Preface

Present book can be considered as a standard university course in complex analysis and special functions (including orthogonal polynomials and basic material of special functions such as Euler's Gamma and Beta functions, Bessel's functions, Weierstrass and Jacobi elliptic functions and some others) for mathematics and physics students. However, some subjects of the book, such as the stationary phase method and Laplace's method, Weierstrass elliptic functions and their applications to nonlinear ordinary differential equations are usually very advanced and often are outside of such type of courses in Complex Analysis. This allows this book to be considered more than just a university textbook, as it has many possible applications in applied mathematics and physics.

This book is mainly based on the courses given by the authors at the University of Oulu in recent years, and given by the first author at Lomonosov Moscow State University in the end of 1990s and the beginning of 2000s.

The book consists of three parts divided with respect to the usual content of complex analysis, orthogonal polynomials, and special functions. The first part includes complex numbers, analytic functions and Cauchy theorem. The second part concerns to the maximum modulus of analytic functions, Phragmen-Lindelöf principle, Liouville's theorem, Taylor's and Laurent's expansions, entire functions, and evaluation of several types of integrals and number series using the residue theory. The third and biggest part includes conformal mappings, Laplace transform, and special functions. We remark that special functions is based on the classical method of Frobenius and includes (as an application) Bessel's functions, orthogonal polynomials, and also Laplace's method. Moreover, the theory of Weierstrass and Jacobi functions and their application to nonlinear ordinary differential equations are considered very carefully together with famous Weierstrass' formula.

This book contains about 500 exercises that are integral part of the text, and more than 150 examples. Each part ends in a selection of exercises that an instructor can use for the exams. They are not only an integral part of the book, but also indispensable for the understanding each part of the book. It might be mentioned here that many exercises were borrowed and reworked from the excellent monographs of Titchmarsh [1] and Whittaker and Watson [2]. Within the text, the reader will also find problems, which range from very easy to somewhat difficult. It can be expected that a careful reader will complete all these exercises.

This book is intended for undergraduate level students majoring in pure and applied mathematics (also in physics), but even graduate students can find here very useful information, which previously could only be detected in scientific monographs.

Despite the fact that this text is standard for universities, however, there are some things that distinguish it from well-known texts in this subject. One difference in this text is the discussion of extended complex plane and the concept of complex infinity including Taylor's and Laurent's expansions at infinity. Another key aspect is the evaluation of improper integrals for multivalued functions of certain special form, and calculation of

https://doi.org/10.1515/9783111632278-201

number series by residue theory. In regard to orthogonal polynomials (Legendre, Hermite, Chebyshev, trigonometric) and Bessel's functions, they are considered here to a sufficient degree of generality and their asymptotic behavior with respect to different parameters are proved. In addition, we prove theorems on expanding continuous functions that have piecewise continuous derivative associated with an orthogonal system of eigenfunctions that correspond to given polynomials. But the major difference in this text (compared with known books) is in consideration of the method of stationary phase for real integrals and of the Laplace's method (saddle point method) for the complex curve integrals. It can be also mentioned here that the Cardano's formulae are considered in their general form for polynomials of degree three.

The systematic and careful consideration of the Weierstrass and Jacobi elliptic functions, and their many applications, can be considered as one of the most important features of this book. We have partly used here the approach of Whittaker and Watson [2].

Last but not least, we want to say that when writing the Laplace's method we were greatly influenced by the excellent book by Sveshnikov and Tikhonov [3] and when writing the part of special functions (especially some orthogonal polynomials and their properties) we were inspired by very advanced book by Nikiforov and Uvarov [4].

September 2024

Valery Serov, Lahti
Markus Harju, Oulu