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RUM: Reconfigurable Umbrella Mesh

Abstract: We propose Reconfigurable Umbrella Meshes (RUMs), a new class of pre-assembled deployable structures. A RUM has a compact rest state that can be easily altered to deploy into different bending-active 3D target shapes. RUMs consist of elastic beams and rigid plates connected by hinge joints and can be assembled in a stress-free fabrication state. The key principle of a RUM is that it encodes the desired deployment shape in the height distribution of its constituent cells. Reconfigurability is achieved by introducing sliding joints to connect the elastic beams with the rigid plates to allow adapting the cell heights. We demonstrate that even for small variations in the cell heights, RUMs can capture a diverse range of shapes. Assembled from identical cells that can be mass-produced, a RUM can deploy into several desired shapes, which makes them well suited for re-usable temporary structures. We provide a computational design tool based on physical simulation, where users can interactively edit cell heights to explore achievable deployed shapes. Numerical optimization enables designers to easily navigate between the reconfigurable design parameters and the deployed shape space. We validate our approach with a physical prototype and demonstrate its various deployed states.

Keywords: geometry, reconfigurable structures, deployable structures, bending-active, optimization

1 Introduction

Shape-morphing structures are physical systems that have the ability to change their spatial configuration. Such systems are of significant interest at multiple scales ranging from micro-scale metamaterials to large-scale architectural designs. Here, we present a new shape-morphing structure that can be easily assembled from identical, mass-produced components in a compact, stress-free state, and that deploys to a wide range of free-form shapes.

Specifically, we propose an extension of *umbrella meshes* recently introduced by Ren et al. (2022). Umbrella meshes are a class of deployable structures that can be optimized to deploy to one pre-defined target surface. We generalize umbrella meshes to be fully reconfigurable, enabling deployment to a continuous range of target shapes for a single physical model.

Umbrella meshes are composed of a regular array of scissor-linkage cells, where each cell's height is optimized to achieve a specific local expansion during deployment. This leads to incompatibilities among neighboring cells causing the elastic structure to deform into the desired curved target surface. As a consequence, each cell is different and needs to be fabricated to measure. Reconfigurable Umbrella Meshes (RUMs) replace

the custom cells by identical copies of a single, adaptable cell. This not only allows mass-production of the cells, but also enables programming many different deployed target geometries *after* fabrication and assembly, thus creating a truly reconfigurable and adaptable system.

We use physics-based simulation to handle the complex force coupling of the elastic scissor linkage and provide accurate predictions of the deployed shape of the global bending-active structure. The design handles, i. e., the programmable heights of individual cells, are directly exposed to the user for physical manipulation. We facilitate design exploration both via a forward simulation tool that computes the deployed shape based on direct user manipulation of the design handles, and an inverse algorithm to optimize design handles for specific target shapes.

2 Related Work

In recent years, many efforts have been conducted on the use of rigid-beam scissor mechanisms for deployable structures in architecture and engineering. A review of recent developments can be found in Moy et al. (2022). Maden et al. (2019) presented a study focusing on the terminology and classification of scissor mechanisms used in the research of deployable structures. The predominant categorization of scissor structures in the literature typically relies on three fundamental planar cells, referred to as translational, polar, and angulated, that can be combined to shape 3D scissor cells. Several recent studies have proposed geometric approaches for exploring the design space of scissor structures using polar and angulated cells (Roovers and De Temmerman 2017; Roovers et al. 2013; Dinevari et al. 2021; Arnouts et al. 2018, 2020).

Besides deployable structures, special attention has also been devoted to the topic of reconfigurable structures. Akgün et al. (2007) proposed a new type of planar cell based on adding two revolute joints and beams attached to a basic scissor cell. The principle was further extended in the study by (Akgün et al. 2011) to develop 3D cells. This advancement introduced unique extension and rotation capabilities that enable dividing the behavior of the entire scissor structure into sub-structures. These sub-structures can then transform locally without impacting the other parts permitting to develop a wide variety of geometrical shapes. Rosenberg (2010) introduced the concept of a double scissor-pair cell, which utilizes a unique combination of two polar cells to transform the scissor structure into various shapes. Additional transformation capabilities based on the double scissor-pair cell are enabled through the introduction of notches. Incorporating notches on the beams makes it possible to convert translational cells into polar ones, thereby introducing an extra degree of freedom to the mechanism. More recently, the concept of a universal scissor component, which can be easily reconfigured to achieve all basic scissor cells and manufactured on a large scale, was introduced by Alegria et al. (2016).

Research on active-bending systems proposes a different approach for constructing deployable scissor structures that do not rely on rigid beams. Active bending is defined as a controlled shaping process for inducing curvature through elastic deformations (Lienhard 2014). By incorporating flexible beams into deployable scissor structures, a greater range of shapes can be achieved, as the beams are able to bend and twist. Recent studies on deployable gridshells aim to finely tune geometrical incompatibilities of irregular grids to achieve both deployability and shaping goals. Panetta et al. (2019) presented X-shells as a generalized type of deployable gridshell in which geometrical incompatibilities during deployment push the structure to buckle out-of-plane to achieve a 3D curved shape. An optimization process is proposed to address the design of X-shells based on the use of physical simulations. Related efforts have also been conducted to investigate geometric approaches based on the design space of geodesic grids (Pillwein et al. 2020; Pillwein and Musialski 2021).

3 Reconfigurable Umbrella Meshes

Our work builds upon and extends *umbrella meshes* as proposed by Ren et al. (2022). Umbrella meshes are deployable structures composed of a regular tessellation of scissor linkage cells assembled in a compact rest state. Each cell consists of three pairs of arms joined as vertical scissor mechanisms around X-joints. The arms connect to rigid plates (top and bottom) through T-joints. The cell is deployed by pushing the top and bottom plates towards each other. This causes the arms to rotate and expand the lateral footprint of the cell (see Fig. 1).

Cells can have varying expansion factors of their lateral footprint, indicated by the dashed lines in Fig. 1b. When fully deployed, the expansion factor of a cell depends on the separation between the two plates in the rest state, defined as cell heights. Deploying two connected cells of different heights creates a mismatch in their planar footprints. When several such cells of incompatible expansion factors are combined, the umbrella mesh deforms out of plane during deployment into a curved, surface-like structure.

3.1 Reconfigurable Umbrella Cell

In the physical realizations presented by Ren et al. (2022), cell arms are cut to size to match the heights of one specific optimized deployed shape. Our key observation is that we can easily avoid this irrevocable, shape-specific fabrication of each cell and instead adapt the heights of the cells dynamically (see Fig. 1). Specifically, our new cell design allows adjusting its height at the T-joints. During reconfiguration, the arms of the scissor linkage can slide through the T-joints and be blocked at the desired height
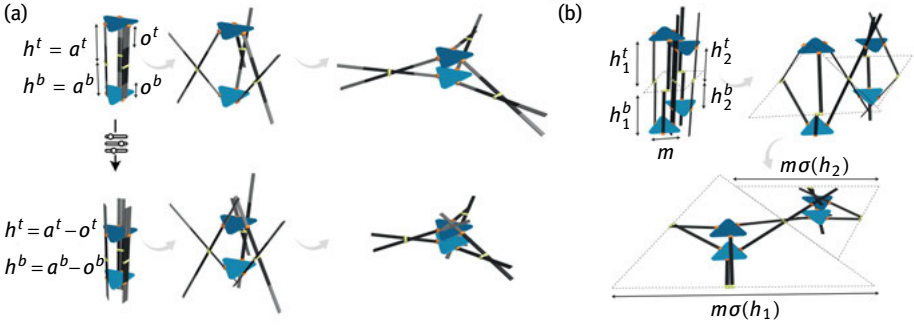
Fig. 1: Reconfigurable Umbrella Cell. Three scissor linkages are connected at the top and bottom triangular plates. Deployment of the cell is achieved by pushing the top and bottom plates towards each other. (a) By sliding these rigid plates downwards in the compact rest state, we can control the length of the scissor arms and correspondingly the lateral expansion during deployment. (b) Two adjacent cells with an identical footprint in the rest state have different expansion (dashed lines) when configured at different cell heights.

value. This reconfiguration becomes possible because the compact state stores no elastic energy, i. e., the scissor arms can easily slide through the T-joints in their unbent rest state. We can thus adjust the corresponding lengths of the active scissor segments h^t and h^b. We call these values the *heights* of a cell. If we denote the fixed length of the top and bottom scissor arms as a^t and a^b, respectively, and the sliding range by o^t (and o^b), then the range of values the height h^t of the top arm of the cell can take is given by $[a^t - o^t, a^t]$, and analogously $[a^b - o^b, a^b]$ for the bottom arm.

Let m be the edge length of the triangular top and bottom plates and $H = h^t + h^b$ be the total height of the cell. H gives the separation of the plates when the cell is in its compact, undeployed state. When deployed, the edge length of the triangular cell footprint increases by $2\sqrt{3}h$, where $h = \min(h^t, h^b)$ as shown in Fig. 1. When the cell is deployed towards a separation s, the length of the edge of the footprint cell increases from m to $m + \sqrt{12h^2 - 3s^2}$. Since we use a regular tessellation of cells with constant triangle size m, the length expansion factor is a function of h and s only:

$$\sigma(h) = \frac{\sqrt{12h^2 - 3s^2}}{m} \tag{1}$$

Figure 1a illustrates the deployment of a cell in two different configurations. Note that the configuration with longer heights expands to a larger footprint as expected. We show one possible physical realization of a reconfigurable umbrella cell in our fabricated prototype in Sec. 7.

A key principle of RUMs is to use identical reconfigurable cells across the entire umbrella mesh, which facilitates mass fabrication and re-use. Figure 1b shows how two identical cells configured to different height values deploy towards planar footprints of different lateral expansion. Identical reconfigurable cells can be assembled to form a RUM which can be configured to different height distributions.

4 Design Exploration

To explore design alternatives, we provide two complementary tools built upon the computational approach presented in Ren et al. (2022): (i) A forward exploration where the designer can directly control cell heights and receives interactive visual feedback on the deployed shapes. This allows building intuition on the range of possible spatial configurations of a given RUM. (ii) An inverse method that computes precise cell heights for several given target shapes in an offline optimization.
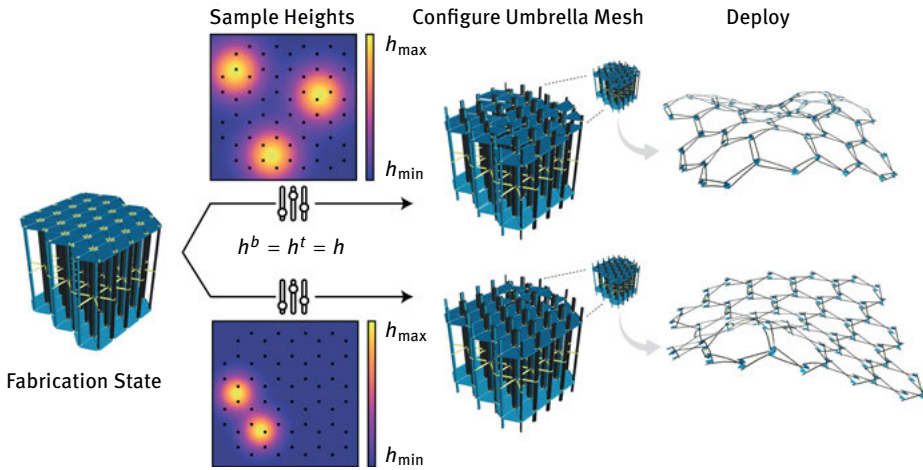

Fig. 2: Direct control of the cell heights allows exploring design variations. A painted density map (center left) is converted to umbrella cell heights (center right). Our algorithm then provides visual feedback of the corresponding deployed shapes (right).

Figure 2 illustrates the forward exploration framework. A user can sample the umbrella cell heights within the reconfigurable range of a pre-determined cell. Here we use a mixture of radial basis functions to generate a continuous height distribution from user input. We overlay the umbrella cell topology to sample the discrete umbrella heights. Alternatively, the user can directly manipulate individual cell heights to generate new configurations of the umbrella mesh. With the chosen height distribution, we use our simulation framework to compute equilibrium state of the deployed structure, actuated by pushing the top and bottom plates of each cell towards each other.

Figure 3 illustrates how inverse design can optimize the cell heights for specific given target shapes. Here we solve for the reconfiguration parameters to achieve a spherical cap (positive Gauss curvature), a saddle (negative Gauss curvature), and a cylinder (zero Gauss curvature). All three of these states can be achieved with the same RUM model. During the optimization we enforce bounds on the design parameters to

ensure that the umbrella heights h^t, h^b are within the permissible range ($[a^t - o^t, a^t]$, $[a^b - o^b, a^b]$) defined by the reconfigurable cell. More details on the optimization algorithm can be found in Ren et al. (2022).
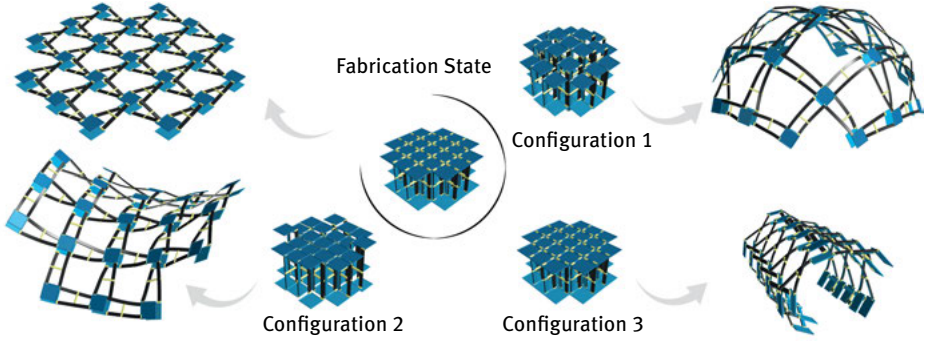

Fig. 3: A single RUM composed of identical cells can be reconfigured to deploy to a variety of target shapes.

5 Shape Space

A critical factor when designing a RUM cell is to select an appropriate range of reconfigurability. For a constituent umbrella cell with reconfigurable ranges $[a^t - o^t, a^t]$, $[a^b - o^b, a^b]$ of the top and bottom heights respectively, we define $h_{min} = \min(a^t - o^t, a^b - o^b)$ and $h_{max} = \min(a^t, a^b)$. These values represent the minimum and maximum possible heights achievable by the chosen reconfigurable cell. Subsequently, we can compute the minimum and maximum expansion factors σ_{min}, σ_{max} from Eq. (1). A larger range requires longer arms and thus more material. If the range is more limited, the space of achievable shapes is reduced. The optimization will then solve for the closest possible surface deployable from a height distribution within the allowed range.

Following Konaković et al. (2016), we can understand the space of all possible deployed shapes of a given RUM using conformal geometry. A conformal map from a design surface to the plane preserves angles and only allows locally isotropic scaling and rotation. This behavior is reflected in the design of our umbrella cells that expand isotropically when deployed, as shown in Fig. 1. Since a conformal map of the surface is invariant under global uniform scaling, the ratio $\frac{\sigma_{max}}{\sigma_{min}}$ defines the maximum conformal scale factor that can be expressed by a specific cell.

The maximum conformal scale factor of an arbitrary surface helps characterize the shape space. Specifically, the Gauss curvature K at a point on a surface is directly related to the local conformal scale factor through the Yamabe equation, $\Delta \log \sigma = -K$, where Δ denotes the Laplace operator. This means that a high *variation* of scale factors,

and consequently a high local incompatibility among neighboring cells, leads to high Gauss curvature. Bounds on the maximal scale factor limit the range of obtainable shapes.

For example, the auxetic structures studied in Konaković et al. (2016) have a maximal conformal scale factor of two. The authors show that within this range, one cannot obtain any spherical patch larger than a hemisphere. In contrast, umbrella meshes can in principle have arbitrary maximal conformal scale factor (Eq. (1) shows $\sigma \propto h$) and can thus approximate any surface. Reconfigurable umbrella meshes on the other hand have limits on their maximal conformal scale factor as a function of their reconfigurable range (fixed once the cell is fabricated), consequently affecting the range of shapes they can accurately attain (see Fig. 4). In Fig. 3 we show that a relatively small range of $\frac{h_{max}}{h_{min}} = 1.5$ allows approximating shapes with significantly different curvature profiles.
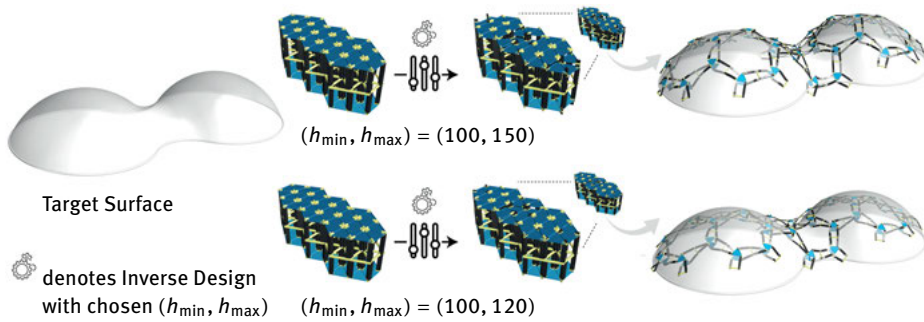

Target Surface

denotes Inverse Design
with chosen (h_{min}, h_{max}) $(h_{min}, h_{max}) = (100, 150)$

$(h_{min}, h_{max}) = (100, 120)$

Fig. 4: Influence of reconfiguration range on the approximation of a target surface during inverse design. A reconfigurable cell with a larger height range (top) approximates the surface better than the cells with reduced range (bottom).

6 Sparse Reconfigurability

Reconfigurable cells are more complex than fixed cells due to the required sliding mechanism that allows adjusting the cell heights. Our computational framework also allows building hybrid assemblies of reconfigurable cells and fixed cells of constant height. Having fewer reconfigurable cells constrains the shape space that the structure can approximate, but reduces fabrication cost and the complexity of reconfiguration. This can be particularly useful when the resolution of the umbrella mesh is high, which leads to a large number of reconfigurable parameters, or when the reconfiguration is automated through motorized actuators.
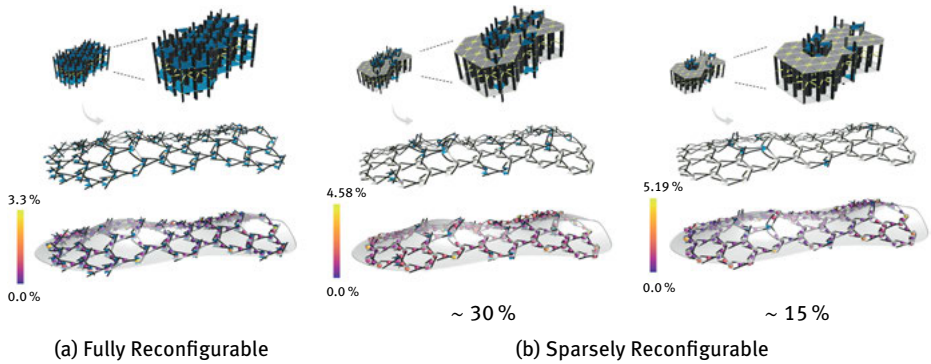
3.3 %

0.0 %

(a) Fully Reconfigurable

4.58 %

0.0 %

~ 30 %

5.19 %

0.0 %

~ 15 %

(b) Sparsely Reconfigurable

Fig. 5: Sparsifying reconfigurability: (Top) demonstrates the optimized deployed structure with the non-reconfigurable cells shown in grey. (Bottom) displays the distance from the target surface relative to the magnitude of the bounding box diagonal. The maximum deviation only increases from 3.3 % to 5.19 % after replacing close to 85 % of the reconfigurable cells with fixed ones.

Figure 5 illustrates reconfigurable umbrella meshes of different extents of sparse reconfigurability approximating a free form surface. Currently, the distribution of reconfigurable and fixed cells is designed manually. An interesting and challenging problem for future work is to automatically find an optimal distribution for a given space of deployable target shapes.

7 Physical Prototype

To observe the physical behavior of RUMs, we designed a prototype cell with sliding joints to allow adjusting the scissor linkage arms. Figure 6 shows the design of the constituent components, reconfiguration by sliding, and deployment of our fabricated cells. The arms are locked in place once the cell's height values are programmed. We discretize the possible values of h for easy control. This simplification did not affect the achievable shape space noticeably. The desired height level is locked using indents on the sides of the arms. Since each cell's h^t and h^b are equal across the three arms, the discrete levels make it easier to ensure this constraint too. Continuous sliding of h values requires a more sophisticated control and locking system but offers higher shape fidelity. The top and bottom plates feature a compliant T-joint that is CNC milled from a 6 mm thick polypropylene sheet. The arms are CNC milled from a 4 mm thick POM-C sheet. The deployment spacing is enforced through 3D printed spacers that are positioned between the plates.

We assembled a RUM consisting of 37 identical cells to demonstrate reconfiguration in practice. In the initial assembly state, all heights are equal and the structure deploys
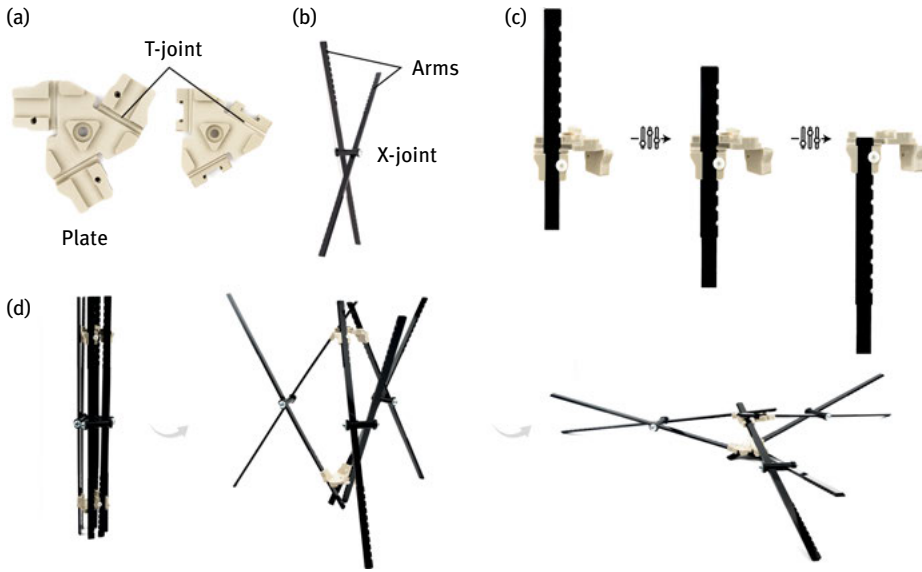
Fig. 6: Components of our Reconfigurable Umbrella Cell: (a) (Left) The top plate in the fabrication state with the T-joints milled and the sleeve for the sliding arm, (Right) The compliant T-joints rotate to form the compact assembly state. (b) The arms assembled with the X-joint (c) The sliding mechanism at the T-joint blocking the heights at three different levels/values. (d) Deployment of a single cell.

into the flat state. Figure 7 illustrates that even such a simple model can be reconfigured to deploy into significantly different shapes of positive and negative Gauss curvature.

8 Applications

Our work focuses on the underlying principle of RUMs and provides an interactive simulation-based approach for designing the deployed structure and exploring potential transformations. RUMs exemplify transformable structures with a high degree of control. We believe the theoretical implications of such transformable structures permeate through several domains and scales. Here we discuss some applications of RUMs in architectural scenarios.

One of our preliminary postulations suggests that RUMs permit adaptability and reuse of architectural structures. Individual cells can be pre-fabricated and assembled off-site to simplify and reduce on-site construction processes. It is also assumed that the compact state of the structure facilitates construction logistics, including transport and storage. The structure is assembled in its compact state and realized through deployment. More fundamental is the capability of RUMs to be taken apart and reconfigured
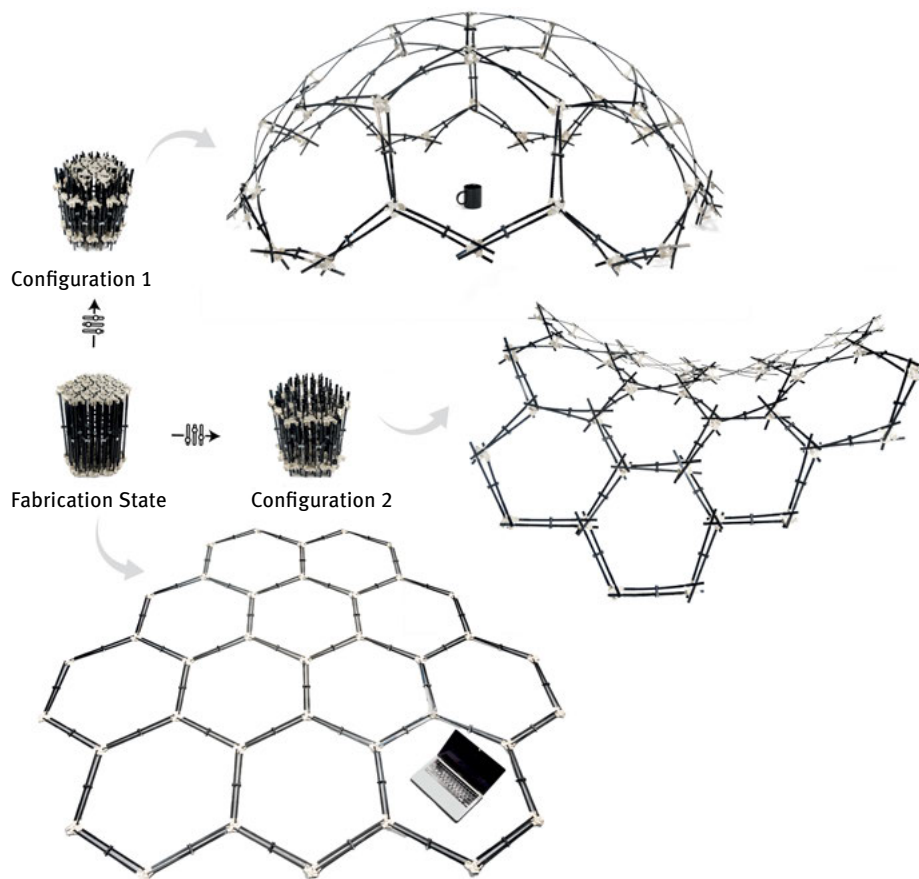
Fig. 7: A fully reconfigurable umbrella mesh composed of 37 identical cells. Sliding the top and bottom plates towards the X-joints effectively shortens the active scissor linkage arms, leading to reduced lateral expansion. In this way, significantly different deployed target shapes can be achieved in a single model. Note how the parts of the arms that are not active, i. e. above the top and below the bottom plates, avoid collisions and form a reciprocal arrangement.

to deploy into a different geometrical shape. This implies that a variety of architectural solutions can be created using the same kit of parts. Between deployments, the structure is returned to its compact state to incorporate a new set of cell heights, or to be disassembled and re-assembled in a new configuration. The inherent modularity of RUMs also allows for topological changes to be easily introduced by altering the number of cells during re-assembly. This type of flexibility permits to introduce major changes in the shape and span of the structure. Under these conditions, it is possible to relocate the structure to a different site and reconfigure its cells to adapt to the needs of that specific location. Eventually, functional, spatial and environmental demands can then drive the transformation of the structure. Inspired by the Great Court at the British
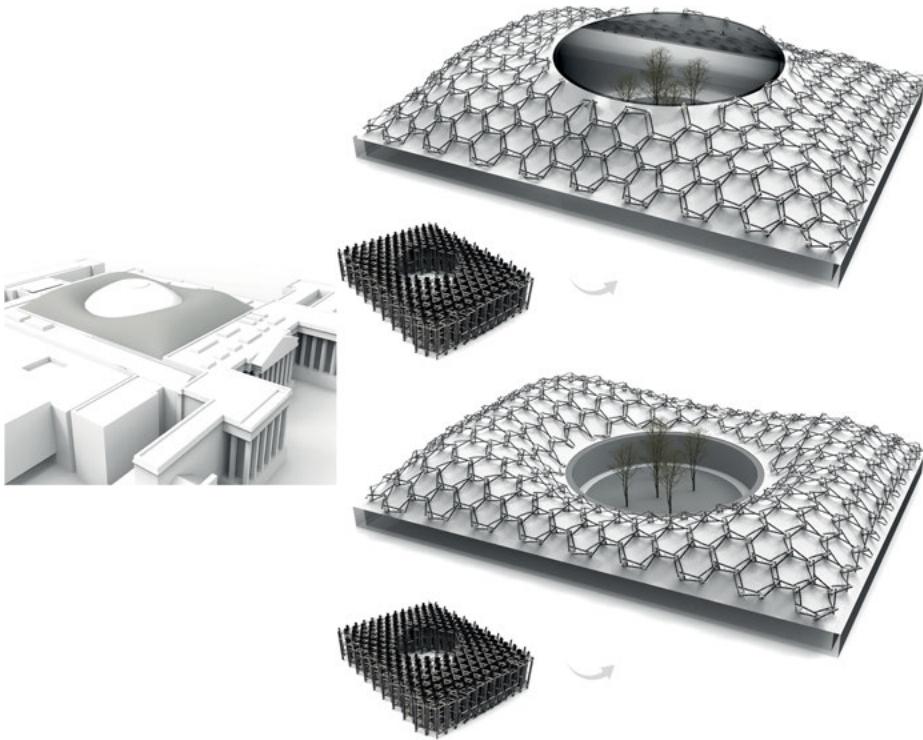
Fig. 8: Concept employing a RUM with two configurations inspired from the geometry of the Great Court at the British Museum.

Museum (Foster and Sudjic 2011), Fig. 8 envisions two possible configurations of a RUM structure. The two configurations provide different spatial layouts for different use cases.

With rapid advances in robotic actuation control, it becomes possible to envision RUMs as a potential type of adaptive structure enabling dynamic transformations. This implies changing the height of the cells without returning to the compact state of the structure. On this subject, we simulated changing the height distribution of the umbrella cells when already deployed to a particular shape. Figure 9 demonstrates a few stages of a transformation of the structure as the initially configured heights corresponding to a saddle shape change to the height configuration for a cylindrical shape. This enables to visualize the concept of adaptive structures controlled by actuators manipulating the cell heights directly in the deployed structure.

In general, efforts are required to explore scalability problems of these types of structures. The materialization of T-joints presents special challenges for the development of cells since there is a need to perform numerous reconfiguration cycles without damage.
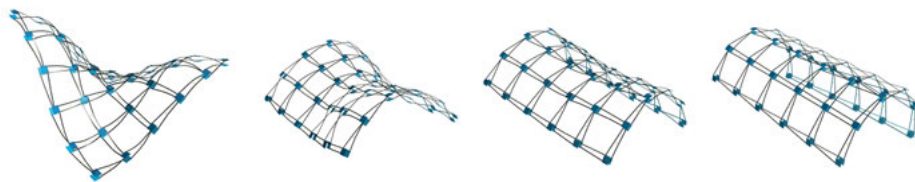
Fig. 9: Active shape morphing: We show a concept where a deployed structure can actively change its shape by reconfiguring the heights when deployed. Here we use a linear interpolation of the height distributions at the ends to generate the intermediate states taking a saddle shape to a cylindrical one.

9 Conclusion

Reconfigurable Umbrella Mesh (RUM) is a new class of deployable structures that features highly adaptable shape configurations, mass-produced, identical, and re-usable constituent cells, and a compact-to-surface deployment that supports efficient transport and storage. Future research will be required to define appropriate material systems when scaling the system up/down and analyze the structural behavior of RUMs. Combinations with other transformable materials, such as fabrics, also provide interesting avenues for future research.

Acknowledgements

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