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Transformable Surface Mechanisms by Assembly of Geodesic Grid Mechanisms

Abstract: The scissors-like grid shells composed of members bent in the out-of-plane direction have structural advantages: the ease of deployment as a one-degree-of-freedom system and the stability as it is modeled as a shell once the scissors' transformation is fixed. However, it is impossible to generate a freeform doubly curved surface from a single grid while maintaining the geodesics. In this paper, we propose an approach based on assembling multiple units of grid systems to obtain a variety of transformable surfaces. We first provide a geometric model of bending active scissors system along the geodesics by interpreting the structures as surfaces that change the intrinsic metric according to the scissors' transformation. Then, we clarify the compatibility conditions between multiple patches and show approaches for designing patterns that satisfy compatibility. Based on the model, we show a simulation of surface transformation using a bar-and-hinge model. In particular, we propose a method for inversely generating a mechanism that deploys to a given target surface.

1 Introduction

Mechanisms that combine scissors' transformation of the grid with the elastic bending of material are used in various fields. Bending active grid shells are an example of such systems and has been used in the architecture as the efficient construction and deployment method [5]. However, the application of grid shells is not restricted to static structures. For example, the tubular braided structures used in McKibben-type artificial muscles, stents for expanding blood vessels, finger traps, and grips for pulling cables can be regarded as a similar grid system on a cylindrical surface and serve dynamic functions. Inspired by these transformable grid shell structures, we are interested in realizing an architectural scale mechanism that exploits the dynamic behavior of transforming surfaces. In this research, we aim to model and simulate the transformation of such bending-active grid shells and freely design the transformation of such mechanisms.

Transformable bending active grids are classified according to the bending directions of the members (Fig. 2b). (1) The most general case is when we allow the member to bend in two directions. In this case, the grid can reproduce part of the given free surface by parameterizing it with an isometric mesh called a Chebyshev net [3,1]. However, the obtained structure is not kinematically determinate, and it is difficult to control the motion of the deployment. Also, they require additional support to make the structure stable. (2) Asymptotic grid shells [10] use members that cannot bend in the normal di-

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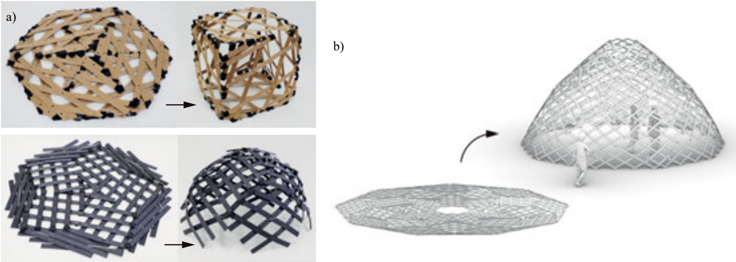

Fig. 1: Mechanisms using transformation of geodesic grids. a) Physical model. b) Architectural scale mechanism.

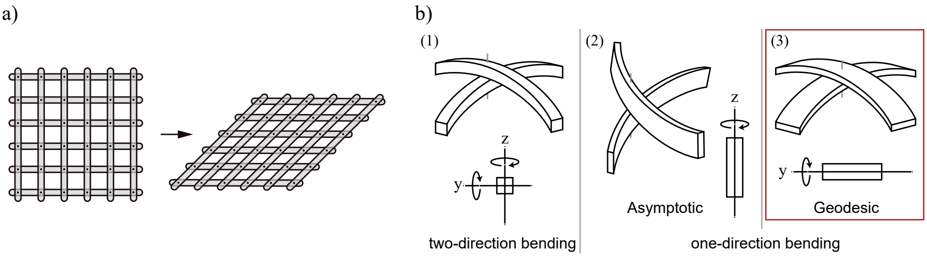

Fig. 2: a) Scissors' transformation of a grid. b) Classification of transformable bending active grids.

rection of the curved surface. Asymptotic grid shells still have a high degree of freedom in transformation and require the control of diagonal braces to make the form into the target shape. Also, the shape is limited to surfaces with negative Gaussian curvature. (3) Geodesic grid shells [7] use members that can only bend in the normal direction of the surface. They have one degree of freedom (DOF) in-plane transformation, allowing for easy deployment control. Therefore, the in-plane shape is expected to be rigid by bracing only a part of the scissors, and the whole mechanism behaves like a shell structure. In this research, we focus on such a grid shell based on geodesic lines.

However, grid shells with geodesic lines cannot generate arbitrarily curved surfaces from a planar state because the degree of freedom of transformation is constrained. Pillwein et al. [7] proposed methods for designing surfaces by introducing the play at the pivots of the scissors by converting holes into slits. This approach would again introduce excessive degrees of freedom due to the sliding that must be fixed after the deployment. Ono and Tachi [2] used an approach by segmenting the curved surface into multiple grids that are kinematically connected without play, so they produce one-DOF in-plane transformation. They generated surfaces with constant negative curvature, but the method was limited to rotationally symmetric shapes. The limitation of the shapes comes from the fact that the transformation is not compatible when multiple grids are combined in general. The conditions of compatibility between grids have not been explored.

This research aims to create a freeform surface as a geodesic grid shell by combining multiple grids so the entire mechanism is one-DOF. We first provide a geometric model of the mechanism based on macroscopic in-plane transformation and clarify the compatibility conditions of the mechanism (Sec. 2). Then, we propose a mechanism deployment simulation method using this geometric model (Sec. 3). We present a mechanism design approach that satisfies the compatibility conditions and applies them to more general curved surfaces. In particular, we propose a method for designing a mechanism that deploys to a given target surface (Sec. 4).

This paper focuses mainly on the system's geometric principles and programmability. However, we intend to extend our research to architectural applications and provide structural verification in the future. Potential applications of this research include building structures that can be constructed by assembling a grid-like shape in the plane and changing only the boundary shape, as well as the development of environmentally controllable adaptive facades, curved formwork, or interior equipment (Fig. 1).

2 Geometry of the Scissors' Transformation

2.1 Grid Transformation of Single Unit

Vector Notation and Scale Factor. When a square grid is transformed in-plane, the direction along the grid does not expand or contract, while the diagonal directions of the grid expand and contract at the maximum and minimum scale factors, respectively. Since the diagonal direction remains orthogonal before and after transformation, this transformation is expressed as a linear transformation with the two orthogonal diagonals as the principal directions. Therefore, the scale factor in any direction can be computed from these two scale factors. Here, we use these diagonal directions as the coordinate system.

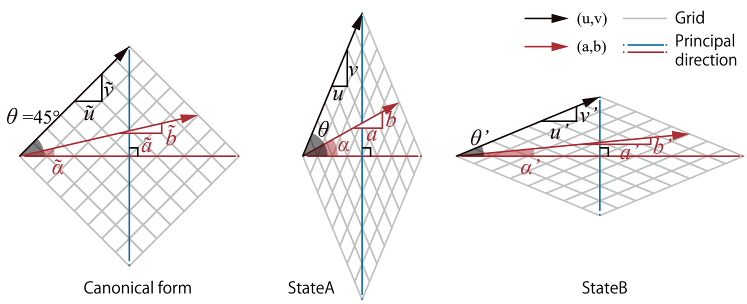

Fig. 3: Transformation of a grid using the principal directions as a coordinate system.

Let σ_1 and σ_2 be the scale factors in the principal directions from the initial state A to a transformed state B. A vector (u, v) along the grid in state A is transformed to (u', v') in state B as

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}. \tag{1}$$

Since the direction along the grid does not expand or contract,

$$\sigma_1^2 u^2 + \sigma_2^2 v^2 = u^2 + v^2. \tag{2}$$

Next, let (a, b) be a vector in an arbitrary direction in state A, and the *direction angle* $\alpha = \operatorname{atan2}(b, a)$ be the angle between the principal direction and (a, b). Then, the transformed vector (a', b') can be described as

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \tag{3}$$

Then, the scale factor S in (a, b) direction before and after transformation is given by

$$S^2 = \frac{\sigma_1^2 a^2 + \sigma_2^2 b^2}{a^2 + b^2}. \tag{4}$$

Notation Using Canonical Form. Because (u, v) is a length-preserving vector, we express it as $(u, v) = (\cos\theta, \sin\theta)$ using the *grid angle* θ. To handle a direction (a, b) independent of the grid angle, we define the *canonical form* of the direction angle $\tilde{\alpha}$ and the direction $(\tilde{a}, \tilde{b}) = (\cos\tilde{\alpha}, \sin\tilde{\alpha})$ to be the direction angle and the direction when $\theta = 45°$.

When the grid angle changes from 45° to θ, the principal scale factor is $(\sigma_1, \sigma_2) = (\sqrt{2}\cos\theta, \sqrt{2}\sin\theta)$. From (3), the direction (a, b) can be represented using the canonical direction $(\tilde{a}, \tilde{b})$ and the grid angle $\tilde{\alpha}$ as

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sqrt{2}\cos\theta & 0 \\ 0 & \sqrt{2}\cos\theta \end{pmatrix} \begin{pmatrix} \cos\tilde{\alpha} \\ \sin\tilde{\alpha} \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\theta\cos\tilde{\alpha} \\ \sin\theta\sin\tilde{\alpha} \end{pmatrix}. \tag{5}$$

Therefore, the direction angle α is

$$\tan\alpha = \frac{\sin\theta\sin\tilde{\alpha}}{\cos\theta\cos\tilde{\alpha}} = \tan\theta\tan\tilde{\alpha}. \tag{6}$$

Now we consider the transformation from state A with grid angle θ to B with grid angle θ'. Then, the direction angle at state B is

$$\tan\alpha' = \tan\theta'\tan\tilde{\alpha}. \tag{7}$$

The scale factor S in the direction angle $\tilde{\alpha}$ when transforming from state A to state B can be expressed as follows:

$$S^2 = \frac{1 + \cos 2\tilde{\alpha}\cos 2\theta'}{1 + \cos 2\tilde{\alpha}\cos 2\theta}. \tag{8}$$

2.2 Compatibility Between Multiple Modules

We call a polygon obtained by cutting a single grid into polygons a *unit*. The edges of a unit expand or contract as the grid angle changes. When two or more units are serially connected by shared edges, they form a one-DOF mechanism because the scale factor of the shared edge need to match, except for the case when the scale factor of one of the edges is 1. If a state for one unit is determined, all states for the units in the series are determined.

However, if we connect the units with interior vertices, the overall structure becomes over-constrained because it forms a closed loop in this chain. Therefore, the motion is generically incompatible, while the whole mechanism is compatible and transformable under special conditions. In the following, we discuss the compatibility condition around such a loop for making the mechanisms work together to form a one-DOF system. When they work together, the sum of α increases or decreases around the vertex; accordingly, a flat pattern transforms into a three-dimensional surface.

Consider the case where n units are connected around a vertex (Fig.4), and let α_i, β_i be the angles between the edge and the principal direction in the i-th unit ($i = 0, 1, 2, \ldots, n-1$, modulo n).

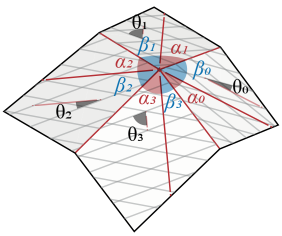

Fig. 4: Unit connection around an interior vertex.

Connection of Two Units. If the parameters of the two units (θ_i, θ_{i+1} and $\tilde{\alpha}_{i+1}, \tilde{\beta}_i$) are given, the i-th transformed grid angle θ'_i determines the $i+1$-th grid angle θ'_{i+1} as

$$x_{i+1} = r_i x_i + s_i, \tag{9}$$

where

$$x_i = \cos 2\theta'_i, \ r_i = \frac{\cos 2\tilde{\beta}_i \left(1 + \cos 2\tilde{\alpha}_{i+1} \cos 2\theta_{i+1}\right)}{\cos 2\tilde{\alpha}_{i+1}\left(1 + \cos 2\tilde{\beta}_i \cos 2\theta_i\right)}, \ s_i = \frac{\cos 2\tilde{\alpha}_{i+1} \cos 2\theta_{i+1} - \cos 2\tilde{\beta}_i \cos 2\theta_i}{\cos 2\tilde{\alpha}_{i+1}\left(1 + \cos 2\tilde{\beta}_i \cos 2\theta_i\right)}. \tag{10}$$

Compatibility Around Vertex. When further connecting units, the i-th grid angle θ'_i is obtained from θ'_0 using (9) as

$$x_i = A_i x_0 + B_i, \quad \text{where} \quad A_i = \prod_{k=0}^{i-1} r_k, \quad B_i = A_i \sum_{k=0}^{i-1} \frac{1}{A_{k+1}} s_k. \tag{11}$$

For n units to be compatible around interior vertices, x_0 and x_n must be identical. Therefore, the compatibility condition around vertices is represented using grid angles at the initial state θ_i, and the canonical direction angles $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ as

$$A_n = 1, \quad B_n = 0. \tag{12}$$

2.3 Special Mechanisms

It is not straightforward to find and design a solution that satisfies the identity equation (12) at each internal vertex. For multiple units to produce a synchronized motion, the scale factor function of the shared edges must be identified during the motion. Therefore, as a special case that satisfies the condition, we introduce mechanisms with *rhombic units* and *single units*.

Mechanism Using Rhombic Unit. A *rhombic unit* is a unit with a rhombic shape such that the principal direction of the grid and the diagonal of the unit coincide (Fig. 5). In this unit, the scale factors of the four edges are always equal and thus (11) holds around every vertex. The mechanism connecting rhombic units form a one-DOF mechanism (in its in-plane motion) which can be controlled by the common edge scale factor S. The exception is when the unit edge and the grid direction coincide (when $\theta = \alpha$) when the scale factor of the edge is 1, so the adjacent units cannot synchronize.

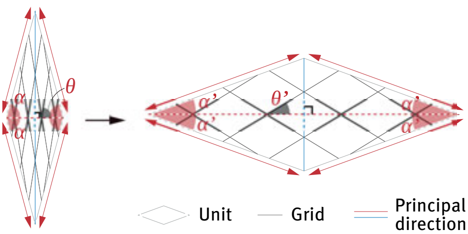

Fig. 5: Transformation of a rhombic unit. All four edges of the unit have an equal scale factor.

Mechanism with Single Unit. An alternative method is to construct a mechanism by repeating the same unit, so adjacent units share the edge of the same scale factor function. Since all units share the same direction angle of the edges during the motion, the grid angle is also the same. Therefore, it forms a one-DOF mechanism that can be controlled by the shared grid angle θ as a parameter.

Two universal ways to connect two units sharing the same edge are (1) connecting with 180° rotation around the midpoint of the edge and (2) mirror reflecting at the edge (Fig. 6). Since the grid has 180° rotational symmetry, units connected by 180° rotation can be regarded as a merged unit in a macroscopic viewpoint. This does not produce

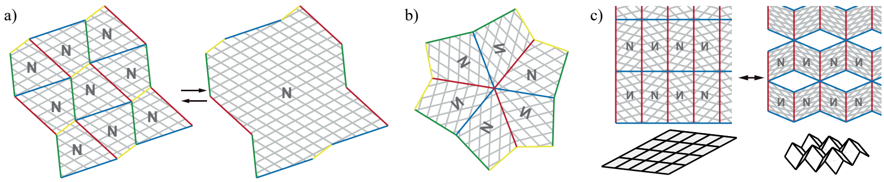

Fig. 6: Tiling of a single unit. Edges of the same color have the same scale factor throughout the transformation. a) Tiling by rotating 180° at the edge midpoint. b) Tiling by mirror reflection. c) A design example that can transform from a sheet to an eggbox surface.

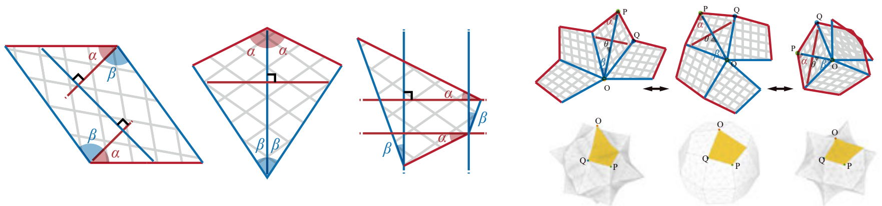

Fig. 7: Examples of units with symmetry. Parallel edges or edges with equal angles from the principal direction have the same scale factor.

angle defects or excess around interior vertices (if starting from a flat state, it keeps 360°) even if the unit is deformed (Fig. 6a). On the other hand, connecting by mirror reflection can generate a mechanism in which the angle around the vertex changes according to the scissors' transformation (Fig. 6b). Figure 6c shows a design example that can continuously change from a sheet to an eggbox surface [9].

Although mirror reflections are limited to the tiling of even-valency vertices, we can extend the single-unit tiling by introducing the symmetry of the tile. For example, using the property that parallel edges or edges with equal angles $\tilde{\alpha}$ from the principal direction have the same scale factor, various types of tiling are possible (Fig. 7). Figure 7 right shows an example using a mirror-symmetric kite shape that can produce polyhedral deformation from a deltoidal icositetrahedron (that requires degree-3 vertex). In this case, the mechanism does not have a planar state; it transforms from a polyhedron to another polyhedra.

3 Simulation

We propose a simulation method for each case (rhombic or single unit system) using the macroscopic model proposed in Sec. 2. This allowed us to confirm the transformation behavior of the entire mechanism before deciding on the specific component layout. We used the Grasshopper [8] and Kangaroo2 [6] for the implementation on 3D-CAD software Rhinoceros.

3.1 Simulation Method

For both unit cases, we first triangulate the unit and the total surface to make a triangular mesh. Then, we construct a dynamic relaxation model by defining several springs at the edges of triangles and minimizing the potential energy of the system as follows. Let $S_{\tilde{\alpha}}$ be the scale factor in the $\tilde{\alpha}$ direction. Here [] represents the goals name of Kangaroo2.

Linear Spring Make the length of the edges the original length times $S_{\tilde{\alpha}}$. [Length (line)]
Angle Spring 1 Keep the fold angle in the unit 0 (represents the out-of-plane stiffness). [Hinge]
Angle Spring 2 Keep the fold angle between units 0 (represents the out-of-plane stiffness). [Hinge]

Here, we use Angle Spring 2 only when adjacent units are joined to transmit the bending moment as in Fig. 16 left.

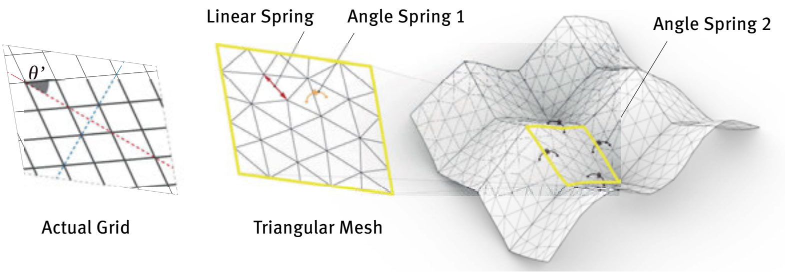

Fig. 8: Simulation model combining linear springs and two types of angular springs.

When simulating mechanisms with rhombic units, we use the scale factor S of all the unit edges as the parameter to control the transformation. When simulating mechanisms with a single unit, we use the common grid angle θ' as the parameter to control the transformation. The scale factor $S_{\tilde{\alpha}}$ for the length constraint of the mesh edges was obtained from θ', θ and $\tilde{\alpha}$ using equation (8) .

3.2 Effect of Triangulation

We observed that the method of dividing the triangular mesh may affect the accuracy of the simulation. Depending on the division, wrinkles may occur at the ridges when the units are deformed out-of-plane. For more accurate reproduction of transformation behavior, it is desirable to use a division pattern that can be easily bent in any direction, i. e., one with high angular resolution. For this reason, pattern E in Fig. 9 was adopted for models with large out-of-plane transformations.

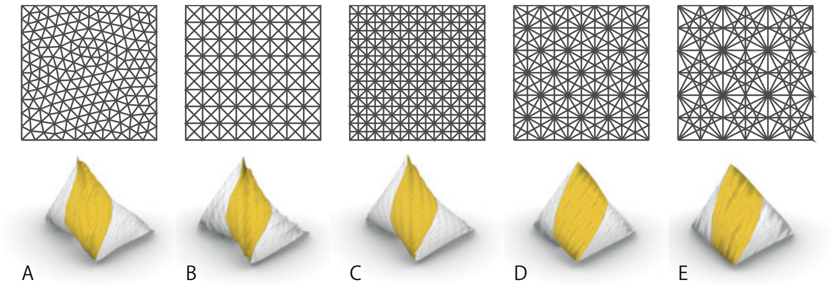

Fig. 9: Comparison of simulation results for different mesh patterns.

3.3 Simulation Examples

Cylinder. Cylindrical mechanisms can be created by connecting the opposite sides of a parallelogram-shaped unit. The transformation is coupled transformation of a twist, squeeze (diameter change), and elongation. If the principal direction and the unit edge are parallel, changing the grid angle θ will not cause twisting because the unit edges remain orthogonal to each other, the behavior observed in braided cords and finger traps. If the grid and unit edge directions are coincident, the diameter of the cylinder is unchanged, and only twisting and elongation occur.

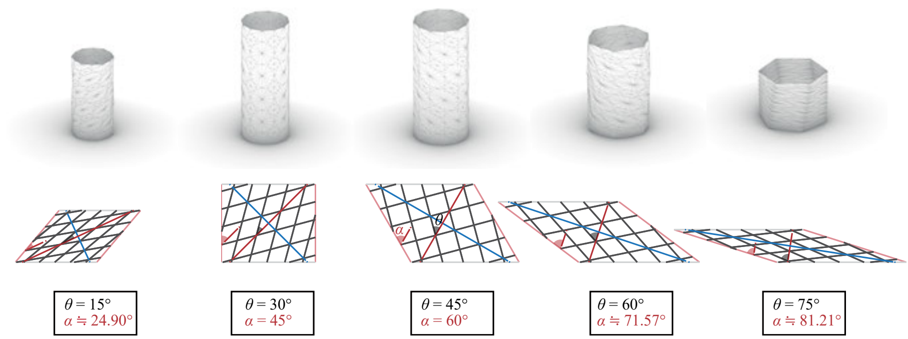

Fig. 10: Simulation result and unfolding of the cylinder model that can twist, squeeze, and elongate.

Tetrahedron. We created a model that transforms from a flat dihedral (double-sided rectangle) to a tetramonohedron (tetrahedron with equal faces). This model was made by connecting the four edges of two units that transform from a square to a parallelogram (Fig. 11). This mechanism has 90° rotational symmetry in the flat state and can be twisted in left and right directions to form tetrahedra in both cases.

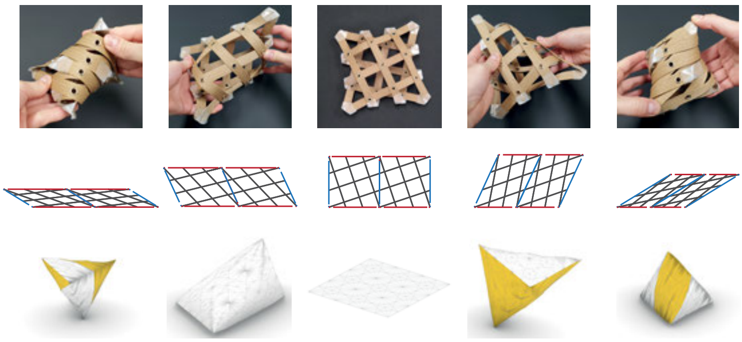

Fig. 11: Tetrahedron design using a single unit. Top: physical model, Middle: grid patterns, and Bottom: simulation results.

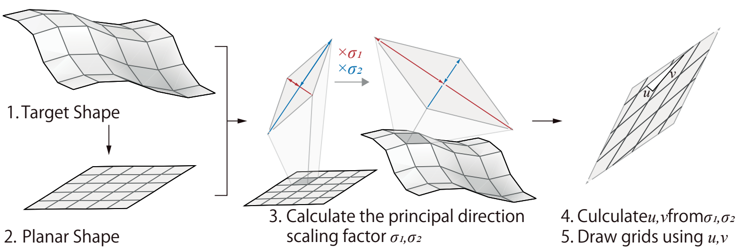

Fig. 12: Design flow of mechanisms that transform to target surfaces using rhombic units.

4 Designing Mechanisms Deploying to a Target Surface

Section 2.3 describes a method to generate compatible mechanisms using the symmetry of units. However, the shape after deployment cannot be predicted and can only be confirmed by simulation. In this section, we propose a method to design a mechanism that can be deployed to a freeform target shape by inversely computing from the target shape. To make interior vertices compatible, the design uses the rhombus unit described in Sec. 2.3. The design flow is as follows (Fig. 12).

1. Define the input target shape consisting of a quadrilateral with equal edge lengths (this may contain non-planar quadrangles).
2. Create a rhombic flat tile with the same connectivity to the target shape.
3. Determine the principal scale factors σ_1, σ_2 from the diagonal lengths of the units of the planar and target shapes.
4. Computing grid direction (u, v) in a planar state from σ_1, σ_2.
5. Draw the actual grid from (u, v) (Sec. 5.1).

4.1 Details of the Steps

Defining Target Shape. First, we define the final target shape. The target shape is given as a surface composed of quadrilaterals of equal edge length. The quadrilateral does not need to be planar, since out-of-plane bending of the final unit is allowed. Each quadrilateral represents a rhombic unit. Such an equal-length quadrangle mesh is characterized by the Chebyshev net on a surface. Existing construction methods of constructing an equal-length grid based on Chebyshev net [3,4] can be used for generating input shapes in our method.

Generating Planar Tile. We generate planar rhombus tiles as unit shapes in a planar state. First, we prepare a planar tile with the same connectivity to the target shape.

When we can find a planar tile with the same connectivity, we can use them as the initial mapping with proper scaling. If tiles with the same connectivity are not trivial, they can be generated by projecting the target shape onto a plane and constraining the length of the edges. After the projection, we need to adjust the edge lengths and proportions of the tiles, so the principal directional scale factor σ_1, σ_2 of each unit satisfies the following equation:

$$\sigma_1 < 1 < \sigma_2 \quad \text{or} \quad \sigma_2 < 1 < \sigma_1. \tag{13}$$

This condition indicates that one of the two principal directions contracts and the other extends. If $S = 1$, the conditional equation (13) is trivially satisfied but the mechanisms do not link together, so this needs to be avoided. Therefore, we need to find tiles where S is reasonably away from 1 but satisfies (13).

Because of this constraint, it is not always possible to construct a mechanism for an arbitrary shape using this method. In a general case, we use constraint-based form finding by solving (1) lengths fixed to the scaled length of the edges by $(1/S)$ and (2) diagonal inequalities through the dynamic relaxation method using springs that represent the constraints.

Computing Grid Direction. The grid inside the unit is calculated from the planar shape and the target shape. First, from the ratio of the diagonal lengths of the units before and after transformation, the principal direction expansion ratios σ_1, σ_2 are obtained. Next, using the property that the vector (u, v) in the grid direction does not expand or contract before and after transformation, (u, v) in the plane can be obtained from sigma by the formula

$$u = \sqrt{\frac{\sigma_2^2 - 1}{\sigma_2^2 - \sigma_1^2}}, \quad v = \sqrt{\frac{1 - \sigma_1^2}{\sigma_2^2 - \sigma_1^2}} \tag{14}$$

The necessary and sufficient condition for this expression is (13).

4.2 Design Example

Examples of mechanisms designed using the above methods are shown in Fig. 13. The target shape of the example *A* (Fig. 13) uses a catenary surface generated by applying an upward load to a three-fold symmetric rhombic mesh and constraining the length of the edges. Planar tiles were generated by scaling the rhombic mesh used as the basis for generating the target shape. The scale factor *S* is $S \simeq 1.21$ in this model. Examples *B* and *C* are a surface near a translational surface and a surface near a polar zonohedron, respectively. Figure 13 shows the grid and the deployment behavior of the mechanism obtained by the above design method. As a result of the deployment simulation, we were able to reproduce a shape close to the given target shape.

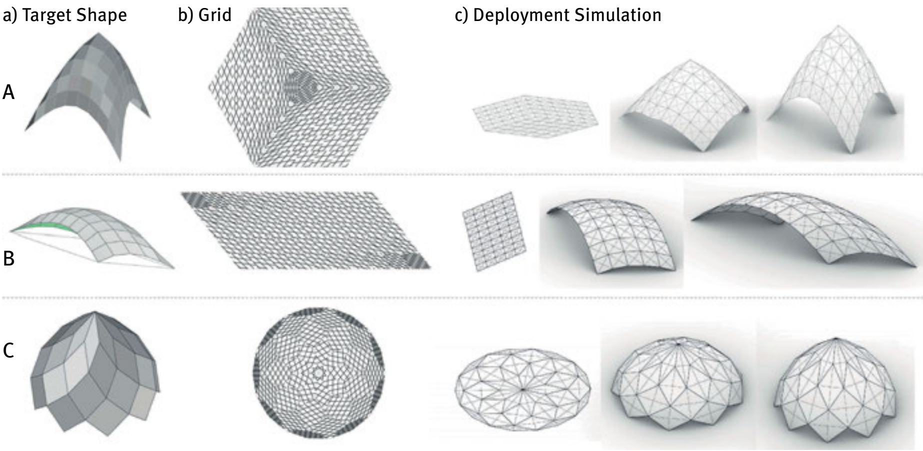

Fig. 13: Design examples using rhombic tiles. A: catenary surface, B: a translational surface, C: polar zonohedron. a) target mesh, b) grid obtained by the proposed method, c) result of the deployment simulation.

5 Joint Design and Fabrication

5.1 Connection of Grid Elements

After the grid angles are determined for units, the actual locations of the discrete grid need to be determined in order to fabricate the mechanisms with actual material. There are two necessary conditions for grid components: a) intersecting within the unit and b) connecting with adjacent units at the ends (Fig. 14).

Intersection within the unit. Depending on the grid layout, the grid elements may not intersect within the unit (Fig. 14a), when the transformation cannot be propa-

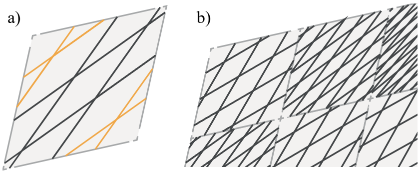

Fig. 14: Problems with grid connections. a) Grid elements not intersecting. b) Endpoints not connected.

gated within the unit. This problem can be solved by increasing the grid density or by changing the unit parameters.

Connection of endpoints. In general, if the grid is defined for each rhombic unit, the members of adjacent units are not connected to each other (Fig. 14b). One solution is to use the common locations of endpoints of the members at all shared edges. Specifically, we can locate the endpoints symmetric around the midpoints of each edge (Fig. 15).

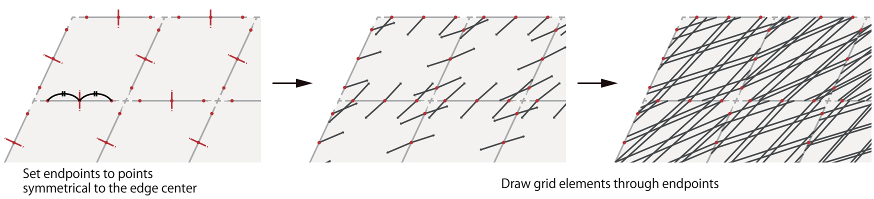

Fig. 15: Adjustment of grid member end point positions.

In the case of a single unit with mirror inversion, the members can be connected regardless of the grid arrangement. However, when the units are connected by parallel translation, the connection of the components must be adjusted.

5.2 Fabrication

The members within a unit are connected by pivots to form a scissors-like mechanism. Between the units, pivot hinges are used when out-of-plane bending was to be transmitted and pin joints are used when the units were to be folded (Fig. 16). We get a bending-active smooth surface when using pivot hinges and creased surfaces when using pin joints. For the model in Fig. 16, the first step is to determine the length, number, and hole locations of the members. Next, the members were cut out, holes were created at the hinge points using a single-hole punch, the members were joined, and finally, the ends of the members were connected. To manufacture the mechanism, 15 mm wide paper bands (craft bands) were used as scissors' members, tag pins or 5 mm diameter eyelets were used as pivot hinges, and the pin joints were realized using tapes.

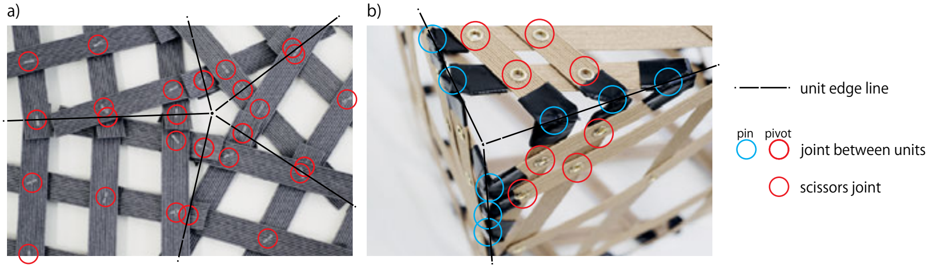

Fig. 16: a) Model with pivots between the units. The joints transfer out-of-plane bending moment, resulting in a smooth curved surface. b) Model with pin joints between units. The joints do not transfer bending moment, resulting in a polyhedral shape with folded hinges between the units.

We showed the conditions for mechanisms to be compatible around a single vertex. However, we have not yet fully solved the problem of how to obtain a general solution with multiple interior vertices, which is left as future work. In addition, the design method shown in Sec. 4 is limited to some families of surfaces, but its characterization is still open. Finally, detailed structural studies under loading conditions are a future requirement for this method to be applied to architecture.

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