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Rationalizing Principal Stress Line Networks Using an Agent-Based Modelling Approach

Abstract: The principal stress line (PSL) network, which consists of two sets of orthogonal curves visualizing internal forces, provides an idealized material distribution scheme for structural design under given boundary conditions. While there have been some excellent cases utilizing PSL in structural designs, the application of PSL has been limited due to a lack of effective methods for generating and rationalizing PSL into efficient structural networks. This study proposes an agent-based modelling (ABM) approach that enables designers to generate and tailor PSL networks during the conceptual design phase. This rationalization process is carried out in two consecutive steps, with two corresponding agent systems being developed. Firstly, a "degeneration agent" system is created to identify degenerate points on the designed surface where the orthogonality of the PSL network becomes abnormal. A topological skeleton is then drawn, dividing the surface into a series of regions containing only conjugate PSL curves, using the identified degenerate points. The second step involves drawing and optimizing the layout of PSL in these regions using the "spacing agent" system, according to user-defined design objectives. The proposed method is validated through the design of floor slabs and freeform shells. This research could provide architects with a more intuitive and flexible approach to dealing with PSL during the design exploration stage.

Keywords: principal stress line, agent-based modelling, degenerate points, density control, shell structure

1 Introduction

Architectural form with both aesthetics and performance is the long-standing pursuit of architects in the form exploration stage. Advancements in structural form-finding methods, such as particle-spring systems [1], topology optimization [2], graphic statics [3], have accelerated the exploration and application of structures that integrate form and performance to a high degree. In this context, principal stress line (PSL) networks offer significant potential for efficient structural design due to their excellent geometric features and structural properties [4,5]. PSL networks consist of two sets of orthogonal curves, one in the maximum principal stress direction and the other in the minimum principal stress direction. These curves form an orthogonal line network, visually representing the internal force flow within a continuum structure under given boundary conditions. PSL networks provide an idealized material distribution scheme for a given structural domain [6]. The architectural design and construction based on PSL

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have been proven to be material-saving and structurally efficient [7], enabling more sustainable ways of building.

1.1 Related works

In terms of architectural applications, the ribbed slab of the Gatti Wool Factory by Pier Luigi Nervi is one of the most famous projects inspired by the visualization of the internal force flow [8] (Fig. 1, left). Another example of a built structure with principal force trajectory is the zoology lecture hall at the University of Freiburg, Germany [9] (Fig. 1, middle). The application of PSLs in architectural design is limited due to architects lacking efficient methods for generating and optimizing them. In recent years, several tools have emerged that allow architects to perform finite element analysis and generate PSLs. Grasshopper Plug-ins such as Karamba3D and Millipede can generate PSLs in a parametric design environment, effectively promoting the application of PSL-based design exploration. In this context, PSLs has become increasingly popular in the digital fabrication research in pursuit of more efficient shells [10] or slabs [11]. Vejrum and Jensen (2022) presented the design and digital fabrication process of a complex ribbed concrete slab that implements the “Nervi system” [12]. The pattern of the CIAB Pavilion, developed by Zaha Hadid Architects, also follows the trajectories of principal stresses [13] (Fig. 1, right). However, the generated PSL networks by these tools are often unfavourable due to problems such as stress line discontinuity and self-intersection. The software’s limited ability to control the PSLs makes it difficult to makes it difficult for designers to directly apply the generated results to structural design.

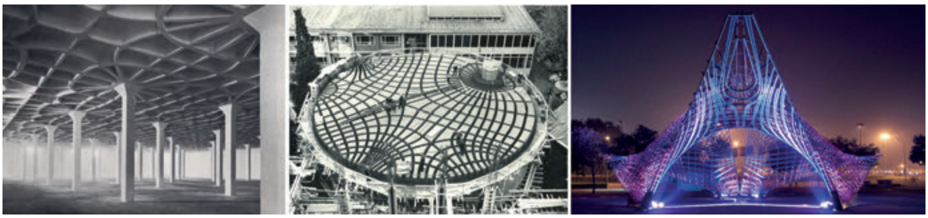


Fig. 1: Design practice inspired by PSL networks includes Gatti Wool Factory Floor System [8] (left), Zoology lecture hall at the University of Freiburg [9] (middle), and CIAB pavilion [13] (right).

The generation of PSL networks heavily relies on the seeding scheme, which determines the placement of seed points to achieve the desired layout of PSLs. Tam and Mueller (2015) optimized the generation of PSLs using an iterative stress line interpolation method and a rule-based correction process [14]. Church (2021) describes two seeding schemes: the neighbor seeding method and the farthest point seeding method, aiming to achieve a relatively evenly spaced network layout [4]. Both studies require an additional post-selection process to control the density of PSLs.

1.2 Research question and objectives

The primary challenge in generating and controlling a PSL network lies in the fact that the network does not always form an orthogonal grid, as desired in the ideal state. Upon observing the PSL network, it becomes evident that certain locations exist where the PSLs in the maximum and minimum stress directions are not orthogonal. These locations are known as degenerate points of the PSL network, which are points where the principal stresses are equal, leading to the absence of principal directions. In other words, all directions can be considered as principal directions [15]. Due to the irregular topology of the PSL network at degenerate points, it becomes challenging to devise a universal solution for both seedings and density control.

Degenerate points in the PSL network can be classified into two basic types: wedge (point A in Fig. 2) and trisector (point B in Fig. 2) [16]. Additionally, compound degenerate points can be formed by merging these basic types, resulting in complex topological PSL networks. For example, a center-type degenerate point (point C in Fig. 2) can be generated in regions where stresses experience discontinuity, such as locations under concentrated loads or supports. This type of degenerate point often results from the merging of two wedges.

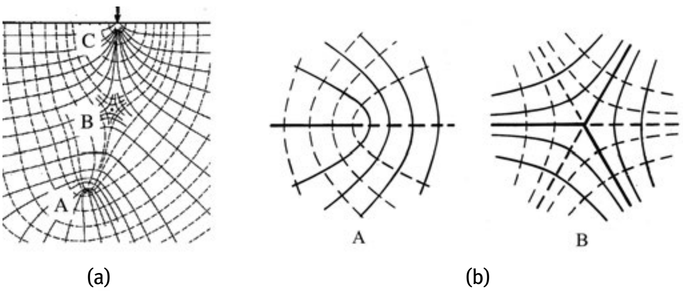


Fig. 2: The degenerate points of PSL network: (a) point A is wedge point, point B is trisector point, C is a degenerate point under concentrated load [15]; (b) the PSLs networks around isotropic point A and B exhibit different geometric features [17].

To address the limitations in generating and controlling PSLs, this research aims to develop a method for designing and regulating PSL networks by further rationalization. The approach employed is an agent-based modelling (ABM) approach, which facilitates the identification of degenerate points in the PSL network and the regulation of PSL density. Through ABM, the generated network density can be regulated by iteratively adjusting the position of seed points, effectively integrating the seeding and density control process. The customizable behavior of agents in the ABM system allows for direct correlation of results with various design considerations, whether related to geometry, structure, or fabrication. This enables enhanced flexibility in regulating PSL networks. Although the proposed method is developed specifically for PSLs, it is equally applicable to other principal trajectories, such as principal bending moments.

2 Methods

The PSL networks only deviate from orthogonality at specific degenerate points. Consequently, the PSLs passing through these degenerate points, which define the topological skeleton of the PSL network, can divide the designed surface into areas without degenerate points, ensuring that the networks within these areas remain orthogonal. Building upon this principle, this research first develops an ABM system to identify the degenerate points within the PSL network. Subsequently, the PSLs passing through these degenerate points are generated to partition the surface. Another ABM system is then devised to facilitate PSL generation and density control within these divided areas.

The ABM systems are implemented using the ABxM framework, an open-source platform designed for exploration with agent-based systems [18]. Geometric modelling, agent system simulation, and visualization tasks are performed using Rhino3D and Grasshopper. The Grasshopper plugin Karamba3D is utilized for structural analysis, enabling the generation of principal stress directions and values.

3 PSL generation

Accurately generated PSLs are the basis of this study. The drawing of the PSLs starts from a seed point and iteratively seeks for the next point at a given step size by estimating the movement direction from the principal stress field. Since the finite element analysis performed with Karamba3D outputs principal stress directions and values corresponding to the input mesh vertices, the generation of PSLs need to first deal with data interpolation – calculate the data for any point with the known discrete values. This research utilized the Inverse Distance Weighting (IDW) interpolation method to calculate the principal stress directions and values for a seed point. The IDW method assigns weights to the values associated with mesh vertices within a certain radius of the seed point, based on their distance. These weighted values are then interpolated to determine the principal stress directions and values at the seed point (Fig. 3a).

For each starting point that is not located on the edges, the seed points are iteratively traced in two opposite directions simultaneously. This tracing process generates a Principal Stress Line (PSL) by sequentially connecting all seed points. The Fourth Order-

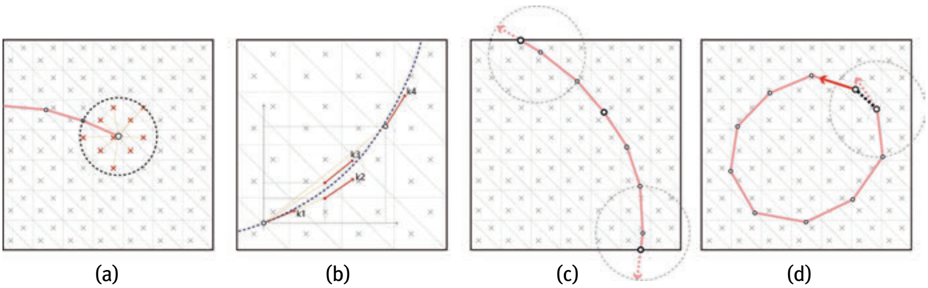


Fig. 3: The generation of a PSL: (a) interpolation method; (b) seed point approximation with RK4 method; (c) edge situation; (d) loop situation.

Runge Kutta method (RK4) is employed to estimate the next seed point at a given step size, based on the principal stress directions at the current seed point (Fig. 3b). While the Euler method is commonly used in PSL generation, the RK4 method is considered to provide better approximation accuracy [19]. The tracing process terminates under two conditions: reaching the design boundary (Fig. 3c), or when the distance from the starting point becomes smaller than the step size, indicating the formation of a loop (Fig. 3d).

4 ABM-based PSL network design

The PSL network is generated through a two-step process, utilizing corresponding agent systems. Initially, the "degeneration agent" system is employed to accurately locate the degenerate points on the designated surface. Subsequently, a PSL skeleton is delineated, dividing the surface into distinct regions characterized by exclusively orthogonal PSL curves. In the second step, the "spacing agent" system is introduced to efficiently draw and optimize the layout of PSLs within the identified regions. This process is meticulously guided by user-defined design objectives, ensuring a tailored outcome. To exemplify the efficacy of the agent-based algorithms, we employ a rectangular slab structure, supported at its four corners and subjected to gravitational forces, as a case study (Fig. 4).

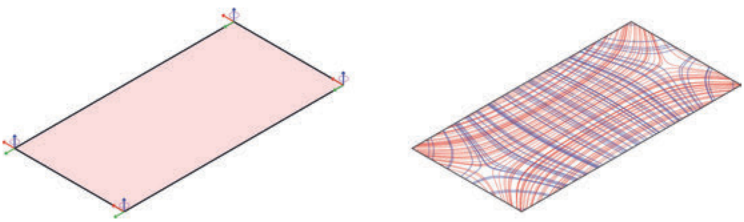


Fig. 4: Left: Selected case slab structure; right: PSL network generated with random seedings.

4.1 Degeneration agent system

The primary challenge of the degeneration agent system lies in identifying a target from the characteristics of degenerate points that can be progressively approached. Degenerate points are characterized by the equality of maximum and minimum principal stresses, resulting in an absolute difference of 0, which is the smallest possible value among all positions. To address this, we frame the problem of identifying isotropic points as a minimization problem. The objective is to minimize the difference, denoted as ΔS, between the maximum and minimum principal stresses at a given point, which is a function of the point coordinates (U, V) within the parameter space of the design surface.

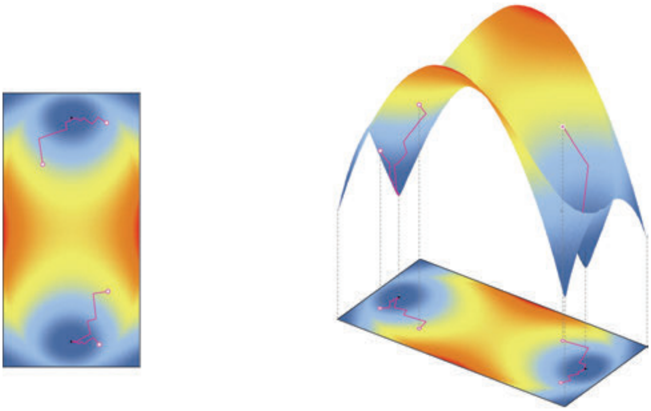
4.1.1 Agent definition

A "degeneration agent" is instantiated as a random point on the design surface. Each agent's position attribute is represented by the coordinates (U, V) in the parameter space of the design surface. This allows us to determine the corresponding ΔS at each agent's current position. In each iteration, every agent calculates a moving vector, V_{m}, based on its defined behavior. The length of V_{m} is determined by a step size, d, weighted by ΔS. Since the objective of this agent system is to minimize ΔS, the length of each agent's movement decreases as it approaches the goal. This behavior is similar to a gradient descent algorithm. Once the length of V_{m} falls below a predetermined threshold, the agent is considered to have reached a degenerate point. Then the agent is terminated and excluded from the subsequent iterations.

4.1.2 Agent behavior

The agent's behavior aims to minimize the ΔS value by continuously seeking the direction where ΔS decreases. To simplify the direction search process, the agent is restricted to move along four predefined options: the maximum and minimum principal stress directions, as well as their opposite directions. Assuming the agent is positioned at a non-degenerate point, it undergoes small step movements in all four directions during each iteration. The direction in which ΔS decreases the most is then selected as the agent's moving direction. This behavior results in the agent's trajectory appearing as a zigzag line with gradually decreasing step sizes. It is important to note that since the data is estimated through interpretation, the final degenerate point found is an approximation of the actual degenerate point.

For the slab case, two degenerate points can be identified. A 3D contour map is employed to visually observe the movement of agents under the aforementioned behavior. In this map, the X and Y axes correspond to the x and y values of the design surface, while the Z-axis represents the ΔS value at corresponding points. By overlapping the design surface with the 3D map, the agent's trajectory can be observed as it gradually descends towards the bottom of the valley, resembling a gradient descent motion (Fig. 5).



(a) (b)

Fig. 5: A 3D contour map is used to better visualize the searching paths of degeneration agents.

4.2 Degenerate points-based surface subdivision

The objective is to partition the surface into regions where only orthogonal networks exist. The topological skeletons formed by PSLs passing through degenerate points are then used to divide the design surface, creating a structured environment for ABM-based PSL network design. This paper focuses on discussing two fundamental types of degenerate points: wedges and trisectors. These types can be easily distinguished based on their geometric features, as observed from the results of the degeneration agents. Wedges exhibit two intersections of PSLs in the maximum and minimum directions, whereas trisectors have only one.

Regarding the wedge points, the PSL network maintains its orthogonal feature, except for the lines that pass through the exact points. Since it is nearly impossible to reach the exact points, obtaining an orthogonal network is relatively straightforward by excluding the area in which these points lie from the design area. However, it's important to note that the topology of the network around wedge points should be further classified into two types: lemon and monster, based on the underlying tensor fields. In the scope of this paper, a detailed discussion of these types is deferred to avoid the need for extensive explanations to tensor fields. Fortunately, the subsequent spacing agent system can effectively operate in areas with wedges.

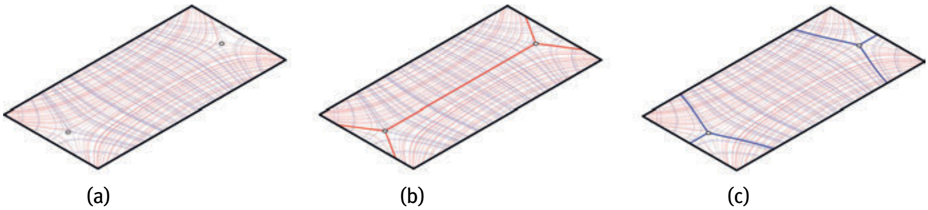


Fig. 6: Surface subdivision with degenerate points: (a) the two trisector points; (b) the maximum PSLs and (c) the minimum PSLs that pass through the trisectors.

For trisector points, both the maximum and minimum PSLs radiate from the point in three directions. After excluding wedge points, these PSLs can divide the surface into regions where only orthogonal networks exist. In the case of the slab, the two isotropic points are both trisectors. Each of these points exhibits three branches in both the principal stress directions, which form the topological skeletons as depicted in Fig. 6.

4.3 Spacing agent system

The spacing agent system is employed to generate PSLs and control their density within the divided area. To illustrate, in the case of the maximum PSL network, the slab is divided by three branches in the maximum principal direction at each isotropic point, while three branches in the minimum direction serve as guide curves for seeding.

4.3.1 Agent definition

The spacing agents are defined as a series of seed points that move along the guide curve. Each agent is associated with a PSL in the maximum direction that passes through the point it occupies. The agents only interact with their neighbors on the same guide curve. Agents located at the starting and ending points of the guide curve maintain their positions unchanged during the simulation. In each iteration, each agent obtains a moving vector, V_S, in the parameter domain of its guide curve through interaction with its neighbors. The system is considered converged once the sum of the V_S values for all agents is smaller than a pre-set threshold.

4.3.2 Agent behavior

The spacing agent's behavior can be customized according to the design requirements. This study takes the goal of evenly spacing the PSLs as an example. For the agents that are not located at the ends of their guide curve, three behaviors are defined to balance the spacing between the PSLs associated with each of the agents.

First, a spacing behavior is designed to directly average the spacing of the PSLs. The calculation of the average spacing D between the PSLs of one spacing agent and its two neighbors is used to determine the difference between the two D values, denoted as ΔD. The spacing behavior aims to reduce ΔD by moving the agent towards the neighbor with a larger D value. The length of the motion vector in each iteration is determined by the step size t, which is weighted by ΔD. As a result, the agent gradually reduces its movement while achieving spacing balance. Second, an adding behavior is developed to introduce a new agent to the system when the D value between any two adjacent agents exceeds a pre-set threshold. The new agent is added in the middle of the two adjacent agents on the guide curve. Third, a removing behavior is used to delete an agent from the system when the D value between one agent and any of its neighbors falls below a pre-set threshold. The removing and adding behaviors work together to maintain the density of PSLs within a desirable range, preventing the generation of layouts that are either too sparse or too tight. The combined action of these three behaviors results in a highly uniform distribution of PSLs in the case slab, as shown in Fig. 7.

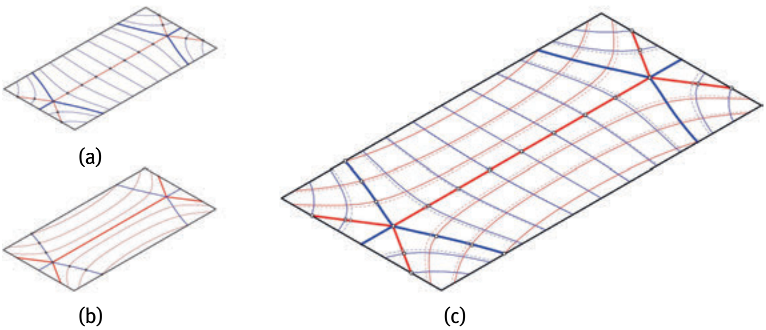


Fig. 7: (a, b) The spacing agents are initialized as evenly distributed points on the guide curves; (c) the balanced layout before (dashed lines) and after (solid lines) the ABM simulation.

4.4 Stress value weighted PSLs spacing

Based on the customizable features of ABM behaviors, different considerations that influence the layout of the PSL network can be incorporated. In this research, we conducted a simple test to regulate the spacing of PSLs based on local stress values. The density of PSLs is higher in areas with larger local stress values and lower in areas with smaller local stress values. This objective is achieved by incorporating the local stress values as weights in the calculation of the D values for adjacent agents. Therefore, the purpose of the spacing agent is to minimize the difference in weighted D values between each agent and its two neighbors. The strength of the effect of local stress values on the layout can be adjusted by a user-defined weight ratio. We further tested this method on a case slab, and the results demonstrate the difference between stress value-weighted and evenly distributed networks (Fig. 8).

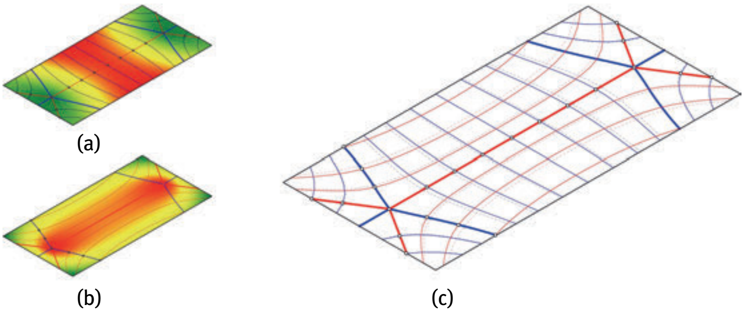


Fig. 8: (1-2) The stress values distribution is used to influence the layout of the PSLs; (3) the difference between evenly spaced (dashed lines) and stress weighted (solid lines) PSL network.

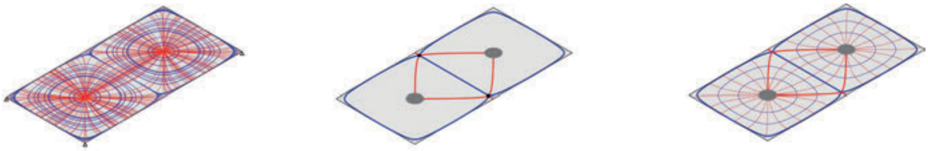
5 Design explorations

5.1 Implementations with different geometries

To test the effectiveness of the proposed PSL network design method, three different scenarios were selected: a floor slab under different boundary conditions, a simple shell, and a freeform shell. The objective was to achieve evenly spaced PSL networks for each scenario.

In the case of the floor slab, it is supported by two columns positioned in the middle. Due to stress concentration at each support, a degenerate point is generated, which is essentially a combination of two wedge points. The notable geometric feature is that the maximum PSL is concentrated at the degenerate point, while the minimum PSL forms a loop around it. By excluding the area where these points are located from the design area (by using a minimum PSL close to the degenerate point), it is easy to obtain an orthogonal network. Additionally, using the degeneration agent system, two trisector points can be identified. By passing through these trisector points, two adjacent loops in the minimum direction and four radial curves in the maximum direction can be obtained as skeletons. The final PSL network consists of closed loops and radial curves, effectively achieving homogenized spacing (Fig. 9).

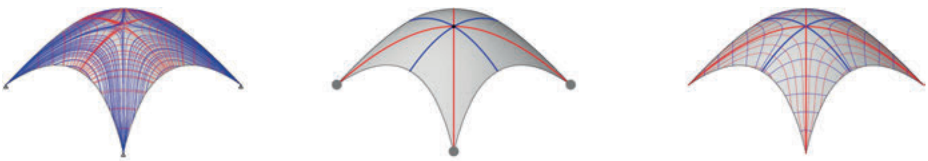
The simple shell is utilized to assess the effectiveness of this method on hyperbolic surfaces. The shell is supported at its four corners under its self-weight. Considering the symmetry of the form and boundary conditions, it is conceivable that there exists



(a) (b) (c)

Fig. 9: Generation of a PSL network for the case slab with two supports in the middle: (a) PSL generated with evenly distributed seed points, (b) topological skeleton derived from degeneration agent system, (c) Evenly distributed PSL generation using spacing agent system.

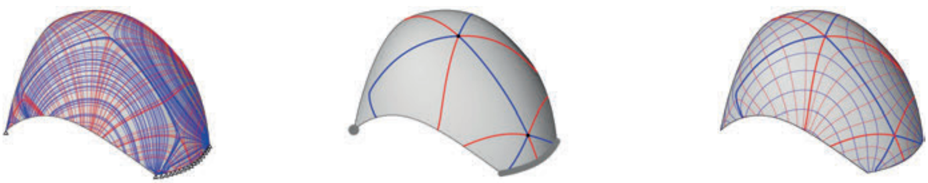
an isotropic point at the center of the shell. This isotropic point corresponds to a saddle-type point resulting from the merging of two trisectors. Consequently, the PSLs passing through this point in both the maximum and minimum directions exhibit four branches (Fig. 10).



(a) (b) (c)

Fig. 10: Generation of a PSL network for the simple shell: (a) PSL generated with evenly distributed seed points, (b) topological skeleton derived from degeneration agent system, (c) Evenly distributed PSL generation using spacing agent system.

The freeform shell is supported at two corners and one edge. Stress concentrations occur at the point supports and edge support, which are excluded from the simulation. Although it may seem complex, the topology of the PSL network on the remaining surface has also been found to be primarily influenced by two trisector points (Fig. 11).



(a) (b) (c)

Fig. 11: Generation of a PSL network for freeform shell: (a) PSL generated with evenly distributed seed points, (b) topological skeleton derived from degeneration agent system, (c) Evenly distributed PSL generation using spacing agent system.

6 Discussion and conclusion

While previous research on PSL network design does not consider the topology of the networks, this research rationalizess the networks by exploring their topology and the underlying formation mechanism. The introduction of the ABM algorithm has made the density control of PSL networks more flexible. The proposed method is intuitive and user-friendly for architects, effectively facilitating the application of PSL in design explorations. Furthermore, the proposed method can be extended to other principal trajectories, such as principal bending moments.

The next step involves exploring the application of this method to a wider range of geometries and boundary conditions. Subsequent studies will also focus on the materialization of PSL-based structures using robotic construction methods, with a specific emphasis on rid-reinforced timber slabs and shell structures (Figure 12).

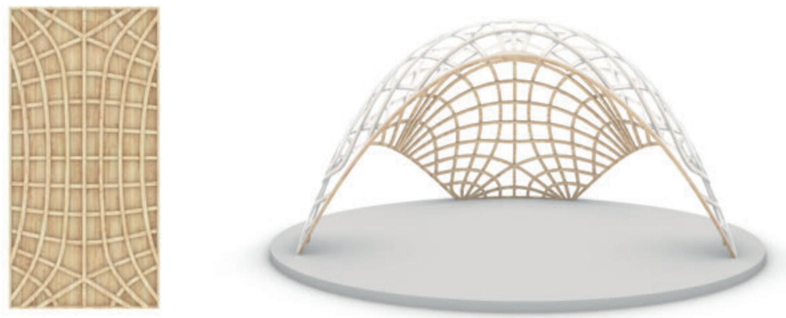


Fig. 12: Application scenarios of PSL in rid-reinforced timber slabs and shell structures.

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