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Bending Cylinders: A Geometric Syntax for Zero-Waste Architecture

Abstract: This research explores the generation of a novel syntax for architectural space driven by a geometric construction strategy that aims to eliminate material waste. The primary goal of the investigations is to avoid off-cuts by creating a vocabulary of varied three-dimensional forms using the full dimensions of standard rectangular sheet material (4 ft × 8 ft) as a building block. By engaging the isometric relationship between flatness and three-dimensional form, the research investigates how waste-conscious approaches to geometric innovation can have typological, topological, and tectonic effects on architectural language. Specifically, the work deploys a subset of curved crease folding that uses planar reflections (sectional mirror operations) to create unique spatial and structural forms through composite cylindrical and conical surfaces. Whereas known studies in curved crease folding typically explore singular (one-off, figural) compositions, this process develops the operation into an aggregation strategy to suggest large inhabitable structures as both figural and field-like conditions. Through geometric literacy, this process attempts to achieve three primary advancements: fusing the design process with constructability, designing the part and whole simultaneously, and creating a workflow whereby complex three-dimensional forms are designed, fabricated, stored, and transported in their compact two-dimensional state. This proposed methodology allows us to consider material economy and labor early in the design process, before construction, and eliminates the additional step of post-rationalizing complex curvature. Finally, the research illustrates a built full-scale prototype that tests the effectiveness of the process in the form of a low-cost shade structure in a small neighborhood park.

1 Introduction: Geometric Aesthetics of the Anthropocene

This research considers methodologies that use 4 ft × 8 ft rectangular sheet material – the most readily available, standardized building material – as the primary tectonic, proportioning, and structural device to create new vocabularies of space. These new geometries serve structural, spatial, ergonomic, and aesthetic roles; their logic of assembly informs a range of constraints, from ideal seating heights to creating novel typological forms or modifying familiar ones.

In this paper, I identify two independent materially driven methodologies as reciprocal venues for exploring the typological, topological, and tectonic potentials of minimizing material waste through an isometric relationship between flatness and

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three-dimensional form: first, curved crease folding, a geometric technique for designing structural surfaces; and second, the aggregation of standard sized rectangular sheet material, a construction strategy for cladding large complex curved surfaces. By pairing these two operations, we create a process that attempts to fuse design with constructability and a strategy to design the part and whole simultaneously. This pairing allows us to consider material economy and labor as part of the design process prior to construction and eliminates the additional step of post-rationalizing complex curvature.

Both methodologies have a long history of productively leveraging their relationship with flatness in their own ways. Curved crease folding is a technique that is inherently materially efficient due to its dependency on developable surfaces to create rigid three-dimensional forms entirely out of flat, two-dimensional sheet material. Developable surfaces, or surfaces that can unroll to flatness without distortion, are uniquely suited to processes that simultaneously design the part and whole as they isometrically map two dimensions onto three. This property means we can immediately establish a workflow between a standard given 2D part (both a material type and dimension) and its three-dimensional implications.

Mapping standardized, mass-produced sheet material onto complex curvilinear forms has become a ubiquitous practice. Most complex curvilinear surface geometries undergo some process of rationalization by being constructed out of planar quadrilaterals (for instance, glass grid shells). The research here seeks to deploy the rectangular sheet not to rationalize complex geometry but rather to generate it. The grammar of flexible rectangular flat sheet material is preserved and translated to map rectilinear Cartesian space onto a curvilinear three-dimensional form, setting new standards of spatial proportions based on the dimensions of the part rather than the whole.

This research studies the spatial, geometric, aesthetic, and typological potentials of using whole rectangular sheets of standard flexible sheet material – specifically, 4 ft × 8 ft – to create composite assemblies of geometric surfaces through the piecewise stitching of individual parts so that they may function at an inhabitable architectural scale. Economical use of the material is paramount, emphasizing the importance of the rectangular geometry of the folded sheets as this tectonic consideration allows for the minimization or elimination of material waste.

2 Curved Crease Folding as a Tectonic Language

Paper folding techniques have been documented and studied exhaustively for centuries. Curved crease folding was first seen through the work of Josef Albers and Irene Shawinsky in the Bauhaus in the 1920s and 1930s. The work of later pioneers in the field (David Huffman, Ronald Resch, and Martin and Erik Demaine) focused mostly on recreating doubly ruled and doubly curved surfaces, such as the hyperbolic paraboloid, out of concentric circular or rectangular pleats on a sheet of paper (Davis et al. 2013).

However, given the ease and economy with which complex calligraphic objects are produced, much of the research, even that which has undergone rigorous mathematical calibration ("computational origami"), has remained purely sculptural (ErikDemaine.org, 2013). Or, in the case of more scientific research, curved crease mechanisms are studied with regards mostly to their relevance to cellular structures, such as intestinal villi and pollen grains (Bende et al. 2015). The studies furnished in this analysis differ from the computational process described by Demaine, in which the three-dimensional outcome is approximated by preferencing the two-dimensional patterning. Instead, this process guarantees a desired three-dimensional outcome by reverse engineering it through the relationship between conic sections in three dimensions and their corresponding conic sections in the transformed flat state. In other words, knowledge of the relationship between an ellipse in three-dimensional space and a hyperbola on a flat plane allows us to precisely derive the type of developable surface (cone or cylinder), its radius, orientation relative to the x-y plane (the ground), and the angle at which a plane that intersects it would need to be to extract an ellipse of a desired eccentricity.

In his seminal work *Structural Systems*, the German architect Heino Engel suggested aggregations of 'bent' or mirrored cylinders for long-span roofs, described simply as "linear structural systems composed of cylindrical surfaces" (Engel 1997a). Structurally, these configurations act as barrel vaults in what Engel refers to as "surface-active" structural systems. However, the execution of these theoretical studies has seldom materialized in the built environment beyond the scale of a tectonic façade detail.

This research looks specifically at a subset of curved crease folding that will be referred to in this paper as "sectional mirrors", also commonly referred to as "reflecting creases" (Demaine et al. 2018) or "curved folding from mirroring operations" (Lebee 2015). While studied extensively, there is not much advancement in understanding the consequential tectonic effects at the architectural scale in the fields of origami and curved crease folding, specifically, on the tectonic translation on modes of habitability and type (materiality, proportion, and structure). Additionally, the difference between *idealized mathematical* curved creasing and *physical paper* creasing does not acknowledge the incorporation of material properties and thicknesses beyond the infinitesimally thin sheet of paper into the computational process (Mundilova 2019).

The application of this research is demonstrated in the design and construction of a full-scale prototype which tests five primary variables of the study: 1) the limitations of a construction sequence of on-site flat-to-form creasing; 2) the structural capacity of the curve-creased membrane geometry; and 3) the time and cost-effectiveness, and ease of installation of a pavilion that deploys this technique.

3 Conic Section Mirroring: Cylindrical Mapping of the Rectangular Sheet Building Block

Any developable surface, such as a cylinder, cone, or tangent developable, can be split in two by an intersecting plane tangent to a curve on the surface, known as an osculating plane.[1] If one part of the original curved surface is mirrored across that plane, the contiguity of the two parts is preserved along the planar curve of the intersection, allowing them to rejoin into a single poly-surface that can unroll coherently into its original shape. The curvature of the surface (direction of concavity) flips about the planar mirroring curve (in this case, a semi-ellipse), creating a single poly-surface of opposing curvatures that share an edge. The result adds a new curve to the surface in the flat state that translates to that planar crease in the 3D state (Fig. 1).

A few aspects of this technique are compelling for considerations of material and structural economy. First, creasing flat sheets in this way adds structural rigidity to singly ruled surfaces which otherwise are not inherently structural unless oriented a certain way (like a barrel vault). So, on the one hand, it becomes a productive way to introduce an extrinsic curve to a mathematical surface (a curve identified as not belonging mathematically to the u- or v- curve fields of a surface). Second, a mathematical relationship can be easily calibrated between the 2D sheet and the 3D form by relating conic sections. In this case, an ellipse of a specific proportion describes a section through a circular cylinder cut at a certain angle, corresponding to a hyperbola of that same angle in the flat state. This approach means a 3D object is designed purely in two dimensions, guaranteeing a pre-determined three-dimensional outcome (the inverse process of Huffman, Resch, and Demaine's work). Lastly, the system allows aggregating and stitching of multiple parts in the flat state which fold into a coherent three-dimensional whole (Fig. 2), introducing a unique and atypical strategy where three-dimensional modules combine in a remote space as 2D objects, using two-dimensional operations on a single plane. The processes necessarily mean we design the part and the whole, and the 2D and 3D – hence the cut-sheet (design) and outcome (construction) – simultaneously.

This line of inquiry attempts to expand on the vocabulary and use of sectional mirrors in two ways: first, to establish descriptive (not analytical or algebraic) mathematical relationships between conic sections derived out of planar intersections with developable surfaces (e. g., the translation of ellipses on a cylinder to corresponding hyperbolas when the cylinder unrolls to a flat plane), which allows for a more intuitive and accessible way of modeling and drawing these geometries without the

1 An osculating plane is defined as a plane that contains the tangent vector and normal vector to a curve at any point along its length, becoming the x-axis and y-axis of that plane, respectively. The normal vector of that plane (z-axis) is defined by the binormal vector of the curve. For a planar curve, the osculating plane, by definition, contains the entirety of the curve.

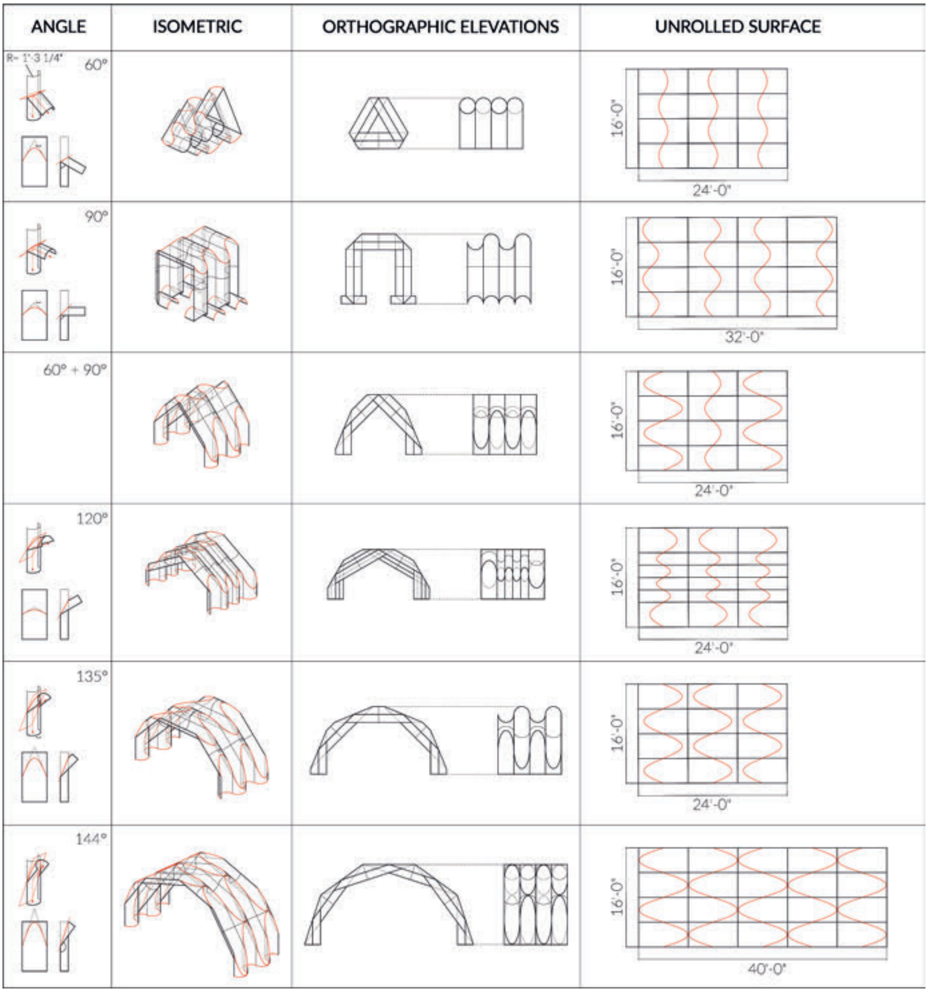

Fig. 1: Assembles of vaulted and gable geometry hybrids using osculating mirrors on cylinders at angles varying from 60° to 144°. Individual cylinders are stitched tangentially with adjacent ones in 2D to produce infinite fields of undulating forms.

need for specialized computational processes. And second, it preserves and maps the original dimensions and rectilinearity of readily available sheet material onto three-dimensional conical or cylindrical surfaces as potential strategies for reducing or eliminating material waste in digitally driven design and fabrication processes. Most existing experiments in curved creasing do not rely on or need to respect the rectilinearity of the original sheet.

The primary interest in these studies investigates formal distinctions between enclosures of different kinds by leveraging the geometric characteristics unique to

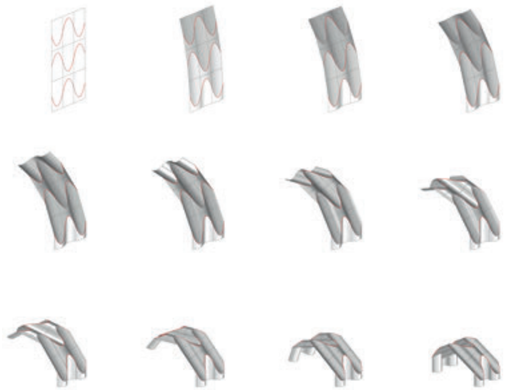

Fig. 2: Assembly of nine individual sheets in the flat state inflated to a three-dimensional gable/barrel vault form.

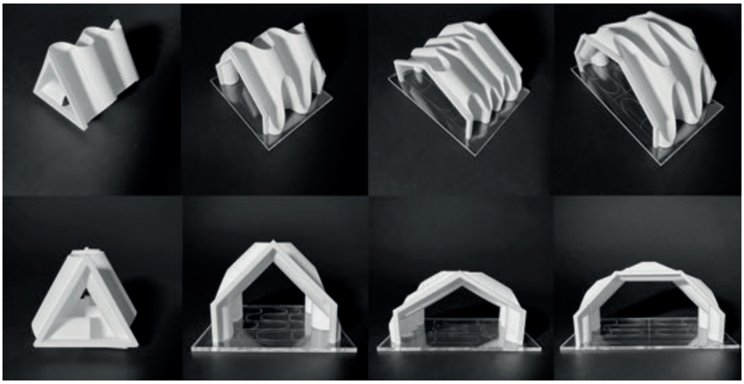

Fig. 3: Assemblies of vaulted and gable geometry hybrids at four different angles and scales.

these surfaces (Fig. 3). Forexample, a cylinder's ability to combine circular geometries and linear edges allows us to ask: can a barrel vault and a gabled roof share the same compositional surface?

4 Bending Cylinders: Shading Structure Prototype for Climate Resilience

A full-scale prototype in the form of a shading structure was constructed in July 2021 in collaboration with the City of Cambridge's Public Space Lab and Community Development Department (Fig. 4) as part of the "Resilient Cambridge" program (CambridgeMA.gov, 2021), which addresses climate inequity in the city. It was imperative

Fig. 4: Left: Aerial photograph of the shading pavilion. Right: The constructed prototype assumes the form of a gable in profile while containing a membrane surface that resembles a barrel vault.

to use construction methods that use recyclable material, produce little to zero waste, and require limited skill and cost in assembly, given a very low budget and the structure's temporary nature. The project addresses three key issues that are important in advancing architectural design in public discourse: social equity, climate equity – two undeniably intertwined issues – and innovation in construction and material technology, in this case, an application of the research on sectional mirror operations to introduce unfamiliar vocabularies of form.

This project uses a cylindrical module of 120° as the primary unit for the "roof-wall". 90° modules transition the vertical wall into the horizontal ground in the form of seating units that visually and structurally anchor the pavilion onto the site. The membrane comprises 1/8th-inch thick sheets of HDPE (high-density polyethylene), a lightweight, translucent, and recyclable plastic that is also relatively cheap. The dimension of each module is calibrated around the most readily available stock size of HDPE to use each in its entirety. No part of any sheet was cut and thrown out. Instead, the sheets are merely scored with hyperbolic curves on a CNC bed and then bent to shape. The same six units create the undulating walls, roof, and seating (Fig. 5). Once dismantled, the plastic can be reused (stored flat or installed at a different location as loose outdoor furniture) or recycled.

While the studies of angular compositions shown in Fig. 1 and 3 show various spatial configurations, the constructed pavilion uses a module of a 60° mirrored cylinder (yielding 120° roof and eave angles) as it proved optimal in terms of four interrelated (dependent) parameters that account for cost, material waste, and dimensional concerns. Up to three 4 ft × 8 ft modules (an entire bay) were stitched together flat and folded into their three-dimensional undulating cylindrical form (Fig. 6). The modules are rotationally aggregated to produce a sine-like wave in the flat state (Fig. 7). This undulation translates visibly in every projection of the three-dimensional object in plan and elevation.

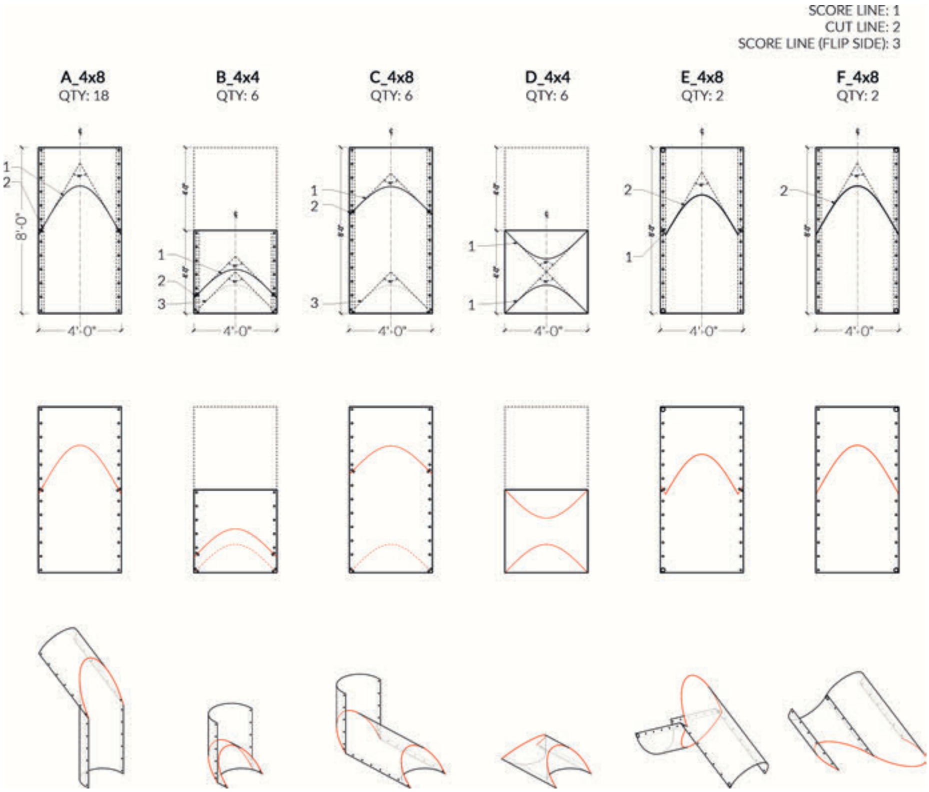

Fig. 5: The six units of construction and their corresponding cutsheets (unrolled surfaces).

5 Observations and Continued Research

The simplicity and ease with which these forms translate from the flat template to the three-dimensional state is beneficial for design and assembly but has limitations. The one-way three-dimensional form emulates an accordion, making it dynamic and indeterminate in one axis. Due to the material's stiffness and the curved crease's inability to hold its shape as effectively as a straight crease, the entire composition retains its memory of flatness and wants to unroll naturally. On the one hand, the curve adds structural stiffness to the planar surface but only if it is held in that curved position by external members. As a result, extrinsic structural systems are required to preserve the integrity of the surface membrane: first, an external structural lumber moment frame that operates as a clamp or bookend to resist the lateral thrust of the curved HDPE sheets; and second, steel tie rods operating in both compression and tension at the roof ridge lines and eaves, and cross bracing in the central bay of the composition, to resist shear.

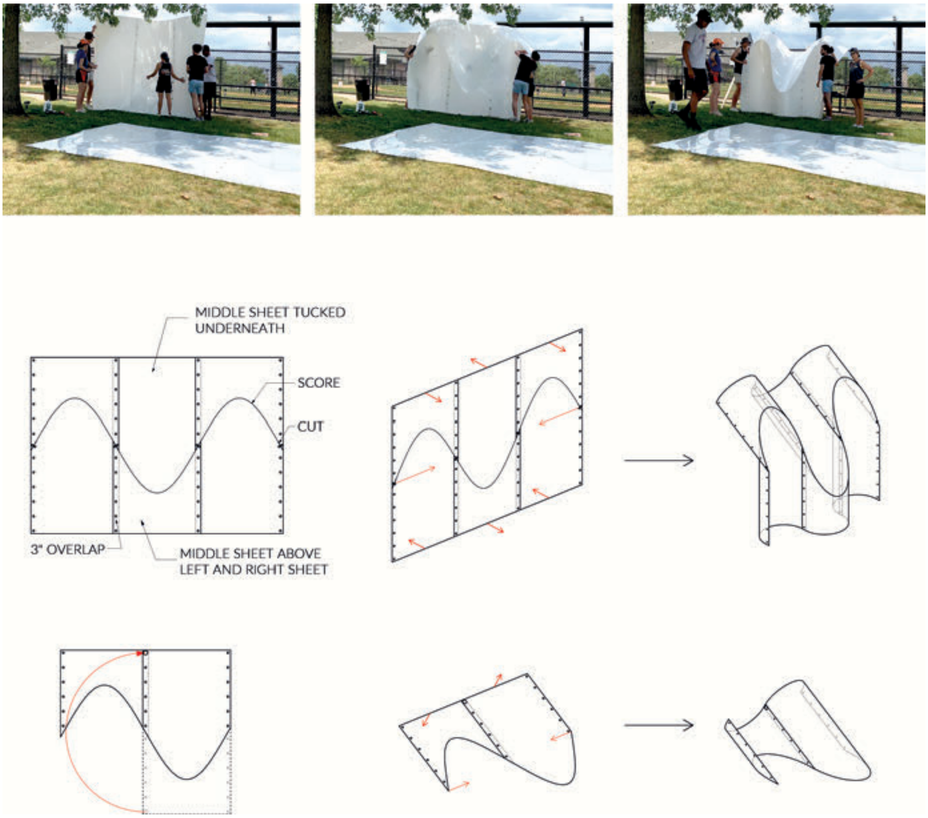

Fig. 6: Flat to 3D transformation process. Three identical Type A modules stitched together with mechanical fasteners along a tangent planar overlap while flat, then curled up into the 3D state collectively.

There are various ways of addressing the structural limitations of the one-way system. The studies in Fig. 8 and Fig. 9 introduce curvature along the perpendicular axis to turn the one-way accordion-like system into a two-way self-supporting system that can turn the corner in two axes. By introducing curvature in a perpendicular axis, the curved sheets assume the dual responsibility of both membrane and structure. This outcome is achieved most effectively by intersecting cylinders in perpendicular planes (via sectional mirroring planes) and recreating the familiar form of a groin vault (Fig. 10). However, the difference here is that the cylinders are not trimmed at the intersection but cut and folded to provide adequate surface area for the adjacent participating cylinders to adhere to one another. Extraneous material left behind belongs precisely to this zero-waste curved crease folding system, elevating the capabilities of surface membranes to behave like reciprocal structures (where individual parts rely on each other in equilibrium) while assuming a sculptural and spatial form of enclosure. Here,

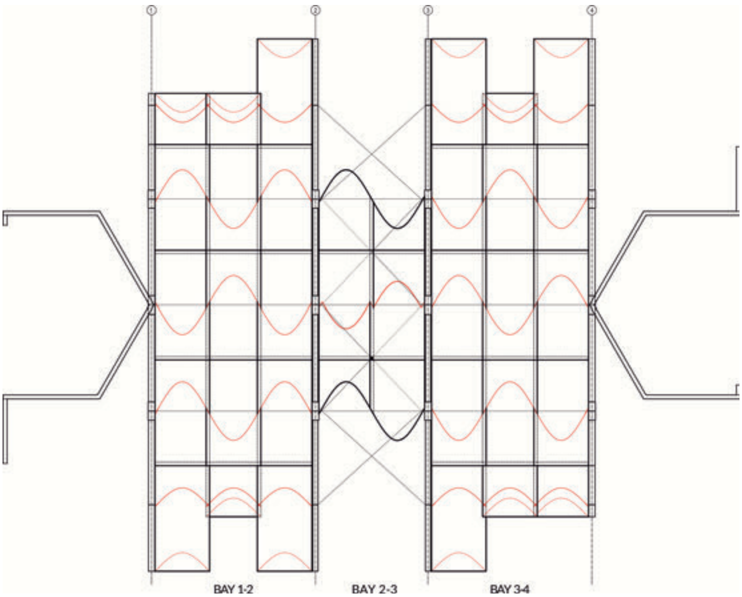

Fig. 7: Unrolled composition of membrane shown with structural members and the 3″ material overlap for fastening sheets with each other (in three-dimensions, this overlap is a planar surface tangent to the cylindrical portion).

we are considering a scenario where membrane, structure, and tectonic assembly sequence are fused into a single vocabulary.

The next phase of the research investigates how the surface can become self-supporting, creating opportunities to attach to itself on either end (as a structural 'loop') or by self-intersecting to introduce curves in the perpendicular plane to counter the torsional effects of the one-way accordion system. An example would be to incorporate structural members within the patterning of the planar mirroring curves. Figure 11 demonstrates a specific proportion of creasing whereby the elliptical curves of adjacent units are almost tangent, thereby acting structurally as arches leaning on one another in mutual compression. If these arches are manifest as structural strips of wood or metal (these would be standard linear members when flat), they can be installed while the sheets are in the flat state and then anchored to one another through tension cables once the membrane takes its 3-dimensional form.

Additionally, eliminating mechanical fasteners to stitch multiple sheets together would be desirable for a more efficient assembly process. Figure 12 demonstrates preliminary studies of more literal "stitching" techniques between two sheets using rope, strips of HDPE, or other similar linear material that can be easily woven and tied without using mechanical fasteners or tools. Further explorations are being developed through a system of additional cuts along the edge that allow the sheets to interlock. Axel Kilian explored a similar endeavor assembling flat sheets to produce double-

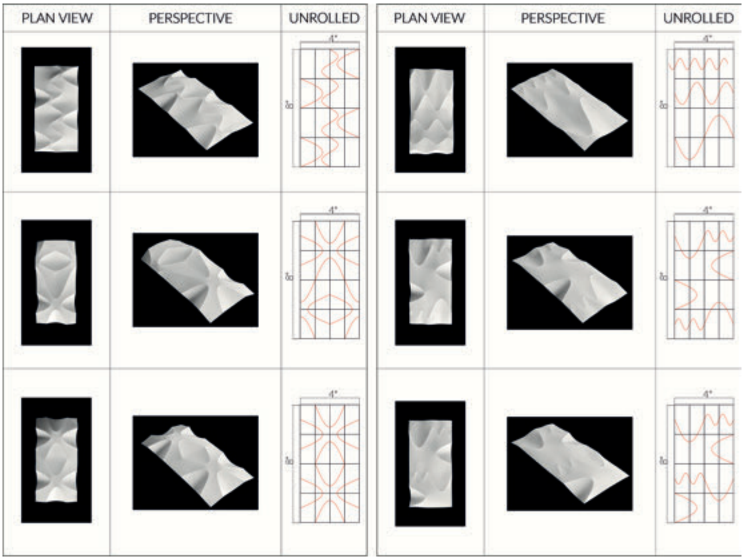

Fig. 8: Inscribing hyperbolic curves on a sheet perpendicular to one another to produce a surface of double curvature to increase the structural rigidity of the surface. These compositions are designed purely in 2D, akin to Huffman and Demaine's work.

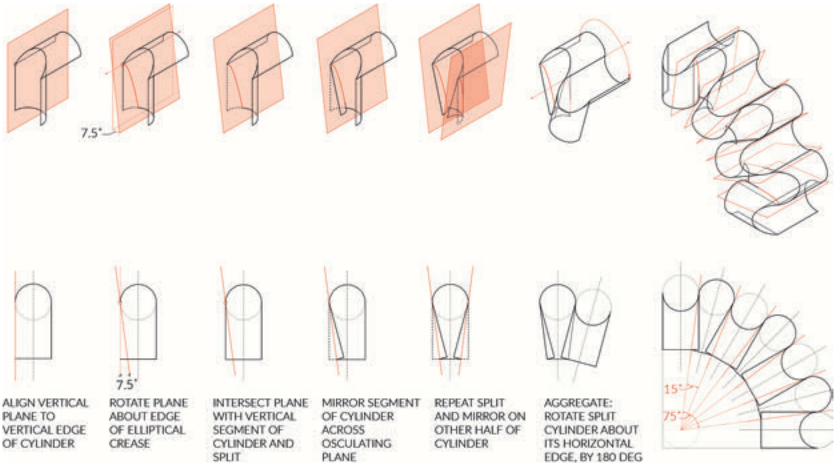

Fig. 9: Introducing sectional mirrors in the perpendicular axis to force the system to rotate in the other direction. This allows the one-way system to evolve into a two-way system by turning the corner in another plane and making the membrane serve structural purposes.

curved surfaces through what he calls a "puzzle joint". This "zipper" system allows adjacent sheets to interlock via friction-fit using the geometry of the surface itself rather than fasteners (Kilian 2003).

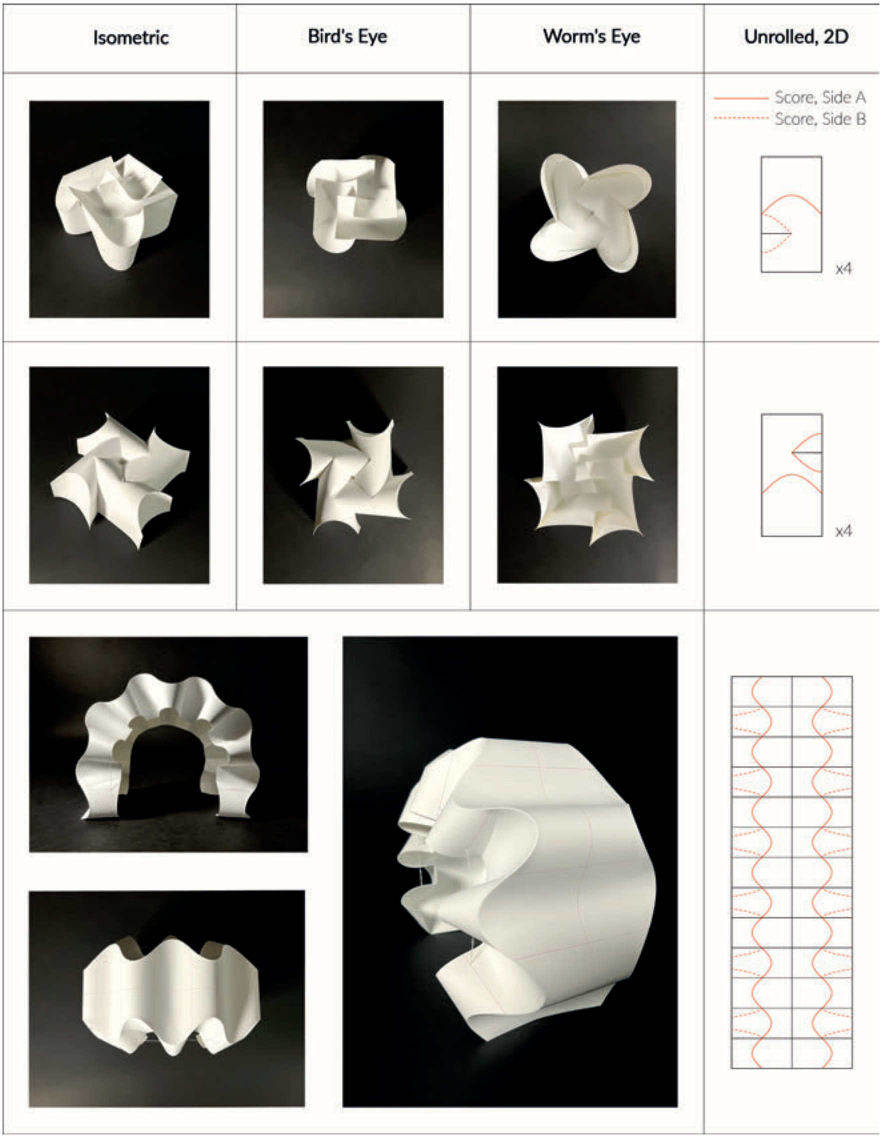

Fig. 10: Drawings and models of groin vault geometries, and a surface using sectional mirrors in the perpendicular axis to allow the system to turn the corner in the perpendicular plane.

Another area of improvement could test the limits of the number of sheets that can be stitched flat and folded up to inflate an entire structure in a single operation. Lastly, this prototype used HDPEsheets but the technique should be tested using various other flexible sheet materials such as metal, wood, fiberglass, other plastics, or a hybrid composite of methods and materials (e.g., sandwiching fabric between two layers of thin plywood).

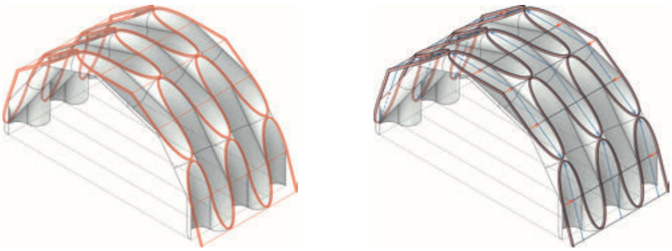

Fig. 11: Strips of wood or metal incorporate structure into the patterning of the creases, acting as arches. The arches lean on each other in mutual compression (blue arrows) while cables would pull them together in tension laterally (red arrows).

This research is continually evolving to advance building practices, develop new geometric vocabularies, and create a system that combines design and construction processes. It requires shifting attention to the geometric intelligence of the individual material part, rather than the complexity of a digital immaterial whole, allowing the tectonic definition to lead the aesthetic, structural, and topological characteristics of the surface. The primary ambition to elevate the tectonic assembly to the design process (rather than remain in the construction process, where it is most typically considered) is to limit or eliminate material waste in the process without sacrificing formal invention.

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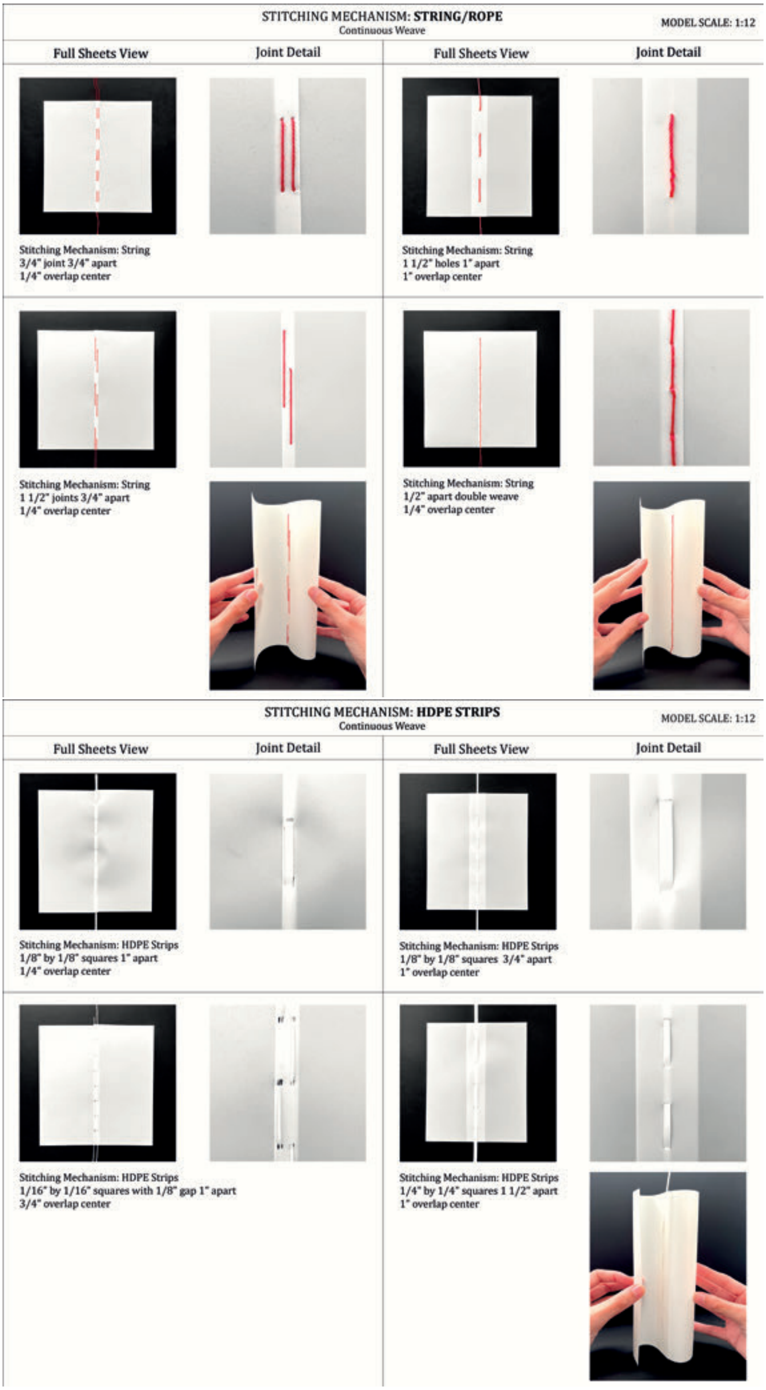

Fig. 12: Scalar studies of alternate sheet fastening mechanisms (*continued on next page*).

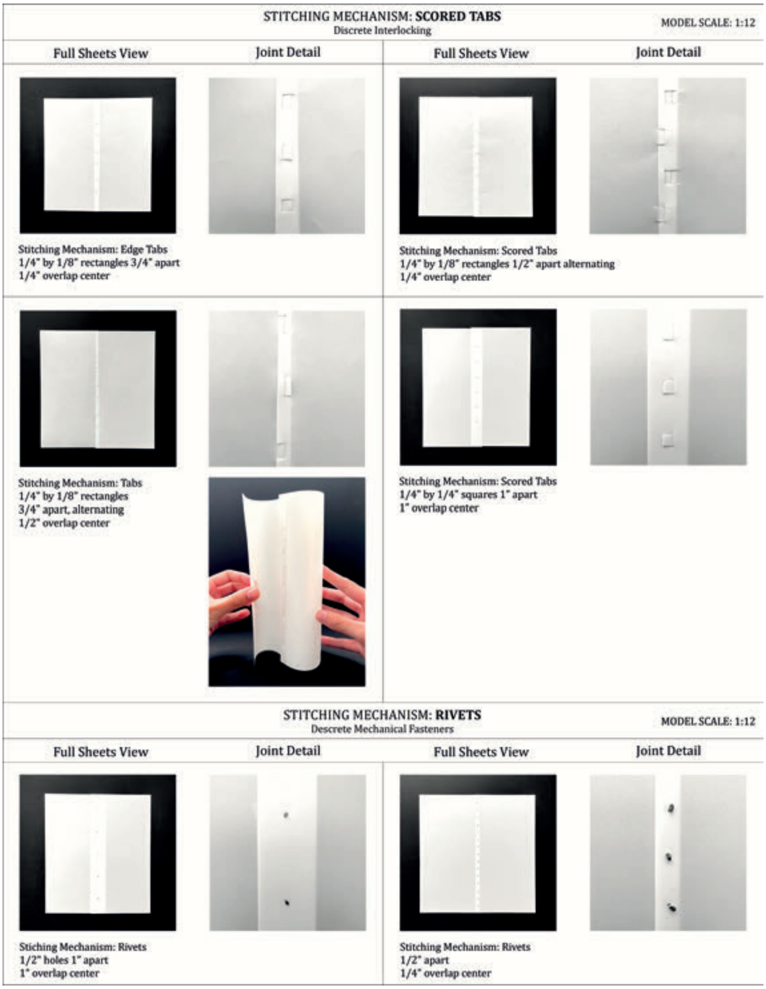

Fig. 12: (*continued*) Scalar studies of alternate sheet fastening mechanisms; weaving techniques through pre-drilled holes using string or HDPE strips (same material as the sheet itself), interlocking mechanism via scored tabs within the sheet itself, requiring no additional or extrinsic material for attachment, and the default metal fastening technique used in the prototype shade pavilion, represented here using rivets.