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Developable Membrane Tensegrity Structures Based on Origami Tessellations

Abstract: In this paper, we propose a design method of membrane tensegrity structures by solving the inverse problem using freeform origami tessellations by generalizing Resch's patterns. We found that if we generate an origami tessellation and replace the mountain crease of the tessellation with struts and the other parts with a membrane under initial tension, the tessellation can be formed as a tensegrity structure. In addition, this structure may snap through and deform significantly, but we have shown that the mean curvature of a given surface is a valid evaluation index as a condition for a surface that does not snap through. When computed origami tessellation patterns are naively used, the shrinking of membranes caused by initial tension results in a deviation from the target shape. We propose an optimization-based framework that resolves the deviation and ensures that the structures match the given surfaces.

Keywords: tensegrity, membrane tensegrity, inverse problem, origami, origami tessellation

1 Introduction

Developable membrane tensegrity structures are tensegrity structures formed from single planar membranes used as the tensile member and struts used as the compression members. They have advantages in the manufacturing process as they can be constructed in planar states by laying out struts on a single pre-tensioned membrane and erected by releasing the pretensions. Several methods for "forward problems" to find the three-dimensional resulting shape from a two-dimensional layout pattern have been proposed.

One example is the MOOM Pavilion, created by students of the Tokyo University of Science under the supervision of Kazuhiro Kojima and Jun Sato (Ratschke et al. 2017). They developed a staggered arrangement of parallel struts and a tunnel-like structure with arches in the direction parallel to the struts and cables pulling in the transverse direction. The pavilion consists of 131 aluminum tubes and an elastic polyester mesh, and was successfully constructed as an 8 × 26 m.

Gupta et al. (2020) extended the designable structure by placing compression struts in a reciprocal pattern on the membrane. They also demonstrated a workflow for accurately producing knits using CNC-produced knittings as the main tensile element. The process of creating these structures is form-finding through mechanical simulation

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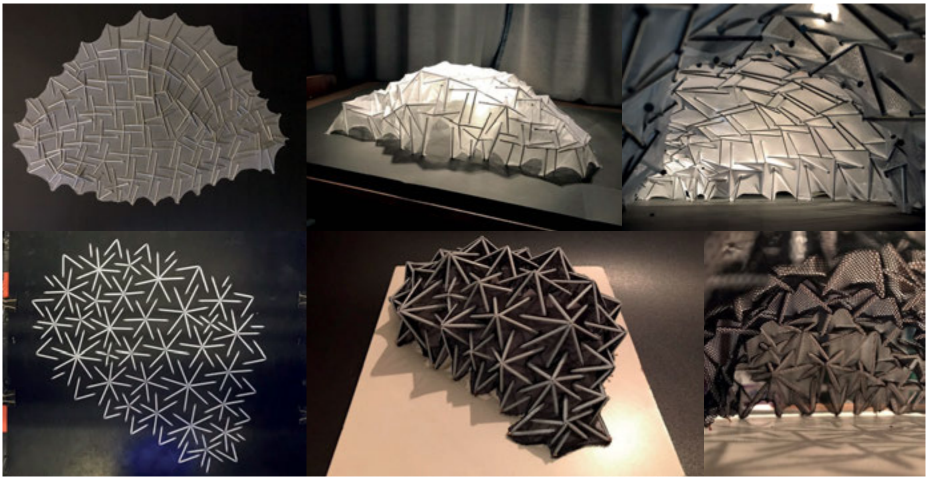

Fig. 1: Process of model making. Top left: Fixing aluminum pipes to stretched stretchable nylon cloth. Top right: When the boundary is cut out and the foot is fixed, the structure stands up in the same shape as the given curved surface.Bottom left: 3D printing PLA directly onto stretched stretchable nylon fabric. Bottom right: When the boundary is cut out and the foot is fixed, a structure with the same shape as the given curved surface rises up.

and model making, solving forward problems to determine what 3D shape can be obtained from a given 2D strut pattern.

However, the inverse problem, i. e., designing the planar pattern when the target shape is given, has not been solved. In this paper, we propose a design method of membrane tensegrity structures by solving the inverse problem using freeform origami tessellations by generalizing Resch's patterns proposed by Tachi (2013). Fig. 1 shows models generated by this method. When the struts are attached to the elastic membrane under tension, or directly 3D printed, and the surrounding area is cut away to release the tension, a structure with the same shape as the given surface rises up. In Sec. 2, we present the basic strategy for converting a polyhedral mesh into a tensegrity structure.

Through experimentation, we have identified two problems and propose the following solution for each problem. (1) Firstly, the obtained structure may buckle into an undesired state depending on the curvature of the input mesh. In Sec. 3, we perform an experiment using constant mean curvature surface to empirically estimate the conditions under which the tensegrity structure can be established without collapse. (2) Secondly, the shrinking of membranes causes a deviation of a surface from the target shape. In Sec. 4, we propose an optimization method to correct deviations due to membrane deformation.

2 Inverse problem of membrane tensegrity structures

Several studies suggest the relationship between origami and tensegrity structures. The analytical model of MOOM pavilion, in which tension members are replaced by cable is very similar to the Yoshimura pattern (Ratschke et al. 2017). The printing pattern on the membrane used in Jourdan et al. (2020) is similar to Resch's pattern. Tachi (2012) points out the theoretical equivalence of an infinitesimal folding mode of a polyhedral surface and the equilibrium of forces to show that the same shape can be used as a shaky polyhedron and as a tensegrity structure.

In this study, we aim to make use of the universality of computational origami design, i. e., origami-based patterns can be computed for given polyhedral meshes Tachi (2009, 2013) for the design of membrane tensegrity structures that can be deployed from flat states.

Specifically, we show that if we generate an origami tessellation by generalizing Resch's patterns proposed by Tachi (2013), and replace the mountain crease of the tessellation with struts and the other parts with a membrane under initial tension, the tessellation can be formed as a tensegrity structure (Fig. 2). The flow of membrane tensegrity structure generation is as follows:

1. Provide a target shape as a polyhedral mesh.
2. *Origamise*, i. e., apply the algorithm of (Tachi 2013) to create a generalized Ron-Resch pattern. We choose star-shaped wrinkle patterns inserted on the edges of the given mesh and let the corrugated surface be developable using an optimization scheme. Here, either *Freeform Origami* (Tachi 2013) or *Crane* (Suto et al. 2022) can be used for optimization calculations.
3. Replace the mountain crease lines of the obtained mesh with compression members and the other creases with tension members under initial tension by setting the target length to its scaled version by some scale factor $s < 1$ (empirically chosen $s \approx 0.90$). Support points are provided around the boundary of the mesh. To obtain the equilibrium position, we use dynamic relaxation. *Kangaroo2* (Piker 2013) was used in our analysis.

After verifying the form under equilibrium, we stretch the membrane in a planar state by some scale factor 1/s and attach rigid material along the mountain creases of the crease pattern. Here, for a small-scale model, the attachment of rigid bars can be replaced by 3D printing directly to the membrane as in Tibbits (2016); Jourdan et al. (2020). We release the prestress by cutting out the boundary of the crease pattern; then the membrane's tension produces the curvature and after a small adjustment and the pinning at the support point. We obtain the desired membrane tensegrity structure.

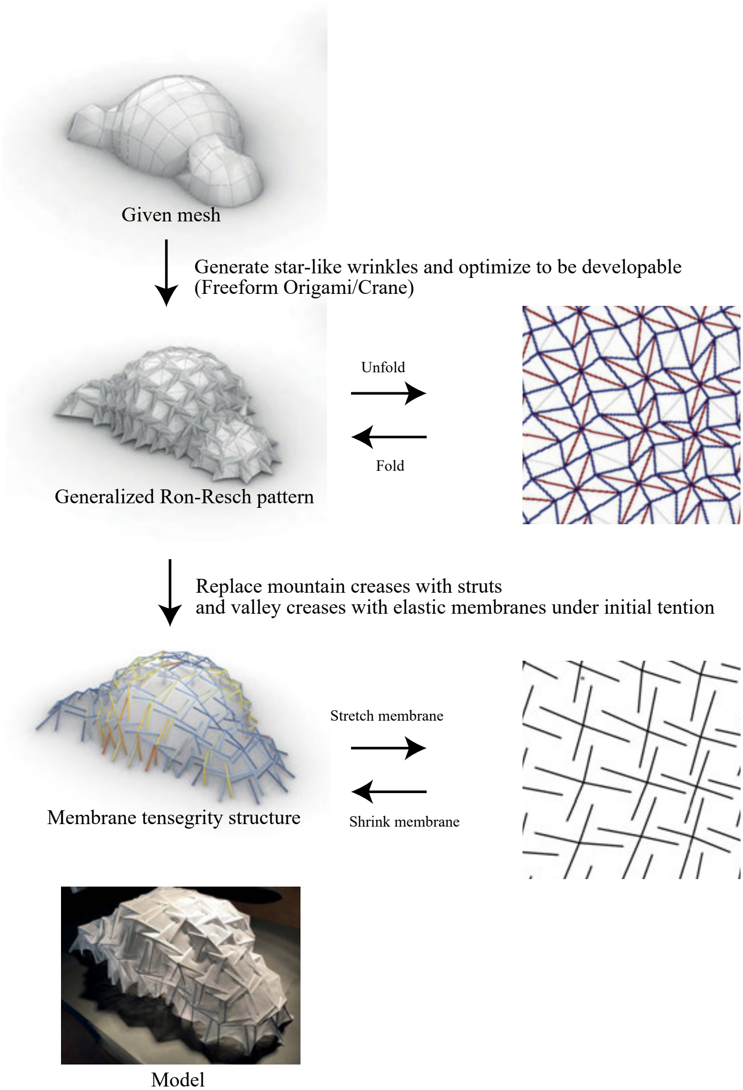

Fig. 2: The process of solving inverse program of a membrane tensegrity structure.

Here, we have obtained two interesting problems. (1) Firstly, the obtained structure may buckle into an undesired state depending on the curvature of the initial mesh and the stretch factor of the membrane. We clarify these conditions in the next section. (2) Secondly, the shrinking of membranes causes a deviation of a surface from the target shape. In Sec. 4, we propose an optimization-based scheme to correct deviations due to membrane deformation.

3 Conditions for membrane tensegrity to maintain its shape

Here, we show the conditions for a surface to which this structure can be adapted. In the simulation phase, i. e., Step 3 in Sec. 2, we apply the equilibrium force by lowering the shrinkage factor s from 1 to a smaller value. Normally, the structure gets stabilized as s lowers; however, we have found that some surfaces with concave parts may lose stability through a snap-through event. Specifically, the star-shaped “spikes” at the concave part of the surface will snap-through when the membrane section is tensioned towards zero length (Fig. 3). When adapting to freeform surfaces, it is important to avoid such snap-through that may lead to the collapse of the entire structure.

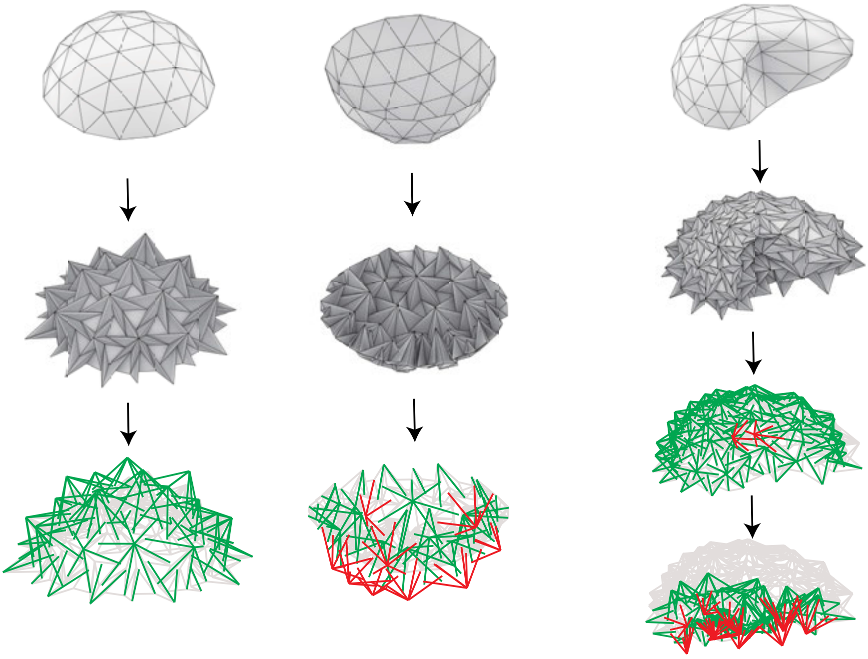

Fig. 3: When the membrane section is tensioned towards zero length, it can be formed as a tensegrity structure if the surface is convex (left), but if it is concave, the vertices will snap through and deviate significantly from the target shape (center). When adapting to freeform surfaces, if concave areas are included, snap-through may occur and the entire structure may collapse (right).

We can observe from the two left figures of Fig. 3 that the system is more stable when the star-shaped spikes orients toward the curvature direction. Also, an example in Fig. 2 shows that even a surface with a concave part can be realized if the surface is “roughly convex”. Therefore, we propose the mean curvature as an empirical criteria for evaluating surfaces that do not snap through. One surface with a constant mean curvature, the unduloid, was discretised and a small mesh consisting of seven degree-6 vertices was taken as the test piece. Generate tensegrity from this small mesh, and then simulated under tension to verify whether snap-through occurs. When the average length of the discretised mesh edges was set to 1 and the mean curvature was 0.05, 0.10, 0.15 and 0.20, it was verified that snap-through occurred at mean curvatures below 0.10 but not at curvatures above 0.15 (Fig. 4).

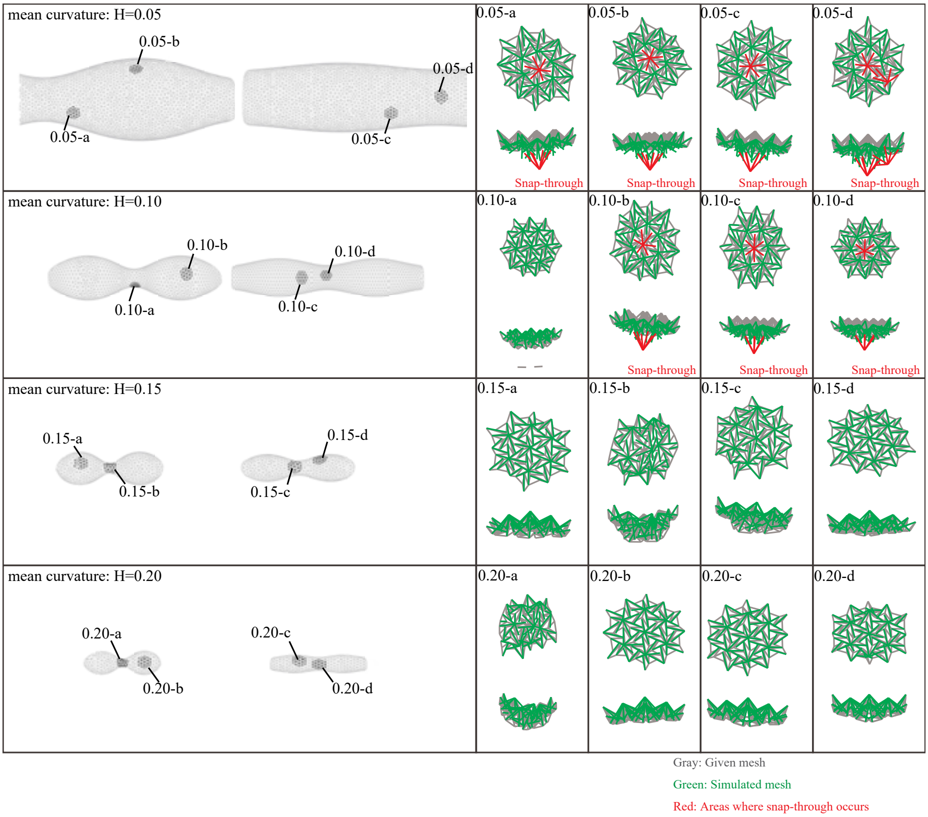

Fig. 4: A small mesh was taken from a surface of constant mean curvature to see if it would snap through under tension on the membrane. Snap-through occurred at mean curvatures below 0.10 but not at curvatures above 0.15.

From the study, it is believed that a stable structure can be obtained by keeping the average curvature above a certain level. In the future, vertices with degrees other than the 6-degree vertex should be considered.

4 Optimization of resolving deviation

We propose an optimization-based framework that resolves the deviation caused by the membrane shrinkage and ensures that the structures match the given surfaces (Fig. 5). The difficulty of the problem is that the actual lengths (relative to original lengths)

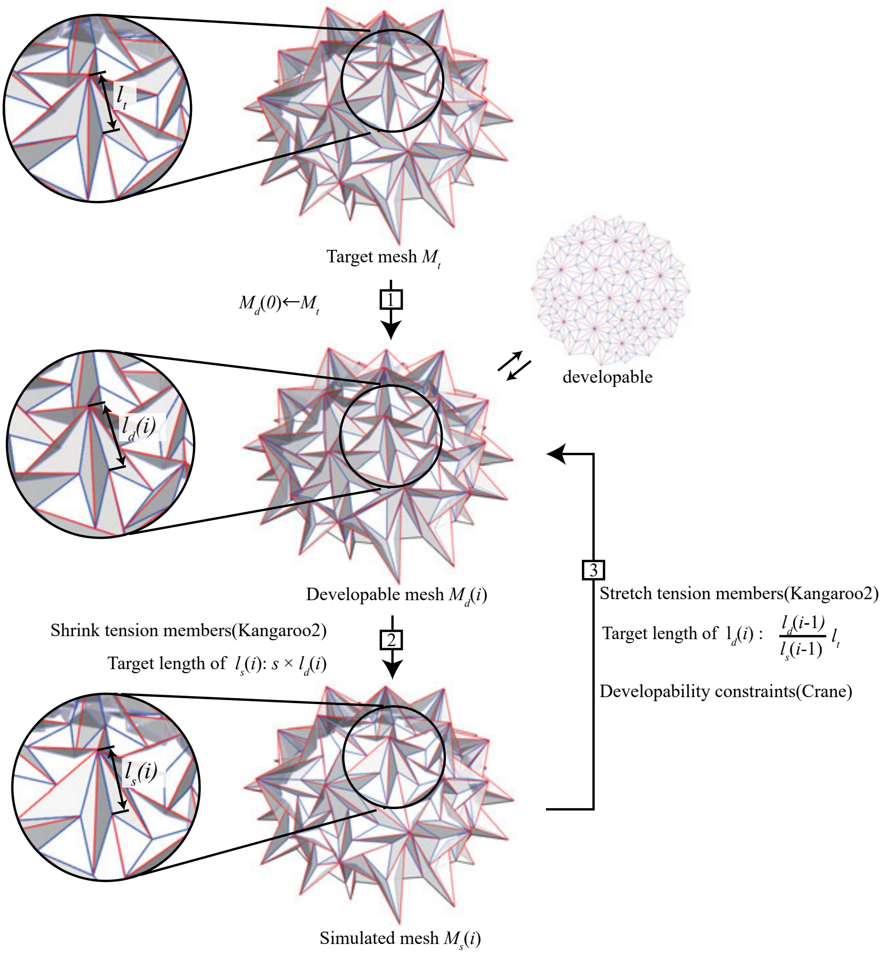

Fig. 5: The process of optimization.

of tension members under equilibrium are chosen between s and 1 as the process of equilibrium and cannot be predefined.

Therefore, our strategy is to iterate the simulation process of the tensegrity structures and correct the scale factor parameters for each step. In each step $i = 0, 1, 2, \ldots,$ we keep three meshes: a target mesh, a deployable mesh, and a simulated mesh.

The target mesh M_t is generalised Ron-Resch pattern of the given surface. The deployable mesh $M_d(i)$ is the membrane stretched under initial tension and is deployable. The simulated mesh $M_s(i)$ is the actual final shape resulting from releasing the tension in the deployable mesh, which should match the target mesh.

In each of the three meshes, the length of the struts (mountain crease lines) should remain unchanged. The lengths of the membrane elements (valley crease lines) are modified during the optimisation process; For each tension edge, we let l_t, $l_d(i)$ and $l_s(i)$ be its lengths in M_t, $M_d(i)$, and $M_s(i)$, respectively. $1/s$ is an initial stretch of the membrane. The optimisation process is as follows,

1. Set $M_d(0) \leftarrow M_t$.
2. Obtain $M_s(i)$ from $M_d(i)$. Specifically, set the target length of $l_s(i)$ to $s \times l_d(i)$ and simulate a dynamic relaxation method.
3. Obtain $M_d(i+1)$ from $M_s(i)$. Specifically, set target length of $l_d(i)$ to $\frac{l_d(i-1)}{l_s(i-1)} l_t$ while applying developability constraints on $M_d(i)$ using *Crane*.
4. Return to step 2 and repeat this until the deviation between the target mesh and the simulated mesh is sufficiently small.

As an example, we tested this process on a tensegrity structure with an average strut length of 1. The average deviation at each vertex was reduced by 54%, from 0.098 to 0.045. The maximum deviation decreased 46% from 0.365 to 0.198 (Fig. 6).

5 Outlook and Future Works

The construction of complex free-form tensegrity can be costly, labor-intensive, and time-consuming due to challenges in reproducing the shape and assembling the structure. Our proposal potentially solves these challenges by enabling various complex three-dimensional forms constructed from a single plane. This method uses a single planar membrane as a guide, and even inexperienced workers can easily construct free-form tensegrity structures. In the future, there is a need to study using full-scale mockups and to clarify the conditions under which tensegrity is established under various loading conditions.

In addition, we tested only the degree-6 vertices case regarding the mean curvature at which snap-through does not occur; we would like to study vertices of other degrees as well. The current algorithm to reduce the deviation converge to a state with non-zero deviation. This may be because we try to equate the intrinsic metric of the equilibrium

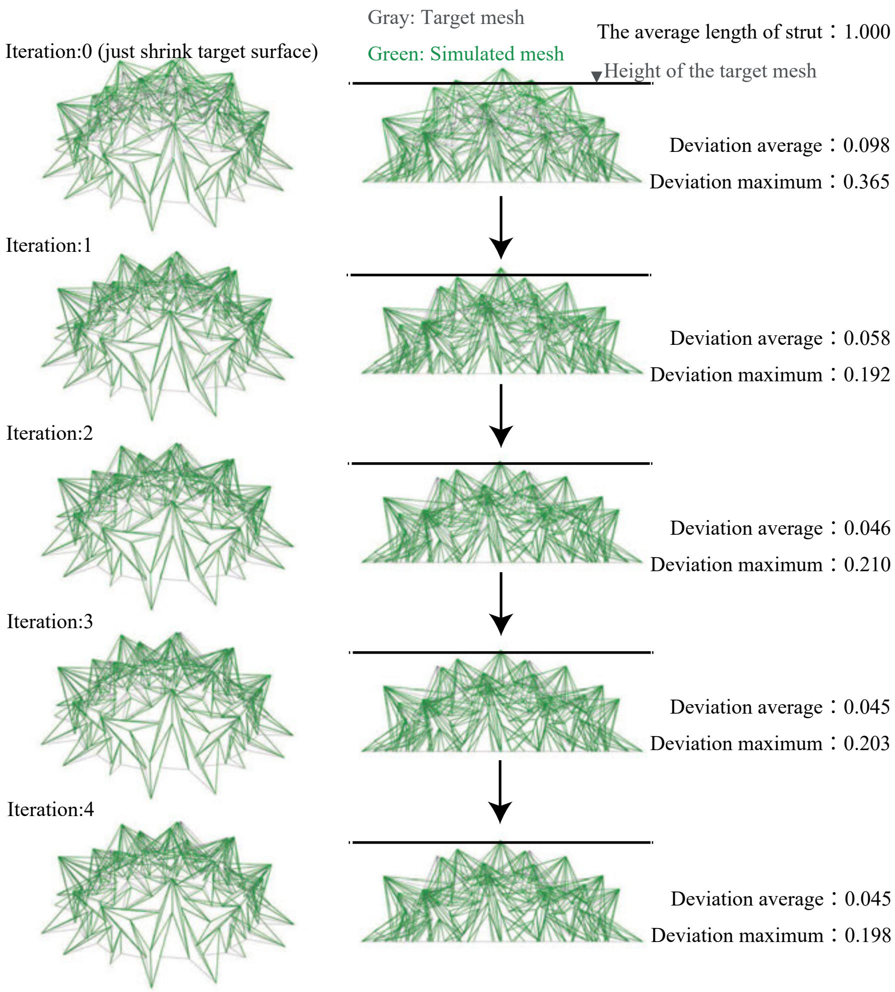

Fig. 6: The result of optimization.

state and the target state already given as a corrugated origami; direct measurement and minimization of the proximity to the given mesh target may improve our result.

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