. and PRICE, A.T., 'Crack propagation under creep conditions in quenched 2¼ Cr-Mo steel', Int. J. Fracture, 1973.

(2) WILLIAMS, J.A. and PRICE, A.T., 'A description of crack growth from defects under creep conditions', J. Eng. Mat. Tech., 1975.

(3) LARDES, J.D. and BEGLEY, J.A., 'A fracture mechanics approach to creep crack growth', Mechanics of Crack Growth, ASTM STP 590, 1976.

(4) ELLISON, E.G. and HARPER, M.P., 'Creep behaviour of components containing cracks - a critical review', J. Strain Analysis, 1978.

(5) FU, L.S., 'Creep crack growth in technical alloys - a review', Eng. Fracture Mech., 1980.

(6) SADANANDA, K. and SHAHINIAN, P., 'Creep crack growth behaviour and modelling - critical assessment', Metal Science, 1981.

(7) SADANANDA, K. and SHAHINIAN, P., 'Review of the fracture mechanics approach to creep crack growth in structural alloys', Eng. Fracture Mech., 1981.

(8) MALIK, S.N., 'Elevated temperature creep crack growth: state-of-the-art review and recommendations', Nucl. Eng. Des., 72, 1982.

(9) WEBSTER, G.A., 'Crack growth at high temperature', Engineering Approaches to High Temperature Design (Eds. B. Wilshire and D.R.J. Owen), Pineridge Press, 1983.

(10) NIKBIN, K.M., SMITH, D.J. and WEBSTER, G.A., 'Prediction of creep crack growth from uniaxial creep data', Proc. R. Soc. Lond., A396, 1984.

(11) NIKBIN, K.M., SMITH, D.J. and WEBSTER, G.A., 'An engineering approach to the prediction of creep crack growth, J. Eng. Mat. Tech., 1986.

(12) HYDE, T.H., KUBBA, B., LOW, K.C. and WEBSTER, J.J., 'Experimental investigation of creep crack growth in a lead alloy', J. Strain Analysis, 1985.

(13) AINSWORTH, R.A., 'Some observations on creep crack growth', Int. J. Fracture', 1982.

(14) HARPER, M.P. and ELLISON, E.G., 'The use of the C* parameter in predicting creep crack propagation rates', J. Strain Analysis, 1977.

(15) AINSWORTH, R.A., 'The initiation of creep crack growth', Int. J. Solid Structures, 1982.

(16) AINSWORTH, R.A., 'The assessment of defects in structures of strain hardening materials', Eng. Fracture Mech., 1984.

(17) HYDE, T.H., 'Estimates of the creep paramerter C* in terms of reference stress', J. Strain Analysis, 1986.

(18) HYDE, T.H. and LOW, K.C., 'Use of the reference stress concept to assess the effect of crack growth increment size on the accuracy of creep crack growth predictions', J. Strain Analysis, 1986.

(19) PENNY, R.K. and MARRIOTT, D.L., 'Design for creep', McGraw-Hill, New York, 1971.

(20) MARRIOTT, D.L., 'A review of reference stress methods for estimating creep deformations', IUTAM Symposium Creep of Structures, 1970, Gothenburg.

(21) BOYLE, J.T., 'The reference stress method and its role in high temperature design'.

(22) KRAUS, H., 'Reference stress concepts for creep analysis', WRC Bulletin 227, June 1977.

(23) KRAUS, H., 'Creep analysis', John Wiley and Sons, New York, 1980.

(24) BOYLE, J.T. and SPENCE, J., 'Stress analysis for creep', Butterworths, London, 1983.

(25) ANDERSON, R.G., GARDNER, L.R.T. and HODGKINS, W.R., 'Deformation of uniformly loaded beams obeying complex creep laws', J. Mech. Eng. Sci, 1963.

(26) MACKENZIE, A.C., 'On the use of a single uniaxial test to estimate deformation rates in some structures undergoing creep', Int. J. Mech. Sci., 1968.

(27) JOHNSSON, A., 'An alternative definition of reference stress for creep', I.Mech.E. Conf. Pub. 13, 1973.

(28) SIM, R.G., 'Reference stress concepts in the analysis of structures during creep', Int. J. Mech. Sci., 1970.

(29) SIM, R.G., 'Evaluation of reference parameters for structures subjected to creep', J. Mech. Eng. Sci., 1971.

(30) SIM, R.G. and PENNY, R.K., 'Plane strain creep behaviour of thick-walled cylinders, Int. J. Mech. Sci., 1971.

(31) SIM, R.G., 'Reference results for plane stress creep behaviour', J. Mech. Eng. Sci., 1972.

(32) SIM, R.G., 'Creep of structures', Ph.D. thesis, University of Cambridge, 1968.

(33) PENNY, R.K. and MARRIOTT, D.L., 'Creep of pressure vessels', I.Mech.E. Conf. Pub. 13, 1973.

(34) HAYHURST, D.R., KELLY, D.A., LECKIE, F.A., MORRISON, C.J., PONTER, A.R.S. and WILLIAMS, J.J., 'Approximate design methods for creeping structures, I.Mech.E. Conf. Pub. 13, 1973.

(35) MARRIOTT, D.L. and LECKIE, F.A., 'Some observations on the deflections of structures during creep', Proc. I.Mech.E., 1963-64, 178 (Pt. 3L).

(36) FESSLER, H., HYDE, T.H. and WEBSTER, J.J., 'Stationary creep prediction from model tests using reference stresses', J. Strain Analysis, 1977.

(37) FESSLER, H., HYDE, T.H. and WEBSTER, J.J., 'Experimentally determined reference stresses for the prediction of creep deformations and life of components', I.Mech.E. Conf. on Recent Developments in High Temperature Design Methods, November, 1977.

(38) HYDE, T.H., 'Use of lead alloy model components in plasticity and creep studies', internal report.

(39) MARTIN, J.B. and LECKIE, F.A., 'On the creep rupture of structures', J. Mech. Phys. Solids, 1972.

(40) GOODALL, I.W. and COCKROFT, R.D.H., 'On bounding the life of structures subjected to steady load and operating within the creep range', Int. J. Mech. Sci., 1973.

(41) GOODALL, I.W. 'The creep of branch connections', I.Mech.E. Conf. on Creep Behaviour of Piping, February, 1974.

(42) LECKIE, F.A., HAYHURST, D.R. and MORRISON, C.J., 'The creep behaviour of sphere-cylinder shell intersections subjected to internal pressure', Proc. R. Soc. Lond. A. 349, 1976.

(43) FINDLEY, G.E. and GOODALL, I.W., 'Some observations on the design of pressure vessels operating in the creep range', I.Mech.E. Conf. on Failure of Components Operating in the Creep Range, April, 1976.

(44) FESSLER, H., HYDE, T.H. and WEBSTER, J.J., 'Prediction of creep rupture of pressure vessels', ASME Energy Tech. Conf., Sept. 1977, Houston, Texas, Paper No. 77-PVP-54.

(45) HENDERSON, J. and FERGUSON, F.R., 'Approximate methods for the prediction of creep rupture in components', Int. Conf. on Structural Mechanics in Reactor Technology,

(46) GOODALL, I.W., COCKROFT, R.D.H. and CHUBB, E.J., 'An approximate description of creep rupture of structures', I. J. Mech. Sci., 1975.

(47) GOODALL, I.W., LECKIE, F.A., PONTER, A.R.S. and TOWNLEY, C.H.A., 'The development of high temperature design methods based on reference stress and bounding theorems', J. Eng. Mat. Tech., Vol. 101, 1979.

(48) CALLADINE, C.R., 'Stress concentration in steady creep: interpolation between solutions in elasticity and plasticity', Proc. I. Mech. E., 1963-64, Vol. 178, Pt. 3A.

(49) AINSWORTH, R.A. and GOODALL, I.W., I. J. Solids Structures, 1976.

(50) HUTCHINSON, J.W., 'Singular behaviour at the end of a tensile crack in a hardening material', J. Mech. Phys. Solids, 1968.

(51) RICE, J.R. and ROSENGREN, G.F. 'Plane-strain deformation near a crack tip in power law hardening material', J. Mech. Phys. Solids, 1968.

(52) RIEDEL, H. and RICE, J.R., 'Tensile cracks in creeping solids', ASTM STP 700, Philadelphia, 1980.

(53) RIEDEL, H., 'The use and the limitations of C* in creep crack growth testing', Proc. ICF Int. Symp. on Fracture Mechanics, Beijing, China, 1983.

(54) RICE, J.R., 'A path independent integral and the approximate analysis of strain concentration by notches and cracks', Trans. ASME, J. App. Mech., 1968.

(55) HUI, C.Y. and RIEDEL, H., 'The asymptotic stress and strain field near a growing crack under creep conditions', Int. J. Fracture, 1981.

(56) AINSWORTH, R.A., 'The assessment of defects in structures of strain hardening material', Eng. Fracture Mech., 1984.

(57) DUGDALE, D.S., 'Yielding of steel sheets containing slits', J. Mech. Phys. Solids, Vol. 8, 1960.

(58) TURNER, C.E., 'Methods for post-yield fracture safety assessment', Post Yield Fracture Mechanics (2nd Ed), Elsevier Applied Science Publishers, 1984.

(59) HYDE, T. H., 'Creep crack growth in 316 stainless steel at 600^{o}C', paper submitted to J. Stress Analysis.

(60) HAIGH, J.R., 'The mechanisms of macroscopic high temperature crack growth, Part 1: Experiments on tempered CrMoV steels', Mat. Sci. Eng., 1975.

(61) AINSWORTH, R.A., 'The initation of creep crack growth', Int. J. Solids Structures, 1982.

(62) HYDE, T.H., LOW, K.C. and WEBSTER, J.J., 'Comparison of finite element predictions of creep crack growth with experimental data', J. Strain Analysis, 1984.

(63) HYDE, T.H., LOW, K.C. and WEBSTER, J.J., 'Finite element predictions of creep crack growth in centre cracked plates and comparison with experimental results', 2nd Int. Conf. on Creep and Fracture of Engineering Material and Structures, Swansea, 1984, Pineridge Press.

(67) AINSWORTH, R.A., 'Approximate blunting solutions for tensile cracks', Applied Solid Mechanics - 1 (Eds. A.S. Tooth and J. Spence), Elsevier Applied Science Publishers, 1986.