MODIFICATIONS IN THE CONCEPT OF THE STRESS INTENSITY FACTORS DUE TO INFLUENCE OF SHEAR

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Based on a recent extensive study of the state of displacements of the flanks of an internal elastic crack in an infinite plate submitted to in-plane loading at infinity it was shown that the influence of shear in the plate imposes various limitations and eventual modifications in our concepts of the components of the stress intensity factors (SIFs).

Since pure shear of the plate results in an immediate non-congruent overlapping of the flanks of the crack, there is a need for a reliable solution tackling the problem of pure shear with the crack flanks closed for defining the real K_{II}-mode. In mixed K_I- and K_{II}-modes of deformation the existing definitions for SIFs are valid only in the domain of loading of the plate, where the contribution of the K_I-mode counterbalances the deleterious effect of the K_{II}-mode, in closing the crack flanks, and guarantees their non-overlapping.

For the plane-stress solution the development of the out of plane deformations makes the introduction of the K_{III}-mode compulsory in describing the state of stress at the crack tip. Moreover as soon as a shear mode of deformation exists, there develops also a round-about displacement of the lips of the crack, called *lip-sliding*, which brings progressively new points from behind the initial crack tip to the instantaneous position of a transient crack tip as the elastic loading of the plate is increased. Simultaneously, the decrease of the curvature at the crack tip of the ideal crack which is deformed to an ellipse results in a reduction of the SIFs and the necessity of introducing instead the notion of a stress concentration factor (SCF). All these phenomena are reviewed in this paper, based on the exact solution of the internal crack as this is described by Muskhelishvili's solution.

Introduction

Little attention was paid up to now in the linear elastic fracture mechanics (LEFM) to the displacement fields of the flanks of an internal crack in an elastic infinite plate submitted to an in-plane biaxial load at infinity, although closed-form solutions exist for this particular problem based on the Muskhelishvili complex stress function theory [1].

Since the definition of the K_I, K_{II} and K_{III} components of the stress intensity factor (SIF) in LEFM is based up-to-now to the expressions of stresses at the crack tip as those are derived from their one-term singular expressions of the series expansion of the $\Phi(z)$, the mathematically ideal crack is assumed at its initial undeformed state and the only displacements evaluated until recently were the displacements of the initial crack-tip based either on the singular, or on the so-called two-term solutions. These displacements gave a rather erroneous picture of the deformed crack since they did not reveal the exact modes of deformation of the flanks of the elastic crack.

The author and his co-workers [2] presented recently a complete and exact study of the shape of the deformed elastic crack based on the exact solution given by Muskhelishvili [1]. They gave the equation of the shape of the ideal crack which when deformed becomes an ellipse with its major axis angularly displaced relatively to the axis of the initial crack. They established also the *lip-sliding* phenomenon of the flanks of the crack due to shear and defined the round-about rotation of the flanks.

The particular characteristics of the case of a central crack loaded under pure shear conditions were described in ref.[3]. It was shown that for a pure shear loading there is always a non-congruent overlapping of the crack flanks which means that opposite points on the lips of the crack before deformation are displaced during deformation in opposite directions so that the touching points of the lips come from different pairs of points of the undeformed crack. The elliptic shape of the deformed crack given by the exact solution was compared with two-branch parabolic shapes derived from the singular and the two-term solutions. The lip sliding phenomenon, which has its cause to the shear loading, and the variation of the curvature of the crack deformed shape at its tip were studied extensively [4].

The deformed crack under tension or compression without shear was studied in refs.[5,6]. In the loading of the crack flanks exempt of shear makes the components of their displacement to depend only on one term. Thus, u-displacements along the crack axis are varying only linearly whereas u-displacements normal to the crack axis are functions of the elliptic term. In this way there is no angular displacement of the crack during deformation but there is always change (positive or negative) of the crack length of the elliptic shape of the crack. The same happens also in the case of shear. The crack length remains unaltered only for hydrostatic tension or compression of the plate. This variation of crack length holds also for shear and the singular solution and this corrects the fallacy that the crack length under pure shear remains unchanged [3].

If the plate is extended the crack flanks are always open with the crack length larger than its initial length for $k>1.0$ (where k is the biaxiality factor, $\sigma_2=k\sigma_1$ the external loading of the plate) and smaller than this length for $k<1.0$. On the contrary, for compressive external loading the crack lips close congruently and there is no stress intensity factor. This phenomenon was proved also experimentally for static and dynamic loading modes [6,9].

Reference [7] indicates that if the plate is loaded under mode II and it is under plane stress condition the deformation of the crack flanks not only follow the phenomena described in refs.[3,4], but also develop out-of-plane shears which engender K_{III}-modes of deformation. Out-of-plane displacements of the flanks were studied, by using the exact solution based on complex stress potentials and conformal mapping, indicating that antisymmetric twisting displacements are appearing in opposite directions for opposite flanks with maxima (positive or negative) at the vertices of the elliptic shapes of the deformed crack.

Experimental evidence with rubber membranes, used in order to show spectacularly the phenomena developed around the cracks due to shear, indicated qualitatively the deformation of the ideal crack to an ellipse with its axis angularly displaced from its initial position by a double rotation, the one due to the linear term and the other due to the elliptic term of the components of the in-plane displacements. The slip-sliding phenomenon was also indicated by scribing an isometric network before deformation and photographing its distorted shape during deformation [8].

In this paper the consequences of these phenomena, especially those due to the shear component of loading, are studied on the accurate evaluation of the components of the stress intensity factor and conditions are established for their validity. Suggestions were also advanced for remedying the incompatible combinations of loading creating overlapping of the crack flanks and thus invalidating the initial elastic solution.

Theoretical Considerations

We consider an infinite elastic plate, containing an internal crack of length $2a$, loaded at infinity with principal stresses σ_0 and $k\sigma_0$ (Fig.1). The origin O of the Oxy-coordinate system is placed in the middle of the crack with the Ox-axis directed along the crack-axis, and the principal loading axis (Oy_0) subtending an angle β with the Ox-axis.

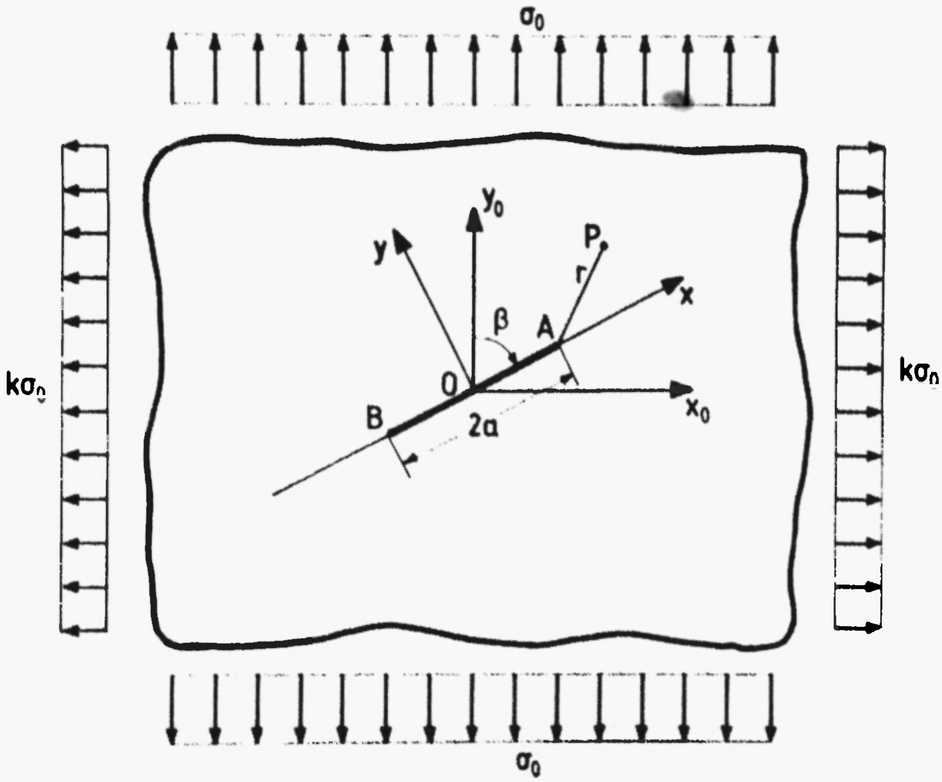

Fig. 1

The geometry of a biaxially loaded internal slant crack of length 2a.

The elastic displacements along the crack-lips for the abovementioned geometry have been derived in ref.[2] from the appropriate complex potentials of Muskhelishvili, assuming no rigid-body rotation at infinity, as follows

$$u^e_{\pm}(x) = c\{(1-k)(\cos 2\beta)x \pm (1-k)(\sin 2\beta)(a^2-x^2)^{1/2}\} \quad (1)$$

$$u^e_{\pm}(x) = c\{(1-k)(\sin 2\beta)x \pm [(1+k)-(1-k)\cos 2\beta](a^2-x^2)^{1/2}\} \quad (2)$$

The pair (u^e_+, u^e_+) denotes, for every value of the variable x in the interval [-a,+a], the Cartesian components of the displacement vector on the upper crack-flank and (u^e_-, u^e_-) the respective components on the lower one. The multiplicative factor c is defined by $c=\sigma_0/E$ and $c=\sigma_0(1-\nu^2)/E$ for plane-stress, or plane-strain conditions, respectively. In these relations E and ν are the Elastic modulus and Poisson's ratio of the material of the plate, whereas k is the biaxiality factor.

Relations (1) and (2) represent the *exact* forms for the displacements at every point along the upper or lower crack flanks without any regional limitation and, therefore, they are suitable expressions for the investigation of the mechanism of deformation and of the shape of the whole deformed mixed-mode crack.

The linear terms of the displacements in relations (1) and (2) have the same sign for both lips. Thus, they lead to a crack deformation which leaves the crack flanks straight, passing through the origin of the coordinate system. On the contrary, the non-linear terms, which have opposite signs for the upper and lower crack flanks, lead to an opening, or eventual overlapping, of the crack flanks and to a curved deformed crack.

It must be pointed out that any solution of the problem predicting overlapping lips must be excluded, because in such a case the initial boundary conditions of stress-free crack flanks is violated.

Figure 2 shows the components of displacement and how an initial crack, AB, of normalized length 2a, deforms. In this figure the three parameters of the crack are $\beta=70^{0}, k=-0.5^{0}$ and c=0.25. The crack tips move to the points A' and B', according to the linear terms, and the generic point C moves to the point C'. Then, the non-linear terms lead to an opening and sliding of the lips and the double point C' splits to the point C_{+} on the upper and C_{-} on the lower deformed flank.

We observe that the opening and sliding of the flanks does not start from the initial crack AB, but from the straight segment A'B'. Thus, the segment A'B' deserves the name *"effective crack-axis"*, and the deformed

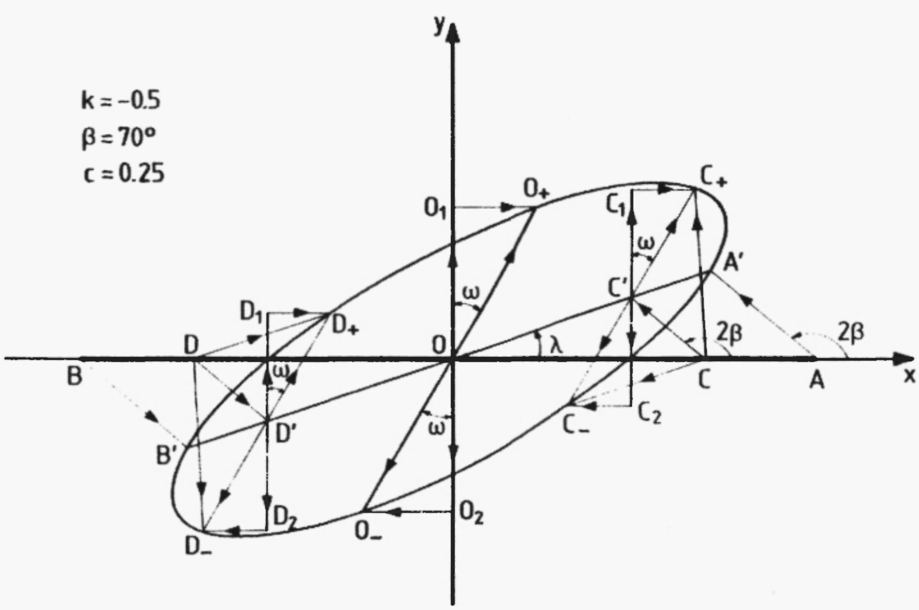

Fig. 2
The geometry of the displacements and the resulting deformed crack for $\beta=70^{0}$, k=-0.50 and c=0.25.

crack is skew symmetric with respect to this effective crack-axis. Moreover, because the effective crack-axis in general presents an angular displacement relatively to the initial crack-axis, both non-linear u- and υ-displacements produce an opening and a sliding effect as well.

While the exact displacements depend on a self-similar manner on the loading level defined by the factor c or, equivalently, on the principal stress σ_0 at infinity the shape of the deformed crack does not follow the same dependence. This is because the parametric equations of the shape of the crack are depending on the displacements through the relationships:

$$x_c^e = x+u^e(x)$$
$$y_c^e = \upsilon^e(x) \quad (3)$$

where the x-parameter takes values in the interval [-a,+a]. Thus, only the y-coordinates of the deformed crack-lips are proportional to the loading σ_0, and this is not true for the respective x-coordinates.

Moreover, the shape of the deformed mixed-mode crack cannot be investigated in terms of the stress intensity factors K_I and K_{II}, which are proportional to σ_0. Such an investigation demands to be made in terms of three parameters, and we have chosen for this purpose the natural parameters of the problem, i.e. c,β and k (see Fig.1).

A systematic analysis of the exact shape of the deformed crack leads to the following results [2]:

i) The exact shape of a deformed crack is always an ellipse. This ellipse degenerates into a straight segment, or even presents overlapping lips for some combinations of the parameters c,β and k.

ii) The major axis of the ellipse is angularly displaced by an angle θ with respect to the initial crack. For non overlapping flanks angle θ is given by [2]:

$$\sin 2\theta = 2c(1-k)\sin 2\beta\{[1-c(1+k)]^2+4c(1-k)[1-c(1+k)]\cos 2\beta+4c^2(1-k)^2\}^{-\frac{1}{2}}$$

Thus, angle θ takes values in the interval [-π/2,π/2] and it is positive for anticlockwise rotations and negative for clockwise rotations. Moreover, the angle θ for open crack flanks is always absolutely greater than the angle λ of inclination of the effective crack-axis with respect to the initial crack.

iii) Points A' and B', where the crack tips are displaced after deformation, do not coincide, in general, with the points of maximum curvature of the ellipse.

The inclination λ of the effective crack-axis is given by [2]:

$$\tan\lambda = \frac{c(1-k)\sin 2\beta}{1+c(1-k)\cos 2\beta} \quad (5)$$

and this slope equals to angle θ only if the deformed crack is a straight segment.

iv) The lengths a_1 and a_2 of the major and minor semi-axes of the ellipse respectively, which represent the exact length and the maximum opening of the crack after deformation, for not overlapping lips are given by [2]:

$$\left.\begin{matrix} a_1 \\ a_2 \end{matrix}\right\} = \frac{1+c(1+k)}{2}a \pm \frac{1}{2}a\{[1-c(1+k)]^2+4c(1-k)[1-c(1+k)]\cos 2\beta+4c^2(1-k)^2\}^{\frac{1}{2}} , \tag{6}$$

where the quantity a_1 refers to the (+) sign and a_2 to the (-) sign.

v) The x-coordinates of the points on the initial crack, which after deformation become the points of maximum curvature of the ellipse, are given by:

$$x_1 = \pm a\cos\theta \tag{7}$$

where θ is defined by Eq.(4). The (+) sign corresponds to a point on the upper crack-lip, if the angle θ is positive, or on the lower one if θ is negative, while the opposite holds for the minus sign in relation (7).

We may remark that the length of the deformed crack and its opening, as well as the slantness of the deformed mixed-mode crack can be defined in a natural way from the quantities a_1, a_2 and the angle θ of the exact solution. These quantities, which concern the whole deformed crack, cannot be derived from the approximative solutions, unless we impose definitions of these quantities in an inadequate way.

Conditions of Overlapping of the Crack Flanks

The analysis of the exact shape of the deformed mixed-mode crack allows the investigation, in a faithful manner, of the overlapping phenomenon of the crack flanks.

The linear elastic solution of the first fundamental problem for a mixed-mode crack, as well as any approximation of this solution, is meaningful as long as the predicted displacements result to an open deformed crack, or, at least, to a deformed crack which is a straight segment and its lips remain in simple contact. In the opposite situation where the predicted displacements yield overlapping crack flanks, the boundary conditions for the stress-free crack flanks are violated and the solution of the problem must be reconsidered.

The analysis of the shape of the deformed crack has shown that the deformed crack may be degenerated to a straight segment for the exact, as well as for the approximate solutions [8], and there is no possibility to obtain a piecewise overlapping of the crack-lips or a partially closed crack. Thus, the limits of validity of anyone of these solutions are given by the respective condition that the deformed crack degenerates to a straight segment presenting touching flanks in its full length.

We define first the angle ω subtended by the Oy-axis and the conjugate axis (O_-OO^+) to the effective axis A'OB' of the ellipse in Fig.2. It may be readily shown that all chords defined by two conjugate points of the ellipse, for example the segments C_+C_- and O_+O_- in Fig.2, are parallel to each other with a constant inclination, ω, with respect to the Oy-axis, given by:

$$\tan\omega = \frac{K_{II}}{K_I} = \frac{(1-k)\sin 2\beta}{(1+k)-(1-k)\cos 2\beta} \tag{8}$$

For the exact solution the lips of the deformed crack are in touch, if and only if the non-linear displacements along the crack flanks take place only along the effective crack-axis. The last condition holds for a positive slope λ of the effective crack, if the angles λ and ω are complementary.

Generally, it may be readily concluded that the touching-flanks condition is equivalent to the condition:

$$\tan\omega\tan\lambda = 1 \quad , \tag{9}$$

where $\tan\omega$ and $\tan\lambda$ are given by relations (8) and (5) respectively. So, the condition (9) in terms of the parameters c,β and k, may be written as:

$$c[(1+k)-(1-k)\cos 2\beta-c(1-k)^2+c(1-k^2)\cos 2\beta] = 0 \ . \tag{10}$$

In another context, it is valid that the deformed crack degenerates to a straight segment, if and only if the area of the ellipse is zero. This happens if the product of the lengths a_1 and a_2 given by Eq.(6) is zero, which, after some algebra, yields again the condition (10).

It is worthwhile mentioning that condition (10) does not imply necessarily that the degenerated deformed crack is situated along the Ox-axis of the initial crack. There are, indeed, combinations of c,β and k, for which condition (10) is fulfilled and for the same values of c,β and k the slope λ of the deformed straight crack is not zero. Only when the initial crack is parallel to one of the loading directions ($\beta=0^0$ or $\beta=90^0$) the degenerated deformed straight crack remains at its initial position [5].

Limiting Conditions for Touching Crack-Flanks

In the followings we shall investigate the regions in the three-dimensional parameter space (c,β,k), where the touching flank-conditions (17) and (19) hold.

Without restricting the generality of the problem we may assume that the *loading factor* c varies in the interval [-1,+1], the angle β takes values in the interval $[0^o,90^o]$ and the biaxiality factor k varies from $-\infty$ to $+\infty$. Large values of the loading factor c above the yield limit of the material invalidate the elastic solution since plastic enclaves start to develop around the crack tips which evolute and eventually occupy the whole stress field. Since the theory did not introduce any restriction concerning the loading level of the phenomena described, it is valid up to the point of plastic deformation.

However, for microcracks and similar defects inside the stress field which are strongly constrained by the surrounding elastic material high loading factors may be operative without violating the conditions for a totally elastic stress field.

The points in the (c,β,k)-space, for which the condition (10) is satisfied, constitute surfaces in this parametric space, which separate regions where the left-hand side (LHS) expression of Eq.(10) is positive or negative. It may be readily concluded that the regions, where the above expression is positive, represent (c,β,k)-combinations for open crack flanks. The opposite is true in the regions where the left-hand side expression (10) negative, and therefore, in these regions, the linear elastic solution invalid. Condition (10) satisfied for $c=0$. This is the trivial evident case of the unloaded crack. So, the aboyementioned surfaces possess a common branch the (k,β)-plane $(c=0)$.

For the exact solution and for $c\neq0$ the touching flank condition (10) is satisfied, if it is valid:

$$(1+k)-(1-k)\cos2\beta-c(1-k)^2+c(1-k^2)\cos2\beta = 0 \ . \qquad (11)$$

This non-trivial part of the touching-flank condition depends on the c-factor, and so it involves not only the k,β parameters, but also implicitly the external load σ_0, the mechanical properties of the material, and the prevailing plane-stress or plane-strain conditions as well. The surface defined by Eq.(11) is not simply connected, but it is composed of three different branches, whose investigation leads to unexpected results.

Figure 3 presents these three branches. In this figure the projections in the (k,β)-plane of the curves are plotted, where the surface intersects planes with c=const. The family of the curves in the right-hand side lower part of Fig.3 corresponds to the first branch of the surface, which is situated in the half space $c>0$, and to the right of the (c,β)-plane.

Each one of the curves of this family with $c>0$ begins at a point $k_i=\left(1+\frac{1}{c}\right)$ of the k-axis and goes asymptotically to $\beta=90^o$ for $k\to\infty$. Thus, the first branch of the surface (11) does not intersect the (k,β)-plane $(c=0)$. The region below the first branch corresponds to cracks with overlapping flanks. It may be readily verified that for $c>0$ and $\beta=0^o$ k-values smaller

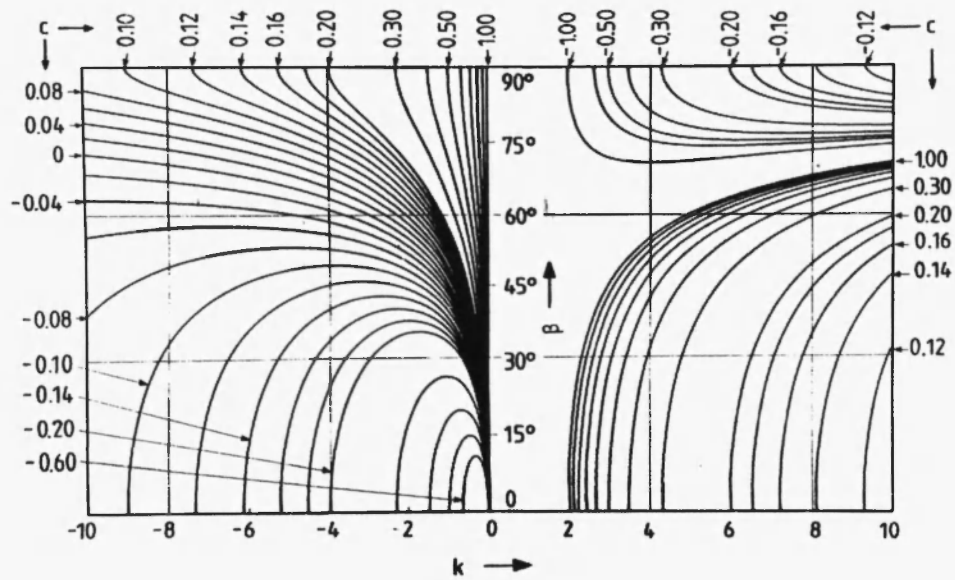

Fig. 3

The overlapping condition in the (k,β)-plane for various c-values, according to the two-term and the exact solution.

than $\left(1 + \frac{1}{c}\right)$ represent open cracks, while when k values are larger than $\left(1 + \frac{1}{c}\right)$ the left and right halves of the respective crack overlap each other.

The family of the curves in the RHS-upper part of Fig.3 corresponds to the second branch of the surface (1), which is situated in the half-space c<0 and to the right of the (c,β)-plane. This second branch, contrariwise to the first one, does not separate regions in the (c,k,β)-space with open and overlapping lips. The (c,k,β)-points below and to the left of this second branch correspond to cracks, for which the upper and lower lips overlap. This happens, for instance, when c=-0.2, $\beta=90^0$ and k=2.0. If k is increasing and surpasses the value k=6.0, while c=-0.2 and $\beta=90^0$ remain constant, namely if the (c,k,β)-point passes through the second surface branch, the crack flanks do not open, but their RHS- and LHS-parts overlap once more to each other. Thus, the region above the second branch corresponds to doubly overlapping cracks, and the whole region c<0,k>0 for every β-value corresponds to unacceptably deformed-cracks.

The family of the curves in the LHS-part of Fig.3 corresponds to the third branch of the surface (11). The third branch intersects the (k,β)-plane along a curve, which is obtained from Eq.(11) if we put c=0. The resulting curve is identical with the intersection of the separating cylindrical surface for open and overlapping cracks as this is given by the singular solution [8]. This cylindrical surface corresponds to cracks of mode-II deformation.

The points on the third branch of surface of Eq.(11) are now, for the exact solution, the points corresponding to mode-II cracks. This new "mode-II" deformed crack has generally a different position from its respective initial crack.

From this extensive study it is clear that for the exact solution the rule if a crack is a mode-II crack or not depends not only on the k- and β-parameters, but also on the c-parameter, that is on the applied stress σ_0, in association with the mechanical properties of the material and the prevailing plane-stress or plane-strain conditions of the cracked plate. This dependence is shown in Fig.3, where now the third branch of the surface of Eq.(11) is not a cylindrical one, parallel to the c-axis. Indeed, we observe in Fig.3 that the third branch of Eq.(11) cuts the plane c=1.0 along the vertical line k=0, then it turns to the left in a helical way, for decreasing c, and intersects the (c=0)-plane along the curve c=0 which coincides with the separating curve for the singular solution [8].

Beyond the plane c=0, the third branch for decreasing c's continues to turn dawnward, but at the same time it warps and evolutes into a sort of tunnel, which is progressively shrinking with decreasing c's. Finally, the tunnel terminates at a single point c=-1.0, k=0 and $\beta=0^0$. This singular point corresponds to a crack, whose length shrinks to a point, as one may readily verify from the relations (1) and (2).

The (c,k,β) points in the half space c>0, that lie on the upper and RHS space of the third branch, correspond to open cracks. On the contrary, the points in the same half space below and on the LHS space of the third branch represent cracks with overlapping lips. The situation is reversed in the half space c<0. In this space the points inside the "tunnel" represent open cracks, whereas the region outside the "tunnel" corresponds to cracks with overlapping lips. The cylindrical surface for c=0 lies below the third branch of the surface (11) for c>0, and outside the "tunnel" for c<0.

The cracks with k=0 and $\beta=0^0$, i.e. the cracks which are subjected to a tensile or compressive load parallel to them, present the only exception of the abovementioned statement. This can be seen in Fig.3, where the third branch of surface (11), as well as the cylindrical surface with c=0, cut the (c,k)-plane along the c-axis. Both stress intensity factors, K_I and K_{II}, for such cracks ($k=0,\beta=0^0$) are zero and no singular stress fields develop at their tips.

In every other case, where $K_I=0$, but $K_{II}\neq 0$, the novel result of this investigation of the crack-overlapping problem demands a reconsideration of the mode-II cracks. In particular, it is an open question whether a singular stress field develops in the case of the pure shear mode-II crack ($k=-1.0, \beta=45^{o}$), where the crack flanks press against each other. Moreover, it becomes now doubtful what a critical K_{II}-stress intensity factor means [10,11].

Then, it is unrealistic to check mixed-mode fracture criteria for cracks presenting overlapping flanks. This has been already done by a lot of authors in recent papers..An interesting remark is given in ref. [12] where the inadequacy of any fracture criterion is stated for cases where overlapping takes place.

For a further study of the surface defined by Eq.(11) we have plotted in Fig.4 the contour lines of Eq.(11) for parametric values of the β-angle. The family of the curves in the right upper part of Fig.4 corresponds to the first branch of Eq.(11). For any β=const. level the respective curve separates a LHS- and a RHS-region, which correspond to open and overlapping cracks, respectively.

The family of the curves in the right lower part of Fig.4 corresponds to the second branch of Eq.(11). These curves lie inside the region $c<0$ and $k>0$, i.e. in the region which corresponds to cracks subjected to compression-compression, and, as we have already mentioned, these cracks present for every β-angle an overlapping or a double overlapping.

The family of the curves in the LHS-part of Fig.4 corresponds to

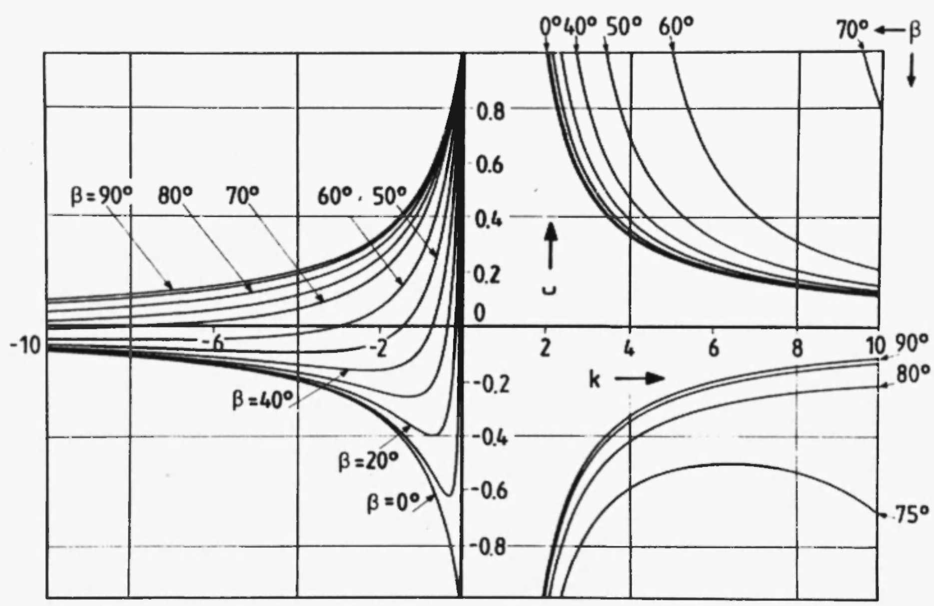

Fig. 4
The overlapping condition in the (k,c)-plane for various β-values, according to the exact solution.

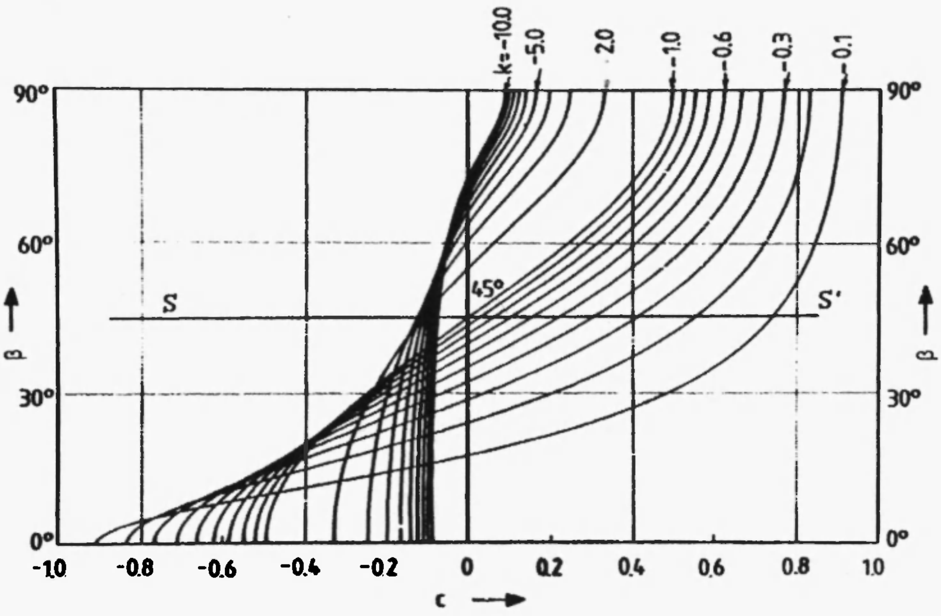

Fig. 5
The overlapping condition in the (c,β)-plane for negative k-values, according to the exact solution.

the third branch of Eq.(11). It is striking how much different is this surface from the cylindrical surface with c=0 corresponding to the singular solution [8], which has its equal level lines parallel to the c-axis. We also observe how abrupt is the slope of this surface close-by the (c,β)-plane. So, small variations of the biaxiality factor k close-by the (c,β)-plane may rapidly lead an open crack to one with overlapping flanks or vice versa.

Figure 5 presents a side-face of the third branch of the surface of Eq.(11). Here the projections of the curves in the (c,β)-plane are plotted, where the surface (11) intersects planes with k=const. and negative. The (c,β)-plane itself (k=0) cuts the third branch along the c-axis and the vertical line c=1.

Every other curve (k<0) intersects the β-axis, and it divides the (c,β)-plane together with the β-axis in four areas. The upper RHS area and the lower LHS one correspond to open cracks for the respective k-values, while the other two areas represent cracks with overlapping flanks.

Every horizontal line intersecting the β-axis at the same point with a $(k=k_0)$-curve lies completely in the areas where for $k=k_0$ the cracks present overlapping flanks. On the other hand, each one of these horizontal lines represents a mode-II crack ($K_I=0, K_{II}\neq 0$), e.g. the line SS' in Fig.5 represents the pure shear mode-II crack ($k=-1.0, \beta=45^0$).

Finally, from this detailed analysis of the internal oblique crack under biaxial loading at infinity it may be concluded that:

Any purely mode-II loaded internal crack presents always overlapping flanks. Therefore, it belongs to physically unacceptable solutions, thus necessitating a reconsideration of the initial boundary-condition problem of the internal crack in an infinite plate.

Fig.6 presents the case of an infinite plate submitted to pure shear with stresses σ_∞ and $-\sigma_\infty$ (k=-1.0) at infinity. It contains an oblique crack AB=2a subtending an angle $\beta=45^0$ with the loading axes. The amount of stresses σ_∞ is expressly taken quite high and equal to $\sigma_\infty/E=0.25$, in order to show the shape of the deformed crack.

The final shape of the loaded crack is the ellipse with the Ox'- and Oy'-axes as major and minor axes respectively, angularly displaced

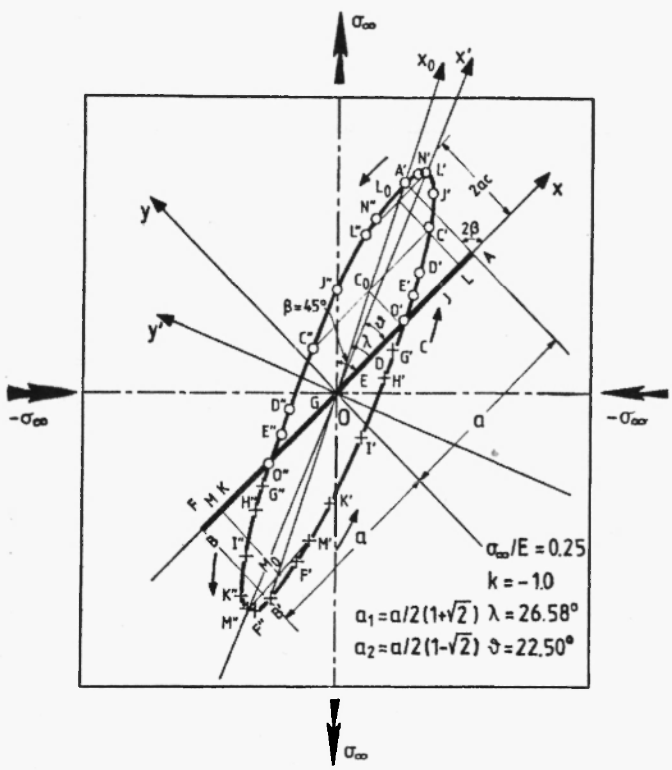

Fig. 6
The geometry of an internal crack in an infinite plate submitted to pure shear under plane stress conditions. The initial crack AB is angularly displaced to its effective crack axis under load, A'B', and then deformed to a final ellipse, by moving its upper lip downwards and its lower lip upwards, thus creating an overlapping of the faces of the crack.

from the Oxy-frame by the θ-angle. The effective crack axis is the line segment A'B', having a half-length, a', given by:

$$a' = a\{1+4c^2\}^{1/2}$$

Then, as the external loading is increased, the ratio c is increased and the length a' of the crack along the effective axis is increased. It suffices an external load of the order of one hundredth of the respective modulus of the material to increase the length of the effective crack-axis by 4 per thousand. Relations (1) and (2) indicate that, in the case of pure shear with $\beta=45^o$ and k=-1.0, there is a single linear displacement, normal to the initial crack-axis. Then, points A' and B' lie on the normals to the initial crack (AB) from its tips A and B. The angle of rotation of the effective crack axis, λ, is given by $\tan\lambda=2\sigma_\infty/E=2c$. Moreover, to every point of the effective crack-axis correspond two points on the final ellipse, defined by line-segments, parallel to the Ox-axis. Thus, the middle point O of the crack goes to points O' and O" respectively, and point C on the initial crack is displaced to point C_0 on the effective axis and then to points C' and C" on the final ellipse. It is obvious that $C_0=2cx_c$ and $C_0C'=C_0C''=$ $=2c(a^2-x_c^2)^{1/2}$.

It is worthwhile indicating that point C' corresponds to the upper flank of the crack, whereas point C" to the lower flank of it. It is then, clear that we have phenomena of *overlapping* of the faces of the crack. It is obvious that this overlapping phenomenon happens for any pure shearing loading.

Discussion and Conclusions

A study of the form of the displacements along the flanks of an elastic internal oblique crack in an infinite plate, submitted to a biaxial load at infinity revealed interesting results and paradoxes for this basic mode of deformation of the crack, which have up-to-now not thoroughly disclosed. The method of analysis was based on the exact solution given in closed form by Muskhelishvili [1].

It was shown that: i) Both flanks of the deformed crack, according to the exact solution, are angularly displaced by an angle λ, due to their linear terms of displacements, thus defining an effective crack-axis, whose slope λ depends on the angle of obliqueness, β, of the crack, on the loading step and the mechanical properties of the plate through c, and, finally, on the biaxiality factor k. ii) The notion of the effective crack-axis disappears for the singular solution, where no linear, common for both flanks, displacements exist. Accordingly, the crack-tips are not displaced, fact which is unacceptable from the physical point of view. iii) For the exact solution the non-linear terms of displacements, which make the flanks of the crack either to open, or to close and overlap to each other, are elliptic and these displacements are smooth and more moderate than the respective displacements derived from the singular or the two-term solutions. iv) The linear elastic crack-flank displacements may result to overlapping crack flanks. However, the overlapping condition in the case of the singular solution which implied that $K_I<0$, differs significantly from the respective one in the case of the exact solution. The latter involves not only the geometry and the loading of the cracked plate, but also the mechanical properties and the prevailing plane-stress or plane-strain conditions.

The overlapping phenomenon is of great importance, since it defines cases, where the basic concept of LEFM, that is the complex stress intensity factor, which is expressed as the vector sum of the K_I- and K_{II}-components, should be reconsidered, since in the cases of overlapping flanks the initial boundary conditions of the problem are strongly invalidated. v) All mode-II loaded internal cracks present from the beginning of the loading of the plate overlapping flanks, and therefore these cracks belong to the physically unacceptable solutions. vi) Addition of friction forces of constant amplitude along the crack flanks, which were assumed as a remedy in geomechanics, is unrealistic, since neither the components of stresses and strains along the flanks may be of constant value, nor they remain constant during loading, since they depend on the three physical parameters β,k and c. vii) All these phenomena are the consequences of the deformations of the crack flanks mainly due to the shear loading of the plate. Then, it becomes evident that a new confrontation of the problem of the sheared internal crack should be undertaken, which may consider the influence of the eventual in-plane and also the out-of plane [7] overlapping of the flanks of a sheared crack.

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