

Contents

Preface — XVII

Acknowledgments — XIX

1 Introduction — 1

- 1.1 Quantum information technology — 1
- 1.2 Quantum physics — 3
 - 1.2.1 Physics as a science — 4
 - 1.2.2 Comparison with classical physics — 5
 - 1.2.3 Planck and de Broglie — 6
 - 1.2.4 Heisenberg uncertainty — 8
- 1.3 Quantized interactions — 8
 - 1.3.1 Fundamental principle of quantum physics — 10
 - 1.3.2 Quantum particles — 11
- 1.4 Quantum states — 12
 - 1.4.1 Fields — 15

Part I: Background material

2 Classical optics — 21

- 2.1 Linear systems — 21
 - 2.1.1 Orthogonality — 22
 - 2.1.2 Fourier analysis — 26
 - 2.1.3 Shift invariance — 27
 - 2.1.4 Transmission function — 28
 - 2.1.5 Unitarity — 28
 - 2.1.6 Multidimensional systems — 29
 - 2.1.7 Convolution kernel — 31
- 2.2 Maxwell's equations — 31
 - 2.2.1 Constituent relations — 32
 - 2.2.2 Properties of the medium — 33
- 2.3 Equation of motion — 34
 - 2.3.1 Wave equation — 35
 - 2.3.2 Monochromatic fields — 35
 - 2.3.3 Helmholtz equation — 36
 - 2.3.4 Scalar fields — 37
 - 2.3.5 Paraxial condition — 38
- 2.4 Solutions of the Helmholtz equation — 41
 - 2.4.1 Cartesian coordinates: Plane waves — 41

2.4.2	Optical beam variables —	42
2.4.3	Orthogonality of plane waves —	44
2.4.4	Completeness of plane waves —	45
2.4.5	Angular spectrum —	45
2.5	Solutions of the paraxial wave equation —	46
2.5.1	Gaussian beam —	46
2.5.2	Polynomial Gaussian modes —	49
2.5.3	Generating functions for polynomial Gaussian modes —	50
2.6	Free space propagation and diffraction —	53
2.6.1	Fresnel propagation —	56
2.6.2	Fraunhofer approximation —	58
2.7	Lens systems —	58
2.7.1	Fourier transformation by a $2f$ system —	60
2.7.2	Imaging properties of a lens and the $4f$ system —	62
2.7.3	Resolution —	63
2.7.4	Gradient index lens —	64
2.8	Angular momentum —	67
2.8.1	Three-dimensional rotations —	68
2.8.2	Angular momentum conservation —	69
2.8.3	Angular momentum of paraxial fields —	70
2.9	Polarization —	71
2.9.1	States of polarization —	72
2.9.2	Poincaré sphere —	73
2.9.3	Orthogonal states of polarization —	74
2.9.4	Anisotropic media —	75
2.10	Coherence —	81
2.10.1	Optical interferometry —	83

3 Quantum optics — 86

3.1	Operator formalism —	87
3.1.1	Fourier theory —	87
3.1.2	Dirac notation —	88
3.1.3	Coordinate bases —	92
3.1.4	Quadrature operators —	95
3.1.5	Noncommuting operators —	96
3.1.6	Ladder operators —	98
3.1.7	Number operator and the Fock states —	98
3.1.8	Fock state coefficient functions —	101
3.1.9	Projection operators —	106
3.1.10	Discussion —	106
3.2	Coherent states —	108
3.2.1	Displacement operator —	108

3.2.2	Unitary operator — 109
3.2.3	Eigenstates of the annihilation operator — 109
3.2.4	Statistical properties of coherent states — 110
3.2.5	Non-orthogonality and over-completeness — 111
3.2.6	Relationship with quadrature variables — 112
3.2.7	Coherent states in term of quadrature bases — 113
3.3	Variance and uncertainty — 114
3.3.1	Schrödinger uncertainty relation — 115
3.3.2	Covariance — 116
3.3.3	Heisenberg uncertainty — 117
3.3.4	Uncertainty in quadrature — 117
3.4	Gaussian states — 119
3.4.1	Moments of Gaussian states — 120
3.4.2	Squeezing — 122
3.4.3	Bogoliubov transformations — 124
3.4.4	Bogoliubov eigenstates — 125
3.5	Phase space — 127
3.5.1	Classical phase space — 127
3.5.2	Classical harmonic oscillator — 132
3.5.3	Quantum phase space — 133
3.5.4	Wigner function — 134
3.6	Quantum dynamics — 136
3.6.1	Schrödinger equation — 137
3.6.2	Spatial evolution — 138
3.6.3	Evolving state vectors — 138
3.6.4	Unitary evolution and the Heisenberg equation — 139
3.6.5	Unitary temporal evolution — 141
3.6.6	Quantum harmonic oscillator — 142
3.7	Discussion — 148
3.7.1	Critical review — 148
3.7.2	The role of the Planck constant — 150

Part II: Functional phase space formalism

4	Combining all degrees of freedom — 157
4.1	Quantization of the electromagnetic field — 159
4.1.1	Ladder operators — 161
4.1.2	Lorentz covariance and Dirac delta functions — 162
4.1.3	Lorentz covariant quantization — 163
4.1.4	Vectorial quadrature operators — 168
4.1.5	Quantization in optical beam variables — 169

4.2	Fock states — 171
4.2.1	Number operator — 171
4.2.2	Notation for contractions — 172
4.2.3	Fixed-spectrum ladder operators — 173
4.2.4	Coordinate bases — 173
4.2.5	Fixed-spectrum Fock states — 175
4.2.6	Discrete basis Fock states — 176
4.3	Finite number of discrete degrees of freedom — 180
4.3.1	Qubits — 180
4.3.2	Bipartite states — 181
4.3.3	Schmidt decomposition — 182
4.3.4	Entanglement — 183
4.4	Fixed-spectrum coherent state — 187
4.4.1	Fixed-spectrum displacement operator — 188
4.4.2	Non-orthogonality — 189
4.4.3	Complete orthogonal Fock basis — 190
4.5	Quadrature bases — 192
4.5.1	Fixed-spectrum quadrature bases — 192
4.5.2	Quadrature bases in all degrees of freedom — 194
4.5.3	Inner products — 197
4.5.4	Quadrature expansion of coherent states — 203
4.5.5	Completeness of quadrature bases — 204
4.5.6	State representation — 206
4.6	Discussion — 206
5	Wigner functional theory — 209
5.1	Functional Fourier analysis — 209
5.2	Functional operator identities — 210
5.3	Definition of the Wigner functional — 211
5.3.1	Marginal probability distribution — 212
5.3.2	Operator-based calculation of the Wigner functional — 213
5.4	Characteristic functional — 214
5.4.1	Operator-based calculation — 216
5.5	Weyl transformation — 216
5.6	Star product — 218
5.7	Covariance kernel — 219
5.7.1	Gaussian Wigner functionals — 220
5.7.2	Finite-dimensional systems — 222
5.7.3	Kernel-based formulation — 222
5.7.4	Complex field variable formulation — 223
5.8	Other functional phase space representations — 225
5.8.1	Glauber–Sudarshan P -functionals — 226

- 5.8.2 Husimi Q -functionals — 227
- 5.8.3 Characteristic functionals for other representations — 228
- 5.8.4 Operator ordering — 229

6 Wigner functionals of states and operators — 232

- 6.1 Coherent states — 232
- 6.1.1 Wigner functional — 232
- 6.1.2 Completeness of fixed-spectrum coherent states — 235
- 6.1.3 Coherent-state-assisted approach — 236
- 6.1.4 Schrödinger cat state — 238
- 6.2 Displacement operator — 239
- 6.2.1 Wigner functional — 239
- 6.2.2 Displacement of an arbitrary state — 241
- 6.2.3 Orthogonality and completeness — 241
- 6.2.4 Completeness of arbitrary displaced states — 243
- 6.3 Fixed-spectrum Fock states — 243
- 6.3.1 One-parameter generating function — 244
- 6.3.2 Two-parameter generating function — 246
- 6.3.3 Multiparameter generating function — 246
- 6.3.4 Number operator — 247
- 6.4 Linear lossless optics — 249
- 6.4.1 Single-photon processes — 250
- 6.4.2 Multiphoton processes — 251
- 6.4.3 Wigner functional of the phase operator — 257
- 6.4.4 Phase transformation of an arbitrary state — 261
- 6.4.5 Phase-only spatial modulation — 263
- 6.4.6 Unitary temporal evolution — 264
- 6.4.7 Phase modulation as a continuous process — 266
- 6.4.8 Free space propagation — 267
- 6.5 Thermal states — 272
- 6.5.1 Density operator of the blackbody thermal state — 272
- 6.5.2 Wigner functional for the blackbody thermal state — 273
- 6.5.3 Physically realistic thermal states — 276
- 6.5.4 Single-mode thermal states — 278
- 6.5.5 General thermal states — 279
- 6.6 Beamsplitter — 279
- 6.6.1 Homogeneous beamsplitter — 280
- 6.6.2 Wigner functional of the homogeneous beamsplitter — 281
- 6.6.3 Applying the homogeneous beamsplitter — 282
- 6.6.4 Inhomogeneous beamsplitter — 283
- 6.7 Loss — 286
- 6.7.1 Homogeneous loss — 287

6.7.2	Inhomogeneous loss —	291
6.7.3	Loss as a continuous process —	293
6.8	Squeezing —	297
6.8.1	Wigner functional of the squeezing operator —	297
6.8.2	Squeezing of an arbitrary state —	305
6.8.3	Bogoliubov transformation —	307
6.8.4	Squeezed vacuum state —	308
6.8.5	Identities for pure squeezed state kernels —	310
6.8.6	Bogoliubov eigenstates and their completeness —	312
6.8.7	Effect of loss on squeezing —	313
6.8.8	Twin-beam squeezed vacuum state —	314
6.9	Post processing —	315
6.9.1	One aperture or half-space —	316
6.9.2	Two apertures —	317
6.10	Discussion —	321
7	Measurements —	325
7.1	Wigner functionals in measurement calculations —	326
7.1.1	Generating functions in measurement calculations —	328
7.1.2	Construction operation —	329
7.2	Detectors —	329
7.2.1	Single-mode detector kernel —	331
7.2.2	Convolution detector kernels —	332
7.2.3	Detector array —	332
7.2.4	Detection systems —	333
7.2.5	Infinite resolution detector kernel —	334
7.3	Intensity —	336
7.3.1	Particle-number degree of freedom only —	336
7.3.2	Localized measurements —	337
7.3.3	Squeezed state intensity —	342
7.4	Photon statistics —	343
7.4.1	Ideal photon-number projection operators —	343
7.4.2	Localized photon-number projection operators —	344
7.4.3	Thermal state photon statistics —	347
7.5	First-order correlations —	350
7.6	Second-order correlations —	353
7.6.1	Hanbury Brown-Twiss —	353
7.6.2	Quantum correlations —	354
7.6.3	Fock states —	357
7.6.4	Antibunching —	358
7.6.5	Wigner functionals —	359
7.6.6	Second-order correlations in squeezed vacuum states —	365

7.6.7	Hong–Ou–Mandel effect —	374
7.7	Noise —	375
7.7.1	Quantum noise —	375
7.7.2	General state with noise —	377
7.8	Heralding —	382
7.8.1	Generic process —	383
7.8.2	Single-photon Fock state preparation —	384
7.8.3	Photon-subtracted states —	387
7.8.4	Photon-added states —	391
7.9	Homodyne measurement —	395
7.9.1	Generic homodyne process —	396
7.9.2	Homodyne tomography —	399
7.9.3	Example: Single-mode Fock state —	408
7.9.4	Naive homodyning —	411
7.10	Discussion —	412

Part III: Parametric down-conversion

8	Modelling parametric down-conversion —	419
8.1	Introduction —	419
8.1.1	Nonlinear processes —	420
8.2	Classical nonlinear wave equation —	422
8.2.1	Nonlinear Maxwell equations —	422
8.2.2	Nonlinear Helmholtz equation —	423
8.2.3	Nonlinear paraxial wave equation —	427
8.2.4	Classical evolution equation —	428
8.3	Propagation operator —	432
8.4	Wigner functional evolution equation —	434
8.4.1	Wigner functional of the propagation operator —	434
8.4.2	Evolution equation —	436
8.5	Simplifications —	437
8.5.1	Coherent state pump —	438
8.5.2	Strong-field perturbation theory —	438
8.5.3	Semiclassical approximation —	439
8.5.4	Pump evolution beyond the semiclassical approximation —	441
8.6	Expressions of the kernel functions —	442
8.6.1	Bilinear vertex kernel —	442
8.6.2	Phase-matching conditions —	442
8.6.3	Phase-matching function —	448
8.6.4	Thin-crystal and plane wave approximations —	450
8.6.5	Leading-order down-converted intensity —	452

- 8.6.6 Higher-order terms — **453**
- 8.6.7 Kernels with walk-off — **458**
- 8.6.8 Dispersion effects — **459**
- 8.7 Stimulated parametric down-conversion — **462**
- 8.7.1 Equations for Bogoliubov kernels — **463**
- 8.7.2 Solutions of the Bogoliubov kernels — **465**
- 8.7.3 Consistency with respect to the spontaneous process — **466**
- 8.8 Upconversion — **466**
- 8.8.1 Second-harmonic generation — **467**
- 8.8.2 Sum-frequency generation — **475**

9 Applications in parametric down-conversion — 481

- 9.1 Imaging the down-converted field — **481**
- 9.1.1 Integrating over optical beam variables — **483**
- 9.1.2 Integrating over z — **484**
- 9.1.3 Summations — **485**
- 9.1.4 Parameters and variables — **486**
- 9.1.5 With dispersion — **487**
- 9.2 Second-order correlations — **490**
- 9.2.1 Integrating over optical beam variables — **491**
- 9.2.2 Summations and z -integrations — **492**
- 9.2.3 With dispersion — **497**
- 9.3 Parametric amplified coherent state — **498**
- 9.3.1 Intensity measurement — **499**
- 9.3.2 Detector kernel for infinite resolution — **500**
- 9.4 Mode transformations — **501**
- 9.4.1 Transformed by the even kernel — **502**
- 9.4.2 Transformed by the odd kernel — **508**
- 9.4.3 Transformed parameter function — **511**
- 9.5 Pump shaping — **516**
- 9.5.1 Laguerre–Gauss modes — **516**
- 9.5.2 Demonstration — **518**
- 9.6 Arbitrary pump state — **519**
- 9.6.1 Semiclassical approximation — **520**
- 9.6.2 Without the semiclassical approximation — **521**
- 9.7 Upconversion teleportation — **523**
- 9.7.1 Input state — **525**
- 9.7.2 Joint measurement—single photons — **531**
- 9.7.3 Teleportation channel limitations — **534**

Part IV: Epilogue

10 Summary and outlook — 541

- 10.1 The aim — 541
- 10.2 Development — 542
 - 10.2.1 Outstanding issues — 543
- 10.3 Applications — 544
 - 10.3.1 Beyond quantum optics — 546
- 10.4 Mathematical foundation — 547
 - 10.4.1 Outstanding issues — 548
 - 10.4.2 Fundamental mathematical properties — 548

A Generating functions — 551

- A.1 Mathematical tagging — 551
- A.2 Basics — 551
- A.3 Generating functionals — 552
- A.4 Applications — 552
- A.5 Statistical calculations — 553
- A.6 Auxiliary representations — 554

B Dispersive birefringence — 557

- B.1 Effective refractive index — 557
- B.2 Sellmeier equations — 557
 - B.2.1 Barium borate — 557
 - B.2.2 KTP — 558
- B.3 Dispersion — 558

C Functional integration — 561

- C.1 Dirac delta functionals — 561
- C.2 Change of integration field variables — 562
- C.3 Isotropic Gaussian functionals — 563
- C.4 Anisotropic Gaussian functionals — 566

D Inverses and determinants — 569

- D.1 One idempotent kernel — 569
- D.2 Inverses with multimode kernels — 570
- D.3 Determinants with multimode kernels — 573
- D.4 Determinants of diagonal kernels — 573
- D.5 Derivatives of determinants and inverse — 574

Index — 577