Theoretical Consideration of the Two-Dimensional Temperature Field in a Cylindrical Three-Layered Conductive Objects under an Electric Cross-Current

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ABSTRACT

The temperature field caused by a direct electric cross-current through a cylindrical three-layered conductive medium is given. The 3D temperature distribution drawing is reconstructed for such a medium. Special consideration is given for analysis of the influence of the current density distribution on temperature field. The numerical analysis is conducted for two systems: Zr-Na-C(graphite) and Cu-Sn-C(graphite). The results obtained might be useful for analysis of behavior of composition materials with cylindrical inclusions (for example, fibres) under electric current (superheating, heating less, thermo-strains, electroconvection mass transfer under conditions of technological processing or exploitation of some heterogeneous objects).

Key words: two-dimensional temperature field, cylindrical three-layered medium, Zr-Na- graphite system, Cu-Sn-graphite system, electro-heating, precondition of electro convection.

1. INTRODUCTION

Unlike the well-known solutions of the classical heat transfer theory in the case of poly-layered media with generation of heat in volume /1, 2/, similar objects, being influenced by electrophysical actions, are researched theoretically to a much smaller degree. Interest in the processes mentioned does not have only theoretical importance. In particular, series composition materials contain in their structure some foreign inclusions, among them ones in cylindrical form (for example rods or fibers, which can be included purposely into any matrix). Naturally, the inclusions of this kind have, as a rule, the conductivity and some other properties different from the external medium, and more often they have higher melting points. There are cases, which occur during the manufacturing, thermal treatment or operation of the mentioned compositions, when it is necessary to understand the picture and the character of interactions between such foreign inclusions and the surrounding current-carrying medium, where a number of processes of heat- and mass-exchanging character are arising during the

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passage of an electric current. The study of such processes has tahe certain interest, as they, for example, cause such effects as: temperature skew, thermoelastic stresses, thermal crack, electrohydrodynamical flows, and convective diffusional mass transfer (the dissolution of inclusions) /3-6/.

2. MODEL

To begin with let us imagine a cylindrical conductor i unlimited in the axial direction that has radius a (the cross-section is described in Fig. 1). It is located coaxially inside a current-carrying cylindrical layer e with external radius b, which, in its turn, is surrounded by the layer s (an external shell) with the outer radius c. Since the model is not limited in the axial direction, the problem is considered as two-dimensional. A direct electric current passes through the three-layered medium described in a direction perpendicular to axis (cross-current). This current is characterized by density j_0 in layer s far from the inner boundary $r=b$. It is clear that the temperature field thus produced by the constant in time electric current (Joule heating) should be non-uniform in volume.

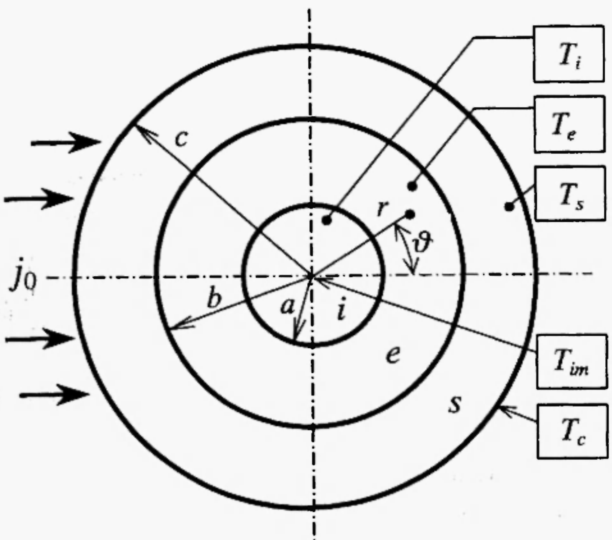


Fig. 1: Scheme of model (cross section): T_i is the temperature in the area i; T_e is the temperature in the layer e; T_s is the temperature in the layer s; T_c is the temperature on the superficial boundary of the outer layer s; T_{im} is the temperature in the centre (r=0).

For the solution of this problem we shall use the polar system of coordinates (r, ϑ). Fig. 1 shows a cross section of the model. We shall perform the analysis for two compositions: Zr-Na-C(graphite) and Cu-Sn-C(graphite). Zr and Cu are used for the internal cylinder i. We shall attribute the specific computations to the conditions when Na and Sn (medium e) are liquid and Zr, Cu, and graphite are solids. Graphite is used, for both cases, as an external conducting medium /5/. The physical properties of all specified materials that are necessary for the numerical calculations are shown in Table 1 /7-10/.

In order to solve this problem the first step was made by analysis of the electric-thermal processes in mediate annular layer $a<r>b$ (see Fig. 1) /11/.

3. FIELD OF ELECTRIC CURRENTS

To determine what temperature distribution arises throughout the model, we shall determine firstly, how the electric current density distributes. For this purpose we shall use an analogy between the dependencies in distributions of the electric induction for the dielectric cylinder unlimited in the direction of the axis, which is immersed in medium with other dielectric permeability /12/, and current density. The potentials in a steady current case must be a solution of the set of two-dimensional Laplace's equations (as is also the case in electric induction). The initial solutions of Laplace's equations are of the forms:

$$\left.\begin{aligned} V_i &= -E_{el}\, r\cos\vartheta + \sum_{n=1}^{\infty} A_{in} r^{-n}\cos n\vartheta, \\ V_e &= \sum_{n=1}^{\infty}\left(B_{en} r^{n} + C_{en} r^{-n}\right)\cos n\vartheta, \\ V_s &= \sum_{n=1}^{\infty}\left(B_{sn} r^{n} + C_{sn} r^{-n}\right)\cos n\vartheta, \end{aligned}\right\} \qquad (1)$$

where E_{el}, A_{in}, B_{en}, C_{en}, B_{sn}, C_{sn} are the constants which are subject to determination.

Let us take into account continuity of the potentials and the normal components of the current densities on the boundaries of layers (i - e) and (e - s):

$$\left.\begin{aligned} V_i\big|_{r=a} &= V_e\big|_{r=a}, \quad \lambda_i \frac{\partial V_i}{\partial r}\bigg|_{r=a} = \lambda_e \frac{\partial V_e}{\partial r}\bigg|_{r=a}, \\ V_e\big|_{r=b} &= V_s\big|_{r=b}, \quad \lambda_e \frac{\partial V_e}{\partial r}\bigg|_{r=b} = \lambda_s \frac{\partial V_s}{\partial r}\bigg|_{r=b}, \end{aligned}\right\} \quad (2a)$$

where $\lambda_{i,e,s}$ are the conductivities, which we shall consider as constants.

We suppose the thickness of the outer layer *s* much larger than dimensions of the radius of cylinder *i* and layer *e*: *c-b>>b-a*, *c-b>>a*. Thus we shall formulate, in place of the boundary condition for surface *r=c*, the condition for the infinity

$$\lim_{r\to\infty} V_s = -E_{el} r\cos\vartheta, \quad (2b)$$

where E_{el} is the electric field intensity in the remote parts of layer *s*.

The use of boundary conditions (2a) and (2b) for the total solutions (1) of Laplace's equation allows the solutions sought for to be obtained:

$$\left.\begin{aligned} V_i &= -\frac{2E_{el}\,\varepsilon}{(L_i+1)} r\cos\vartheta, \\ V_e &= E_{el}\,\varepsilon\left(-r+\frac{a^2}{r}\right)\cos\vartheta, \\ V_s &= E_{el}\left(-r+\gamma\frac{b^2}{r}\right)\cos\vartheta, \end{aligned}\right\} \quad (3a)$$

where

$$\left.\begin{aligned} \varepsilon &= \frac{2k^2 L_s(L_i+1)}{(k^2-1)(L_i L_s+1)+(k^2+1)(L_s+L_i)}, \\ \gamma &= \frac{(k^2-1)(L_s L_i - L_s - L_i)+k^2+1}{(k^2-1)(L_s L_i+1)+(k^2+1)(L_s+L_i)}, \\ \Lambda &= \frac{\lambda_e-\lambda_i}{\lambda_e+\lambda_i}, \quad L_i = \frac{\lambda_e}{\lambda_i}, \quad L_s = \frac{\lambda_e}{\lambda_s}, \quad k = \frac{b}{a}. \end{aligned}\right\} \quad (3b)$$

Further, the solution of this problem, when one proceeds from potentials (3a) and takes into account boundary conditions (2a), allows to determine components of current density vectors for all layers:

$$\left.\begin{aligned} j_{ir} &= -\lambda_i \frac{\partial V_i}{\partial r} = \frac{2j_0}{L_i+1}\varepsilon\cos\vartheta, & j_{i\vartheta} &= -\lambda_i \frac{1}{r}\frac{\partial V_i}{\partial\vartheta} = -\frac{2j_0}{L_i+1}\varepsilon\sin\vartheta, \\ j_{er} &= -\lambda_e \frac{\partial V_e}{\partial r} = j_0\,\varepsilon\left(1-\Lambda\frac{a^2}{r^2}\right)\cos\vartheta, & j_{e\vartheta} &= -\lambda_e \frac{1}{r}\frac{\partial V_e}{\partial\vartheta} = -j_0\,\varepsilon\left(1+\Lambda\frac{a^2}{r^2}\right)\sin\vartheta, \\ j_{sr} &= -\lambda_s \frac{\partial V_s}{\partial r} = j_0\left(1+\gamma\frac{b^2}{r^2}\right)\cos\vartheta, & j_{s\vartheta} &= -\lambda_s \frac{1}{r}\frac{\partial V_s}{\partial\vartheta} = -j_0\left(1-\gamma\frac{b^2}{r^2}\right)\sin\vartheta. \end{aligned}\right\} \quad (4)$$

The knowledge of the current densities components (4) enables us to calculate numerically the whole picture of the complete distribution of the electric current density in the whole model. For computations related to the specific chosen compositions, data from Table 1 are used. These two compositions chosen differ from each other (see Table 2): a) $\Lambda>0$ (for system Zr_{sol}-Na_{liq}-C_{sol}), and b) $\Lambda< 0$ (for system Cu_{sol}-Sn_{liq}-C_{sol}). In this paper any thermal expansion is neglected, and the case of an immovable liquid medium is considered. The conditions leading to the appearance of convection in it will be analysed separately, not here. It should be noted that a liquid current-carrying conductor in the two-dimensional problem is "susceptible" to a lesser degree to the action of the spatial Lorentz-force that arises here than it is in the three-dimensional problem, for example, at the electroconvection in a current- carrying liquid surrounding a spherical solid inclusion /13/. In particular, if a liquid conductor has a uniform conductivity then the spatial Lorentz-force in the three-dimensional case results in hydrodynamic flows /14/, but in the two-dimensional case it does not result in such flows (this will also be analyzed later in another paper). Along the vertical axis *z* the reduced components and complete reduced values of the electric current densities in the relative dimensionless units:

Table 2

Numerical values of parameters of equations

Parameters	System			
	$Zr_{(sol)}$-$Na_{(liq)}$-$C_{(sol)}$		$Cu_{(sol)}$-$Sn_{(liq)}$-$C_{(sol)}$	
λ_e-λ_i	8.03		-23.03	
Λ	0.711		-0.857	
L_i	5.92		0.0769	
L_i	122.75		20.42	
	Min.	*Max.*	*Min.*	*Max.*
J_r	0	1.977	0	3.376
J_ϑ	-3.141	0	-3.375	0
J	0.384	3.144	0.388	3.377
W_i	4.95		4.82	
W_n	2.07	671.05	2.66	405.98
$\Phi_n(r,\vartheta)$	0	1.304	0	2.664
$\Phi_n^*(r,\vartheta)$	0	1.276	0	3.030

Table 1

Properties of components

Temperature	Component	Conductivity λ_n	Heat capacity c_n	Heat conductivity k_n	Density ρ_n
400 K (127 °C)	Zr	1.40×10^6	0.27	21.10	6.44×10^3
	Na	5.23×10^6	1.37	147.32	0.93×10^3
	C (graphite MPG-6)	0.08×10^6	1.00	82.00	2.00×10^3
600 K (327 °C)	Cu	28.21×10^6	0.42	370.30	8.93×10^3
	Sn	2.00×10^6	0.26	33.70	6.93×10^3
	C (graphite MPG-6)	0.10×10^6	1.40	70.00	2.00×10^3

$$J_{nr} = \frac{j_{nr}}{j_0}, \quad J_{n\vartheta} = \frac{j_{n\vartheta}}{j_0}, \quad J_n = \sqrt{J_{nr}^2 + J_{n\vartheta}^2}. \quad (5)$$

are plotted (Fig. 2)

The fourth plots (the lowest ones) in Fig. 2 are vertical sections of a 3D plot for J_n at ϑ=0. The extreme values of the electric current density in the reduced units are shown in Table 2. All calculations here and further on are made only for the first quadrant; for other quadrants the results are mirror symmetrical. The general picture of the current density distribution depends on the numerical values (λ_e- λ_i), Λ, L_i and L_s (3b). Using Eqs. (4), it is possible for each point in the areas of the model to construct the vector of current density, which in the aggregate gives the vectorial field (Fig. 3). The vector scales for both cases are equal. The differences in the pictures in Figs. 3(a) and 3(b) are the consequence of the sign and the absolute value of the parameter Λ.

4. TEMPERATURE FIELD

Let us consider further what influence will the found distribution of the electric current density have on a temperature field formed in this way. In accordance with the Joule-Lenz law we may write the function of the density of the thermal capacity (specific generation of heat), i.e. the quantity of energy, which will be allocated in the unit of volume per a time unit in each of three media. In general, this expression can be written as:

$$Q_n = \frac{j_{nr}^2 + j_{n\vartheta}^2}{\lambda_n}, \quad (6)$$

where n= i, e, s. Now, substituting Eqs. (4) in Eq. (6) and taking into account the relation between heat and temperature, it is possible to write down equations for the densities of the "temperature sources":

$$W_i = \frac{Q_i}{c_i \rho_i} = \frac{4 j_0^2 \varepsilon^2}{(L_i + 1)^2 \lambda_i c_i \rho_i}, \quad (7)$$

$$W_e = \frac{Q_e}{c_e \rho_e} = \frac{j_0^2 \varepsilon^2}{c_e \rho_e \lambda_e}\left(1 + \Lambda^2 \frac{a^4}{r^4} - 2\Lambda \frac{a^2}{r^2}\cos 2\vartheta\right), (8)$$

$$W_s = \frac{Q_s}{c_s \rho_s} = \frac{j_0^2}{c_s \rho_s \lambda_s}\left(1 + \gamma^2 \frac{b^4}{r^4} + 2\gamma \frac{b^2}{r^2}\cos 2\vartheta\right), (9)$$

where $c_{i,e,s}$ and $\rho_{i,e,s}$ are the heat capacities of the unit of mass and the densities of materials of which cylinder i and layers e, s are formed respectively.

In the present model we suppose approximate heat capacities and densities are constants. The computer calculation of distribution for the densities of the "temperature sources" W_n with the help of Eqs. (7)-(9) is shown in Fig. 4. The maximum and minimum values of W_n are offered in Table 2 (value W_i for area i is constant (see Eq. (7)). A more rigorous analysis must lead to the possibility of flows of liquid in the area e (but in this paper the questions of the electromagnetic forces and fluid dynamic effects are not considered).

To find the distribution of temperature, it is necessary to solve a system of the heat transfer equations. Here the stationary problem

$$\Delta T_n + \frac{W_n}{a_n} = \frac{\partial^2 T_n}{\partial r^2} + \frac{1}{r}\frac{\partial T_n}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T_n}{\partial \vartheta^2} + \frac{W_n}{a_n} = 0, (10)$$

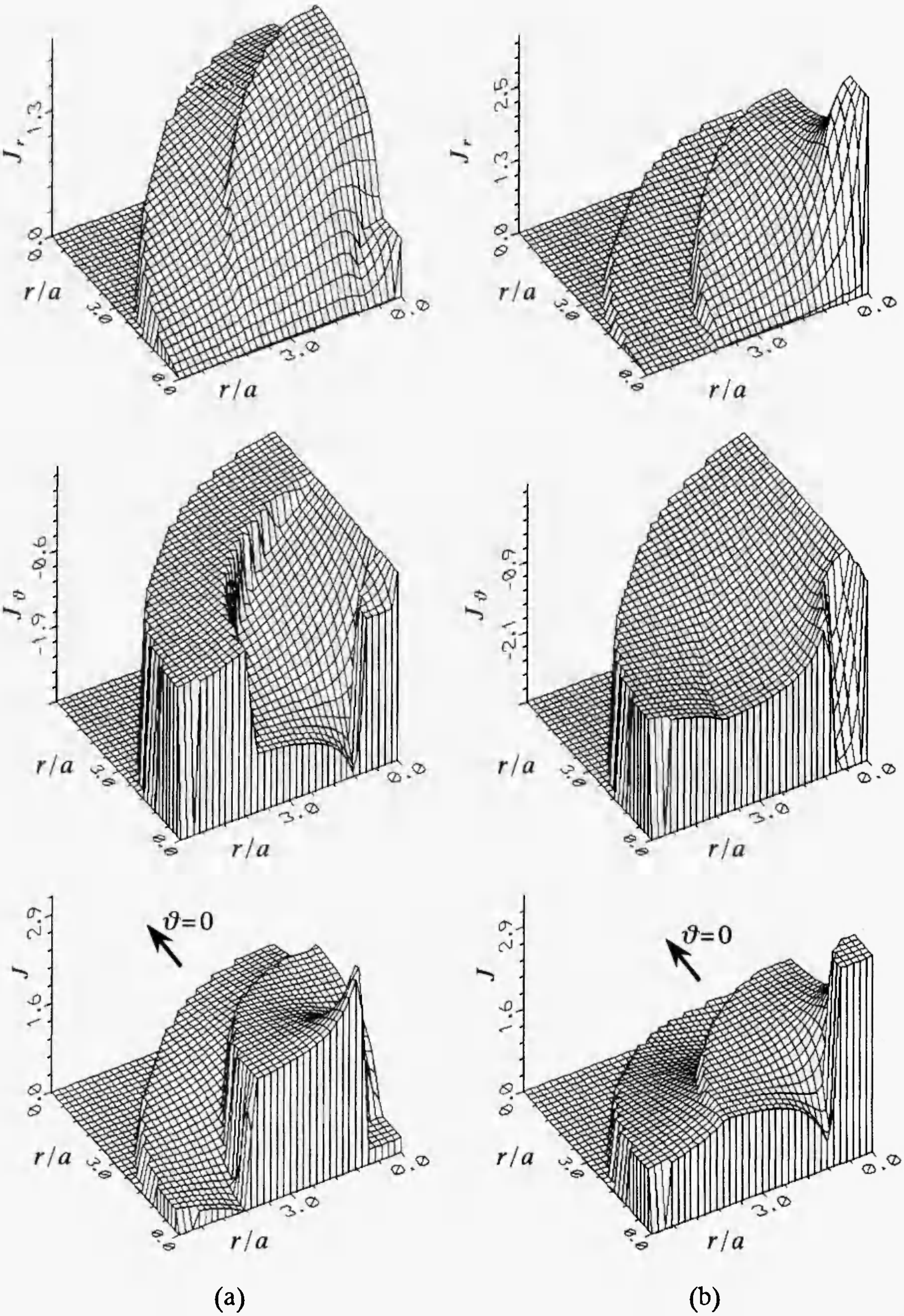
will be solved,

where Δ is the two-dimensional Laplacian, a_n=k_n/($c_n\rho_n$) is the thermal diffusivity, k_n is the thermal conductivity. The boundary conditions for surfaces r=a and r=b are similar to that of Eq. (3a) with obvious replacement of potential to temperature, and electric conductivity for heat conductivity. Let us transform Eq. (10) to the following ones:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \vartheta^2}\right) T_n = -\frac{W_n}{a_n}. \quad (11)$$

Suppose the function sought for T_i is a sum of two functions /2/:

$$T_i = T_{i1}(r) + T_{i2}(r, \vartheta), \quad (12)$$


J_r
1.3
0.0
r/a
3.0
6.0
3.0
0.0
r/a
J_r
2.5
1.3
0.0
r/a
3.0
6.0
3.0
0.0
r/a
J_ϑ
-0.6
-1.9
r/a
3.0
6.0
3.0
0.0
r/a
J_ϑ
-0.9
-2.1
r/a
3.0
6.0
3.0
0.0
r/a
J
2.9
1.6
0.0
ϑ=0
r/a
3.0
6.0
3.0
0.0
r/a
J
2.9
1.6
0.0
ϑ=0
r/a
3.0
6.0
3.0
0.0
r/a
(a)
(b)

continued....

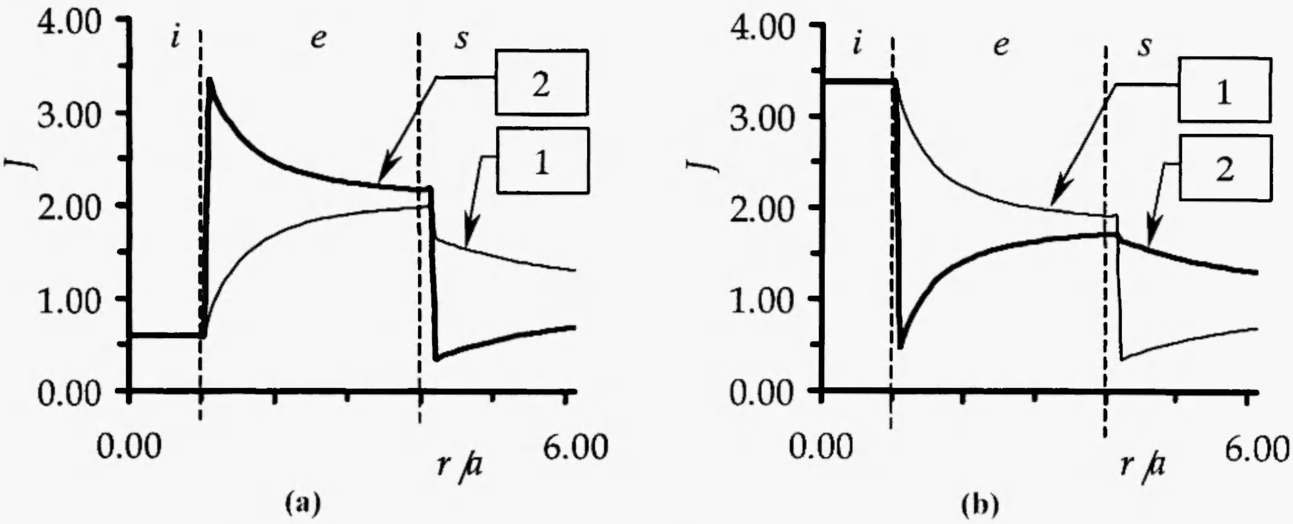


Fig. 2: 3D plot and 2D diagram of the dimensionless current density distribution (for two various angles ϑ for (a) and (b): 1- 0 rad (0^0), 2- $5\pi/10$ (90^0)).
For present drawing and for the next ones: (a) system Zr-Na-C; (b) system Cu-Sn-C.

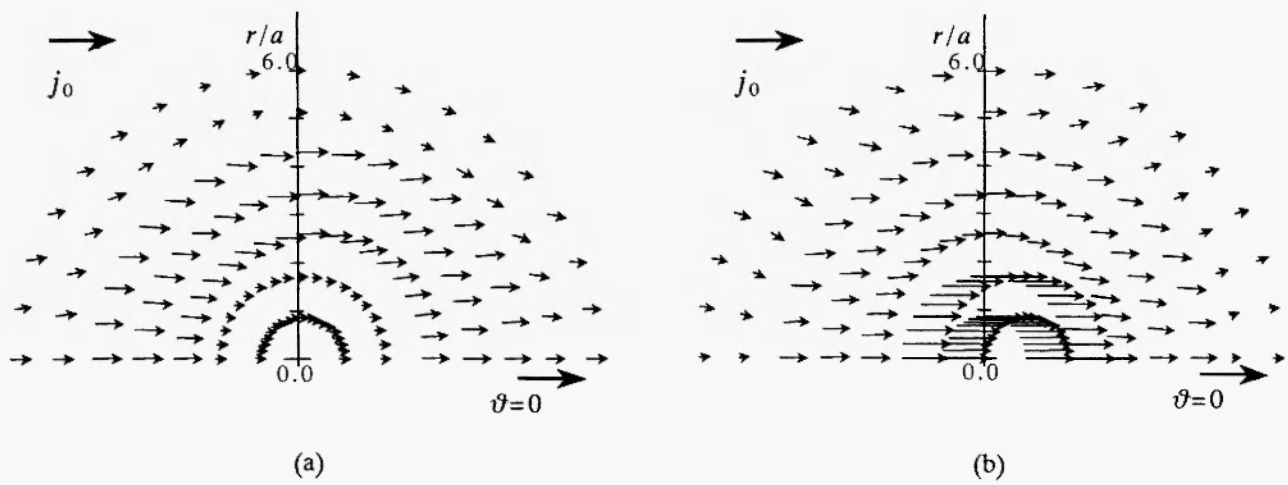


Fig. 3: Plot of distribution of the current density vector.

where $T_{i1}(r)$ depends only on r, and $T_{i2}(r, \vartheta)$ depends on r and ϑ.

Then Eq. (11) splits into two equations:

$$\frac{\partial^2 T_{i1}}{\partial r^2} + \frac{1}{r}\frac{\partial T_{i1}}{\partial r} = -\frac{W_i}{a_i}\,, \quad (13)$$

$$\frac{\partial^2 T_{i2}}{\partial r^2} + \frac{1}{r}\frac{\partial T_{i2}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T_{i2}}{\partial \vartheta^2} = 0\,, \quad (14)$$

where the value W_i does not depend on any argument (see Eq. (7)).

Eq. (13) – the ordinary differential equation – can be solved directly. The solution of the homogeneous

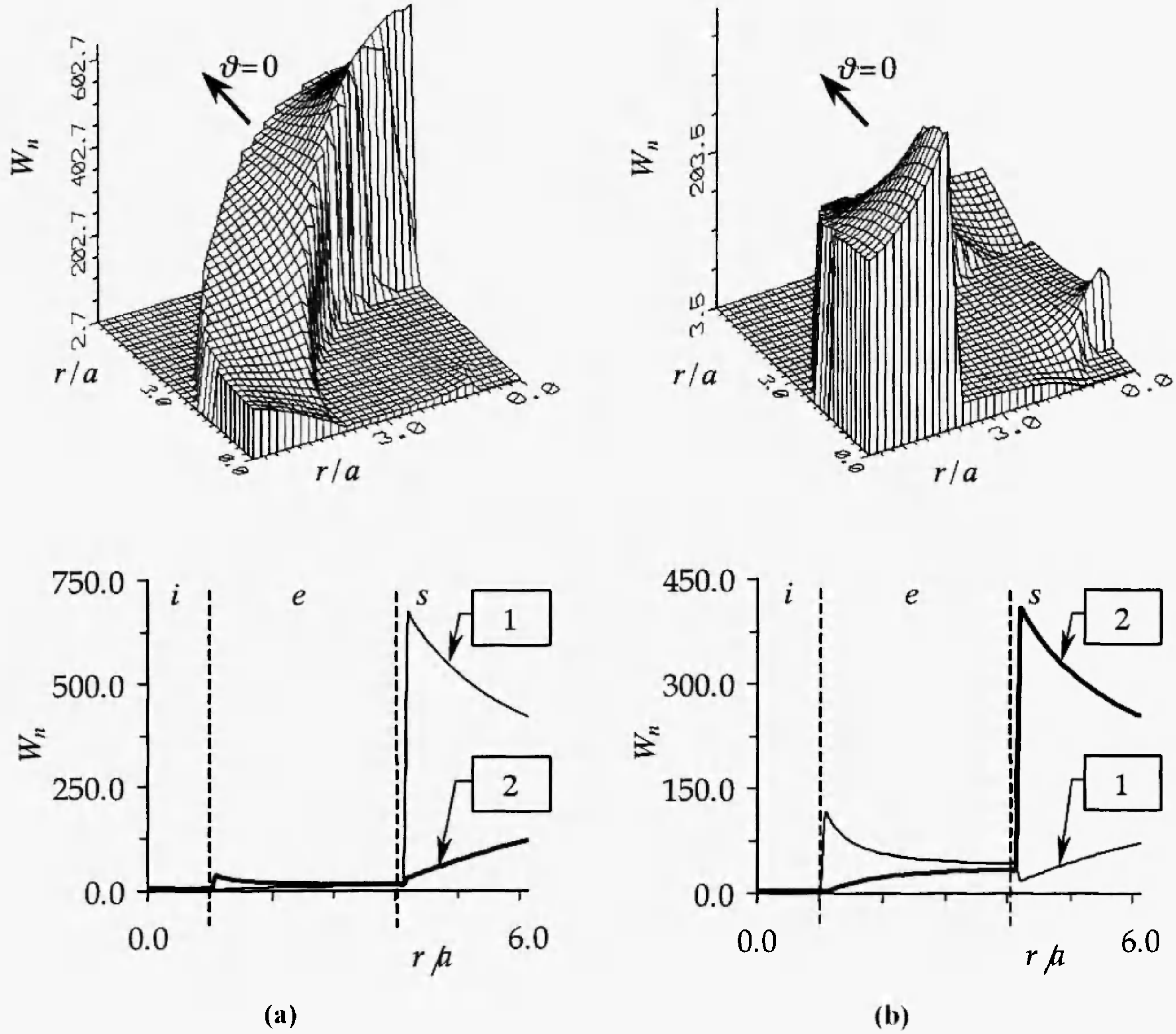


Fig. 4: 3D plot and 2D diagram (below, where: 1- 0 rad (0^0), 2- $5\pi/10$ (90^0)) for spatial distribution of "temperature sources" densities.

equation (14) can be obtained by analogy to the solution of the equation for electrostatic potential in circular areas /12/. The equations for functions T_e and T_s can be solved in another way (see Appendix). As a result the following equations are obtained:

$$T_i = -\frac{W_i}{4a_i} r^2 + C_{i1} \log r + C_{i2} + \left(\frac{B_{i1}}{r^2} + B_{i2} r^2 \right) \cos 2\vartheta , \quad (15)$$

$$T_e = -\frac{E}{4} \left(r^2 + \frac{\Lambda^2 a^4}{r^2} \right) + C_{e1} \log r + C_{e2} + \left(\frac{B_{e1}}{r^2} + B_{e2} r^2 - \frac{E a^2 \Lambda}{2} \right) \cos 2\vartheta , \quad (16)$$

$$T_s = \frac{G}{4} \left(r^2 + \frac{\gamma^2 b^4}{r^2} \right) + C_{s1} \log r + C_{s2} + \left(\frac{B_{s1}}{r^2} + B_{s2} r^2 + \frac{G \gamma b^2}{2} \right) \cos 2\vartheta , \quad (17)$$

where $G=\dfrac{j_0^2}{k_s\,\lambda_s}$, $E=\dfrac{j_0^2\,\varepsilon^2}{k_e\,\lambda_e}$, and C_{n1}, C_{n2}, B_{n1}, B_{n2} are the integration constants. These constants can be determined from the boundary conditions. These conditions are analogous to Eqs. (2a) with the respective replacement of the potential for the temperature, and of the electric conductivity – for the thermal conductivity. There is only distinction in the condition for external boundary of outer layer *s*. It is natural to suppose the boundary condition of the so-called third kind /2/

$$-k_s\left.\frac{\partial T}{\partial r}\right|_{r=c}+\alpha\left(T_c-T_s\big|_{r=c}\right)=0\,, \tag{18a}$$

where α is the heat exchange coefficient. If α is very large quantity, then we suppose $k_s/\alpha\to 0$, and in this limit case we may formulate the last boundary condition:

$$T_s\big|_{r=c}\equiv T_c\,. \tag{18b}$$

Since the solution (15) in point r=0 should not turn to the infinity, $C_{i1}=B_{i1}=0$, the temperatures and the radial heat flows on the boundaries of media *i–e* (at $r=a$) and *e-s* (at $r=b$) must be equal (these equations are omitted here for short). Thus, as it is seen from Eq. (17), the factor at $\cos 2\vartheta$ (when $r=c$) disappears:

$$\frac{B_{s1}}{c^2}+B_{s2}\,c^2+\frac{G\gamma b^2}{2}=0\ . \tag{19}$$

Taking into account the conditions (18b) and (19), we can derive the expressions for temperature in all areas:

$$\begin{aligned}
T_i-T_c={}&\frac{W_i\,a^2}{4a_i}-\frac{E\,a^2(1+\Lambda^2)}{4}+\frac{E}{4}\left(b^2+\frac{\Lambda^2a^4}{b^2}\right)+\frac{G\,b^2(1+\gamma^2)}{4}+\\
&+\left[\frac{Q_i\,a^2}{2k_e}-\frac{E\,a^2(1-\Lambda^2)}{2}\right]\log\left(\frac{b}{a}\right)+\left[\frac{Q_i\,a^2}{2k_s}-\frac{E\,a^2(1-\Lambda^2)\,k_e}{2k_s}+\right.\\
&\left.+\frac{E\,k_e}{2k_s}\left(b^2-\frac{\Lambda^2a^4}{b^2}\right)+\frac{G\,b^2(1-\gamma^2)}{2}\right]\log\left(\frac{c}{b}\right)-\frac{G}{4}\left(c^2+\frac{\gamma^2b^4}{c^2}\right)-\frac{W_i}{4a_i}r^2+\\
&+\left\{\frac{E\,\Lambda\,k_s\,k_e}{2K(k_i+k_e)}\left[2\left(\frac{b^2}{c^2}+\frac{c^2}{b^2}\right)-\left(\frac{c^2}{a^2}+\frac{a^2}{c^2}+\frac{b^4}{c^2a^2}+\frac{c^2a^2}{b^4}\right)\right]+\right.\\
&\left.+\frac{E\,\Lambda\,k_e^2}{2K(k_i+k_e)}\left(-\frac{c^2}{a^2}-\frac{a^2}{c^2}+\frac{b^4}{c^2a^2}+\frac{c^2a^2}{b^4}\right)+\frac{G\gamma\,b^2k_e\,k_s}{a^2K(k_i+k_e)}\left(\frac{c}{b}-\frac{b}{c}\right)^2\right\}r^2\cos 2\vartheta\,,
\end{aligned} \tag{20}$$

$$\begin{aligned}
T_e-T_c={}&\frac{E}{4}\left(b^2+\frac{\Lambda^2a^4}{b^2}\right)+\frac{G\,b^2(1+\gamma^2)}{4}-\frac{G}{4}\left(c^2+\frac{\gamma^2b^4}{c^2}\right)+\left[\frac{Q_i\,a^2}{2k_s}-\frac{E\,a^2\,k_e(1-\Lambda^2)}{2k_s}+\right.\\
&\left.+\frac{E\,k_e}{2k_s}\left(b^2-\frac{\Lambda^2a^4}{b^2}\right)+\frac{G\,b^2(1-\gamma^2)}{2}\right]\log\left(\frac{c}{b}\right)-\frac{E\,a^2}{4}\left(\frac{r^2}{a^2}+\frac{\Lambda^2a^2}{r^2}\right)+\\
&+\left[\frac{Q_i\,a^2}{2k_e}-\frac{E\,a^2(1-\Lambda^2)}{2}\right]\log\left(\frac{b}{r}\right)+\left\{\left[\frac{E\,\Lambda\,a^2\,k_i(k_e+k_s)}{2K(k_i+k_e)}c^2+\frac{E\,\Lambda\,a^2\,k_i(k_s-k_e)}{2K(k_i+k_e)}\frac{b^4}{c^2}-\right.\right.\\
&\left.-\frac{(E\,\Lambda\,a^2+G\gamma\,b^2)k_s}{2K}\frac{k_i-k_e}{k_i+k_e}\left(\frac{b^2}{c^2}+\frac{c^2}{b^2}\right)a^2+\frac{G\gamma\,b^2\,k_s}{K}\frac{k_i-k_e}{k_i+k_e}a^2\right]\frac{1}{r^2}-\\
&-\left[\frac{E\,\Lambda\,a^2k_i(k_e+k_s)}{2K(k_i+k_e)}\frac{1}{c^2}+\frac{E\,\Lambda\,a^2k_i(k_s-k_e)}{2K(k_i+k_e)}\frac{c^2}{b^4}-\right.\\
&\left.\left.-\frac{(E\,\Lambda\,a^2+G\gamma\,b^2)k_s}{2K}\left(\frac{b^2}{c^2}+\frac{c^2}{b^2}\right)\frac{1}{a^2}+\frac{G\gamma\,b^2k_s}{K}\frac{1}{a^2}\right]r^2-\frac{E\,\Lambda\,a^2}{2}\right\}\cos 2\vartheta\,,
\end{aligned} \tag{21}$$

$$
\begin{aligned}
T_s - T_c = &-\frac{G}{4}\left(c^2 + \frac{\gamma^2 b^4}{c^2}\right) + \frac{G}{4}\left(r^2 + \frac{\gamma^2 b^4}{r^2}\right) + \\
&+\left[\frac{Q_i a^2}{2k_s} - \frac{E a^2 k_e (1-\Lambda^2)}{2k_s} + \frac{E k_e}{2k_s}\left(b^2 - \frac{\Lambda^2 a^4}{b^2}\right) + \frac{G b^2 (1-\gamma^2)}{2}\right]\log\left(\frac{c}{r}\right) + \\
&+\left\langle\left\{\frac{E \Lambda a^2 k_i k_e}{K(k_i+k_e)} c^2 - \frac{(E\Lambda a^2 + G\gamma b^2) k_e c^2}{2K}\left(\frac{b^2}{a^2} + \frac{k_i - k_e}{k_i + k_e}\frac{a^2}{b^2}\right) + \right.\right. \\
&\left. + \frac{G\gamma b^4}{2K}\left[-(k_s - k_e)\frac{b^2}{a^2} + \frac{k_i - k_e}{k_i + k_e}(k_s + k_e)\frac{a^2}{b^2}\right]\right\}\frac{1}{r^2} - \\
&-\left\{\frac{E \Lambda a^2 k_i k_e}{K(k_i+k_e)}\frac{1}{c^2} - \frac{(E\Lambda a^2 + G\gamma b^2) k_e}{2K}\left(\frac{b^2}{a^2} + \frac{k_i - k_e}{k_i + k_e}\frac{a^2}{b^2}\right)\frac{1}{c^2} + \right. \\
&\left.\left. + \frac{G\gamma}{2K}\left[(k_s + k_e)\frac{b^2}{a^2} - \frac{k_i - k_e}{k_i + k_e}(k_s - k_e)\frac{a^2}{b^2}\right]\right\} r^2 + \frac{G\gamma b^2}{2}\right\rangle \cos 2\vartheta ,
\end{aligned}
\tag{22}
$$

where $K = \left(\frac{c^2}{a^2} - \frac{k_i - k_e}{k_i + k_e}\frac{a^2}{c^2}\right)(k_s + k_e) + \left(\frac{b^4}{c^2 a^2} - \frac{k_i - k_e}{k_i + k_e}\frac{c^2 a^2}{b^4}\right)(k_s - k_e).$

The expressions for $T_{i,e,s}$ at condition (18a) will otherwise contain parameter α and be of more cumbersome form. The value of the temperature difference in the centre (r=0) and on the outer surface (r=c) can be determined easily from Eq. (20):

$$
\begin{aligned}
T_{im} - T_c = &\frac{W_i a^2}{4a_i} - \frac{E a^2 (1+\Lambda^2)}{4} + \frac{E}{4}\left(b^2 + \frac{\Lambda^2 a^4}{b^2}\right) + \frac{G b^2 (1+\gamma^2)}{4} + \\
&+\left[\frac{Q_i a^2}{2k_e} - \frac{E a^2 (1-\Lambda^2)}{2}\right]\log\left(\frac{b}{a}\right) + \left[\frac{Q_i a^2}{2k_s} - \frac{E a^2 (1-\Lambda^2) k_e}{2k_s} + \right. \\
&\left. + \frac{E k_e}{2k_s}\left(b^2 - \frac{\Lambda^2 a^4}{b^2}\right) + \frac{G b^2 (1-\gamma^2)}{2}\right]\log\left(\frac{c}{b}\right) - \frac{G}{4}\left(c^2 + \frac{\gamma^2 b^4}{c^2}\right).
\end{aligned}
\tag{23}
$$

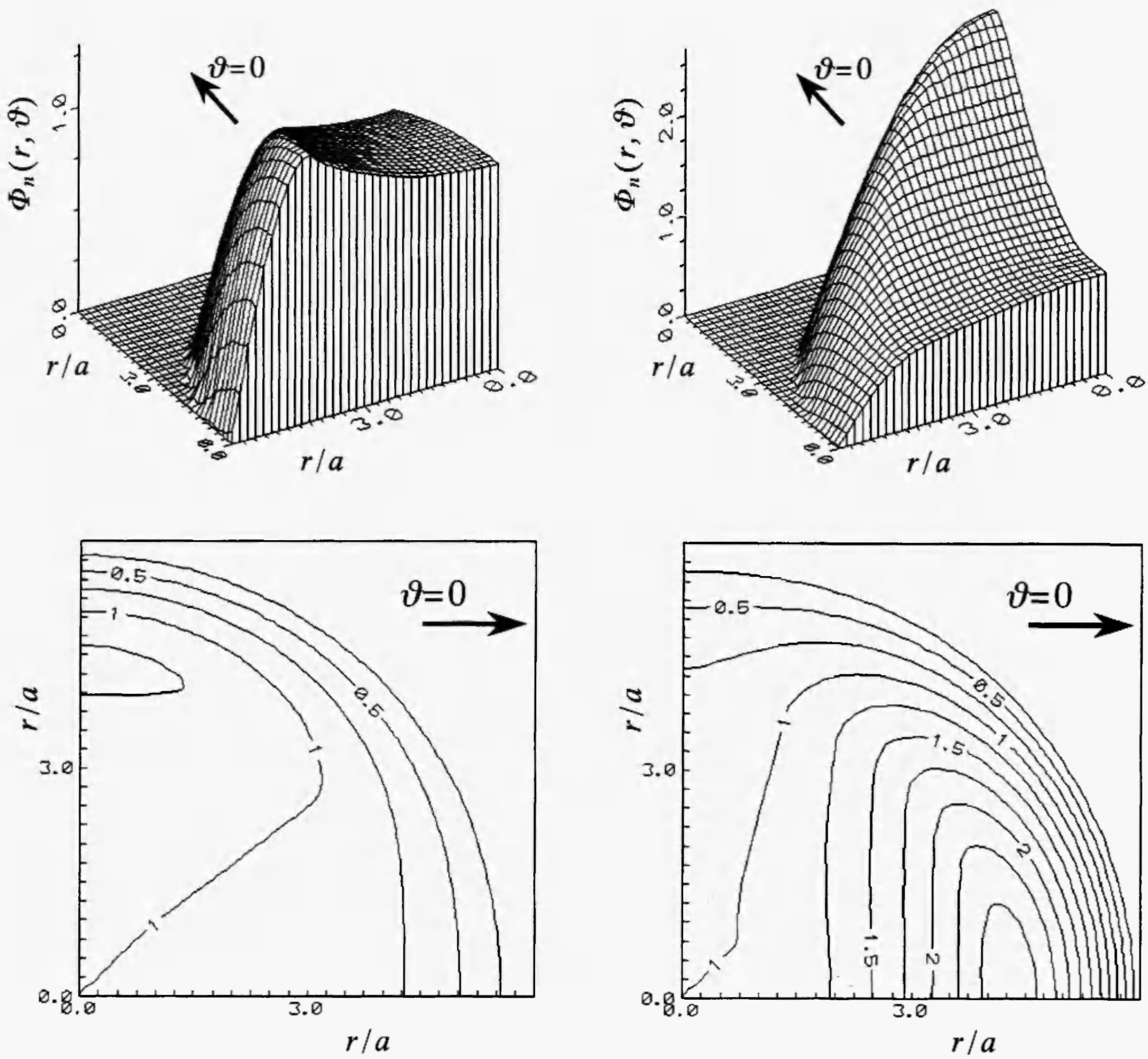
It is possible now to write down in a general form the reduced temperature for each layer, considering the difference (23) as the basic difference of temperatures:

$$
\Phi_n(r,\vartheta) = \frac{T_n(r,\vartheta) - T_c}{T_{im} - T_c} , \tag{24}
$$

where n= i, e, s.

The numerical computation with the help of Eq. (24), taking into account Eqs. (20)-(22) too, yields representation of the spatial dependency of the reduced temperature (Figs. 5, 6). This result is somewhat surprising: the values of the function $\Phi_n(r,\vartheta)$ (see also Table 2) can reach over the upper limit of interval /0; 1/. The existence of areas with $\Phi_n(r,\vartheta)>1$ might be explained by the local change of the electric current density in certain places of the space of the model (see Fig. 2). The level of such "thickenings" will depend basically on values L_i and L_s (see Table 2). These "thickenings" concentrate in the region of the interlayer transitions (see Figs. 2, 5). They are caused by the sign of parameter Λ (see Table 2). The level of temperature in the centre (on the model axis) is lower. If one makes sections of the surfaces in Fig. 5 by the vertical planes at certain angles ϑ, then a certain group of curves can be obtained (Fig. 6). The type of these curves might help to elucidate, in particular, the following questions: what will be the value of the temperature gradients and in what directions should we expect the main heat removal?

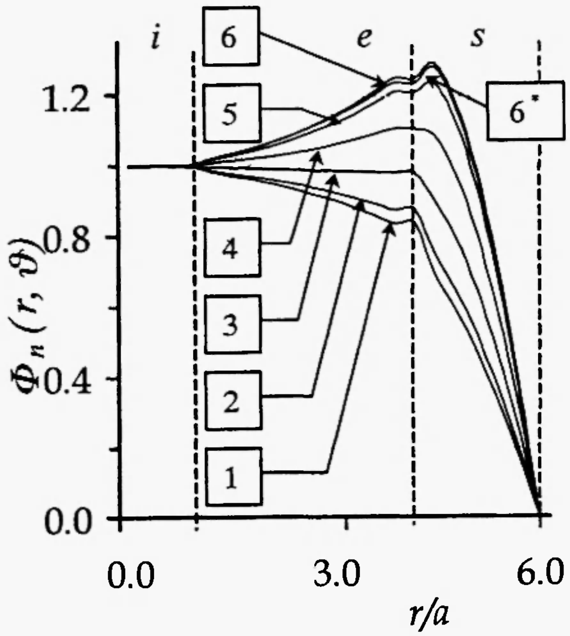
It is interesting to take a look at how the value of the reduced temperature $\Phi_n(r,\vartheta)$ could be influenced by



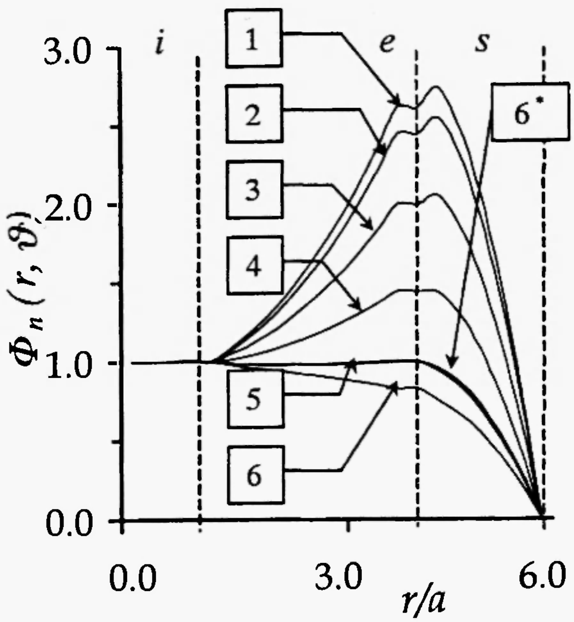
(a) **(b)**

Fig. 5: 3D plot of function $\Phi_n(r,\vartheta)$ for the temperatures reduced and the projection of horizontal sections of this plot onto the (r, ϑ)-plane ("isolines").

using the values of properties (density, heat capacity, and heat conductivity) at the typical temperatures, which were obtained earlier in the calculations where the initial data were used (Table 1). According to the data related to certain sections (see the pair of curves 6-6* in Fig. 6) the maximum change in the level of the reduced temperature $\Phi_n^*(r,\vartheta)$ (Table 2) for the system Zr-Na-C (Fig. 6(a)) is equal to 2.15% and for the system Cu-Sn-C (Fig. 6(b)) it is equal to 12.08%. Such comparisons allow us to ascertain those limits in sparseness of the values obtained which are non-essential, i.e. when one can use the analytic equations.



(a)



(b)

Fig. 6: Profiles of function $\Phi_n(r,\vartheta)$ for sections at some various angles ϑ for (a) and (b): 1- 0 rad (0^0), 2- $\pi/10$ (18^0), 3- $2\pi/10$ (36^0), 4- $3\pi/10$ (54^0), 5- $4\pi/10$ (72^0), 6- $5\pi/10$ (90^0). 6^*- control calculation $\Phi_n^*(r,\vartheta)$ for angle $\vartheta = 5\pi/10$ (90^0) at $\Phi_n(r,\vartheta)_{max}$: (a) – 1.304 and (b) – 2.664 (see Table 2).

5. CONCLUSIONS

The mathematical description of a stationary temperature field, arising in the poly-layered current-carrying medium, illustrated here by the cylindrical three-layered model, is given. The case when the electric currents are passing in the directions perpendicular to the axial coordinate is considered. The dependence in formation of the non-uniform temperature field on the distribution of the electric current density is established. As illustrated by the specific examples, the theoretical analysis of two compositions fully allows one to find specific places, where the direct heat generation is located and the maximum temperatures arise. A similar analysis and proper calculations can also be applied to compositions, existing in other forms, whether natural or geometrical. In particular, the qualitatively similar processes might also exist in media where the layers do not differ so much in the chemical content (for example, in the case of an ice cylinder surrounded by saline water /15/). If a fluid in the intermediate layer remains immovable in spite of any electromagnetic coercions, then the solutions of the type (20)-(22) are true. In the contrary case these expressions must play the role of starting point for development of proper hydrodynamic theory. The analytical research of the heat transfer process which is caused by the convection arisen (when the spatial forces arise due to gravitation /15/ or gravitation together with electromagnetism /6/) and by dissolving of a solid cylinder in a surrounding liquid, might be started and developed using a mathematical formalism, analogous to the one proposed here. The non-uniformity of temperature is a precondition of the origin of the convection flows. The theory of electroconvection in the beginning will be based upon the temperature distribution of the equations of the (21)- and (24)- types similar to how this was done in the case of the liquid horizontal cylinder in the non-uniform temperature field under gravitation /16/.

The next stage will contain analysis of influence of non-uniform temperature on arising of electroconvection and diffusional mass transfer.

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APPENDIX

Consider Eq. (11) at n=e (i.e. for W_e expressed by Eq. (8)):

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \vartheta^2}\right)T_e=-\frac{W_e}{a_e}= -E\left(1+\Lambda^2\frac{a^4}{r^4}\right)+2E\Lambda\frac{a^2}{r^2}\cos 2\vartheta \quad \text{(A1)}$$

Let us present the function T_e as the sum

$$T_e=T_{e1}(r)+T_{e2}(r,\vartheta) \quad \text{(A2)}$$

and substitute expression (A2) into Eq. (A1). Then Eq. (A1) is split into two equations:

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial T_{e1}}{\partial r}\right)\right]=-E\left(1+\Lambda^2\frac{a^4}{r^4}\right), \quad \text{(A3)}$$

$$\frac{\partial^2 T_{e2}}{\partial r^2}+\frac{1}{r}\frac{\partial T_{e2}}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T_{e2}}{\partial \vartheta^2}=2E\Lambda\frac{a^2}{r^2}\cos 2\vartheta \quad \text{.(A4)}$$

The solution of Eq. (A3), that is, the ordinary differential equation is the following:

$$T_{e1}=-E\left(\frac{r^2}{4}+\frac{\Lambda^2 a^4}{4r^2}\right)+C_{e1}\ln r+C_{e2}. \quad \text{(A5)}$$

The solution of Eq. (A4) is

$$T_{e2}=\left(\frac{B_{e1}}{r^2}+B_{e2}r^2-\frac{Ea^2\Lambda}{2}\right)\cos 2\vartheta. \quad \text{(A6)}$$

The addition of Eq. (A5) and (A6) yields, in accordance with Eq. (A2), the sought for solution of Eq.

(A1):

$$T_e = -\frac{E}{4}\left(r^2 + \frac{\Lambda^2 a^4}{r^2}\right) + C_{e1} \ln r + C_{e2} + \left(\frac{B_{e1}}{r^2} + B_{e2} r^2 - \frac{E a^2 \Lambda}{2}\right)\cos 2\vartheta \quad \text{(A7)}$$

The solution for the function T_s may be obtained by the substitutions $E \to G$, $\Lambda a^2 \to -\gamma b^2$, $\Lambda^2 a^4 \to \gamma^2 b^4$ in Eq. (A7).