

Contents

Preface — V

1 Curves in \mathbb{R}^n — 1

- 1.1 Frenet curves in \mathbb{R}^n — 1
- 1.2 Analytical representations of curves — 19
- 1.2.1 Plane curves — 19
- 1.2.2 Space curves — 24
- 1.3 The tangent and the normal plane — 28
- 1.4 Osculating plane — 34
- 1.5 Curvature of a curve — 40
- 1.6 The Frenet frame, Frenet formulae, and the torsion — 46
- 1.7 Advanced practical problems — 56

2 Plane curves — 74

- 2.1 Envelopes of plane curves — 74
- 2.2 The evolute — 81
- 2.3 The complex structure on \mathbb{R}^2 — 84
- 2.4 Curvature of plane curves — 86
- 2.5 Rotation angle of plane curves — 91
- 2.6 The curvature center — 93
- 2.7 The involute — 94
- 2.8 The osculating circle of a curve — 97
- 2.9 Advanced practical problems — 102

3 General theory of surfaces — 110

- 3.1 Parameterized surfaces — 110
- 3.2 The equivalence of local representations — 127
- 3.3 Curves on surfaces — 130
- 3.4 The tangent vector space, tangent plane, and normal to a surface — 131
- 3.5 Differentiable maps on a surface — 143
- 3.6 The differential of a smooth map between two surfaces — 144
- 3.7 The spherical map. The shape operator — 145
- 3.8 The first fundamental form of a surface — 148
- 3.9 Applications of the first fundamental form — 151
- 3.9.1 The length of a segment of a curve on a surface — 151
- 3.9.2 The angle between two curves on a surface — 153
- 3.9.3 The area of a parameterized surface — 157
- 3.10 The matrix of the shape operator — 158
- 3.11 The second fundamental form of a surface — 163
- 3.12 The normal curvature. Meusnier theorem — 165

3.13	Asymptotic directions and lines — 167
3.14	Principal directions and curvatures. Gauss and mean curvatures — 168
3.15	Advanced practical problems — 171
4	Fundamental equations of a surface. Special classes of surfaces — 182
4.1	Some relations — 182
4.2	The Christoffel symbols — 183
4.3	The fundamental theorem in the theory of surfaces — 191
4.4	The Darboux frame — 198
4.5	The geodesic curvature. Geodesic lines — 201
4.6	Geodesics of Liouville surfaces — 203
4.7	Ruled surfaces — 204
4.8	Minimal surfaces — 205
4.9	Advanced practical problems — 206
5	Differential forms — 211
5.1	Algebra of differential forms — 211
5.2	Exterior differentiation — 218
5.3	Properties of the exterior differentiation — 222
5.4	Closed and exact differential forms — 228
5.5	Gradient, curl, and divergence — 232
5.6	Differential forms in \mathbb{R}^n — 235
5.7	Advanced practical problems — 236
6	The nature connection — 243
6.1	Directional derivatives — 243
6.2	Tangent spaces — 248
6.3	Covariant derivatives — 250
6.4	The Lie brackets — 254
6.5	Advanced practical problems — 258
7	Riemannian manifolds — 263
7.1	The notion of a manifold — 263
7.2	Differentiable maps — 264
7.3	Tangent spaces — 265
7.4	Riemannian metrics — 269
7.5	The Riemann connection — 270
7.6	The Christoffel coefficients — 276
7.7	Advanced practical problems — 278
Index	281