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A Design Modeling Framework for Multi-Material Biopolymer 3D Printing

Abstract: This paper presents a novel design-to-fabrication pipeline for multi-material 3D printing. The pipeline is based on a multi-resolution grid-based approach that allows multiple data values to be stored and computed locally. Integrating contextual data, generative algorithms and print path generation, our pipeline incorporates efficient data storage and manipulation, seamless integration with existing digital workflows, advanced volumetric operations, and scalability to accommodate different scales of volumetric data. We demonstrate and assess the application of this workflow in the design and fabrication of Radicant, a biopolymer-composite wall system, exemplifying the possibilities of utilizing diverse materials and efficient modeling and fabrication techniques.

Keywords: volumetric modeling, adaptive design, context-aware modeling, growth algorithms, dynamic density adjustment, multi-material printing

1 Introduction

This paper introduces a digital workflow for multi material 3D printed bio-polymers and describes the application of this workflow to a panel-based interior wall cladding. Biopolymers are polymers that are produced by or derived from living organisms such as plants instead of petroleum. They can be 100 % biodegradable under natural conditions and produced at large scale relatively cheaply. Biopolymer composites are created by reinforcing a biopolymer with bio-fillers and fibers. By varying the choice and ratio of filler, differing mechanical, absorptive, and expressive properties can be achieved (Aaliya et al. 2021). It is the potential of combining this ability to locally differentiate and grade performance with the geometric possibilities of 3D printing that motivates this paper.

Robotic printing of bio-polymer composites offers the opportunity to create multi-material 3D printed bio-polymer elements at architectural scale, in which material properties can be varied in response to different performance criteria related to a design and its environmental and boundary conditions. However, to design and fabricate these elements, material specifications need to be closely coupled to local geometry descriptions as well as robotic control code. This interdependence means that methods typically associated with 3D printing such as slicing, and boundary representation-based modelling cannot be applied. Instead, there is a need for approaches that enable

design, material, and contextual information to be locally and volumetrically stored, queried, and manipulated via generative, parametric and fabrication workflows.

In this paper we present a novel workflow that seamlessly integrates contextual data, generative operations, and print path generation across multiple scales of resolution. We describe the application of this workflow to the case of a customizable 3D-printed biopolymer-composite paneling system, Radicant, and evaluate its performance. By showcasing the capabilities of this workflow in Radicant, we aim to highlight its potential for future improvements and extensions in the realm of modeling for multi-material 3D printing.

2 State of Art

The potential of multi-material 3D printing to produce elements in which material and geometric specifications are tightly coupled has been identified early on (Palz 2009; Oxman 2010; Richards and Amos 2014; Grigoriadis 2015). Multi and graded materials (Bever and Duwez 1972) are studied and applied within fields where it is crucial to provide great material performance, as in aeronautics (Saleh et al. 2020), for producing biological tissues (Watari et al. 2004) or mechanical engineering (Maalawi and Badr 2009), as well for architectural construction (Sinke et al. 2022, Nicholas and Tamke 2012, Pajonk et al. 2022). Attempts for software capable to describe complex variable material distributions have been undertaken within the traditional CAD framework of boundary representations (Luu et al 2022) or more radically, outside of it, such as with voxel-based representation (Michalatos and Payne 2016).

The generation of fabrication code for 3D printing was originally developed on the assumption of 3D boundary models and mono-material prints, with code generated from the slicing of a geometric model. Most current printing applications retain this approach or its extension into multi-filament printing, in which a geometric model is subdivided into parts that are each assigned to specific materials. With more advanced printing technologies such as the Stratasys Polyjet and its predecessors, which can selectively deposit layers of multiple acrylic-based photopolymers to predefined geometries via inkjet printing, it becomes possible to achieve more complex material descriptions that can be varied within a geometric part description. This approach assumes a linear process that applies material assignments onto a pre-existing geometric definition.

The work within the field of soft robotics and especially that of Hod Lipson (Cheney et al. 2014) on soft robots with multiple materials and a powerful generative encoding, demonstrate a yet unachieved potential of designing with multi material composites, and raises the question of how to link geometry design to material assignment to the fabrication process in a more integrated and less linear way. The locomotion that Lipson's multi-material robots achieve (in simulation) is accomplished by connecting

design to materialization. A compositional pattern-producing network (CPPN) generates geometries, these are tested for their performance and the results are fed back to a new iteration of the design process. These interdependencies between performance, overall geometry and varying local material properties have been subject to research projects, such as Complex Modelling (Thomsen et al. 2017) and can be tackled with multi-scalar modelling strategies (Weinan et al. 2011, Nicholas et al. 2015, Faircloth et al. 2018). At the core of these approaches is a digital design pipeline which connects an ecology of models for different scales and functions, such as analysis and generation of geometries. The geometry of the design is herein a variable and not static as in the current modelling approaches for multi material 3D Print. The question is therefore whether a design to fabrication workflow can be established, which spans very different modelling paradigms and simultaneously combines the design-level information from site and architectural concept up to the generation of highly specific fabrication code for multi material robotic 3D printing?

3 Project Context

The workflow we describe in this paper is developed at the intersection of three related research projects which provide technologies as well as research directions:

- Predicting Response: development of the base material recipes,
- EcoMetabolistic Architecture: development of the custom robotic print heads, and
- Living Prototypes: provision of the case to develop a novel type of interior wall cladding.

We demonstrate the developed workflow in a specific design case: Radicant – a built demonstrator of 7×5 m, which was exhibited in the Aedes Gallery in Berlin in 2022/23. The Radicant collagen glue wall is a bespoke wall-paneling system created from a biopolymer composite reinforced with various types of cellulose from waste products. In the exhibition context it demonstrates the design, fabrication, and creation of a bio-based architecture. Each tile was 3D-printed using a different composition of biopolymer composites (Fig. 1), with the resulting variations arranged to create a branched interwoven form extending over six meters that is materially graded from bottom to top. With a design inspired by the silk tree, the biomaterial tiles are printed more densely where they are fixed at their trunks and become more open towards their leafy edges.

To establish a design-to-fabrication framework capable of spanning various modelling paradigms and incorporating high-level information on site and architectural concepts, a comprehensive data pipeline is necessary. This pipeline consists of several components, integrating both frontend and backend tools to achieve the desired multi-material 3D printing capabilities. The frontend was developed using the popular



Fig. 1: Use case: Bespoke wall-paneling system “Radicant”, Aedes Gallery Berlin 2022.

modeling software Rhinoceros 3D and Grasshopper. The backend is built on the top of OpenVDB (Museth, 2013), which is an open-source volumetric data structure and toolkit designed for efficient storage and manipulation of sparse volumetric data. It is particularly useful for representing complex, graded material compositions in a voxel-based format. It enables the backend component of the pipeline to process and handle the complex geometric processes. Svilans et al has demonstrated the application of OpenVDB for the analysis and specification of biomaterials (Svilans et al. 2022). In this research we aim to extend this trajectory to explore how volumetric data can play a significant role in integrating design, material, and fabrication data within digitally driven design workflows. In this context, our framework possesses four main advantages:

- **Efficient data storage and manipulation:** The data structure enables the efficient storage of large volumes of heterogeneous material data, such as density, porosity, or mechanical properties.
- **Integration with digital design workflows:** It can be integrated into existing digital design environments and can be used as a base for further geometric algorithms.
- **Advanced volumetric operations:** The framework provides a range of operations for processing volumetric data, such as filtering, smoothing, and resampling.

- Scalability: The flexible data structure can accommodate different scales of volumetric data, from micro-scale material properties to macro-scale architectural elements, and the transfer of information between these scales.

4 Design Pipeline

The design pipeline for Radicant (Fig. 2) begins with a 3D scan to create an accurate digital model of the wall to which the paneling system will be attached. This model is then used to create a volumetric representation using a high-resolution voxel grid, which serves as the foundation for the creation of information layers, including local height, structural requirements, and pre-existing wall features. After the voxel-based model is populated with this underlying contextual information, it is then utilized as the basis for geometry and property generation that is informed by the material and fabrication properties, geometrical constraints, and design considerations. Instead of relying on an intermediary model to derive the print path, we directly create the print path from the volumetric model and associated information layers. This streamlined process eliminates the need for additional conversions or translations between models.

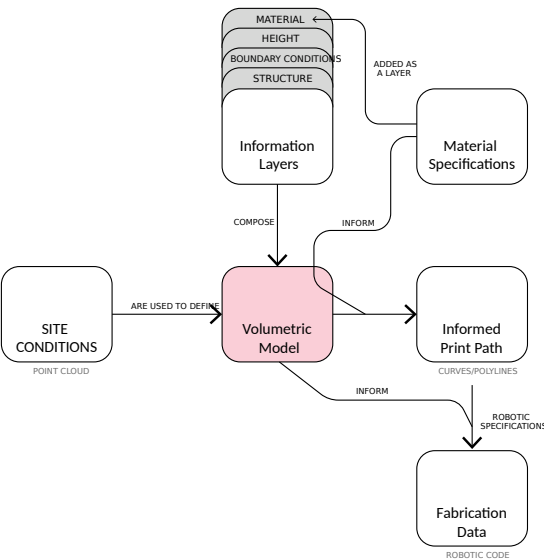


Fig. 2: Overview of the design pipeline.

The computational model follows a non-linear approach, allowing for dynamic updates and adjustments throughout the entire process. This adaptive framework enables a continuous refinement and modification of the design in response to changes in

contextual data, material properties, or design goals. If new information becomes available or project requirements evolve, the computational model is able to seamlessly integrate these updates.

4.1 Volumetric Model

The volumetric model used here is a 3D model that represents an object or environment as a collection of voxels. Each voxel in the model can hold custom attributes, which makes it a useful tool for data storage, organization, and query. The base of the volumetric model is a 3D scan. The derived point-cloud provides accurate size and dimensions of the space, as well as information about existing elements on the site, such as walls, columns, and other architectural features. This data was used to establish the constraints and context for the design process, ensuring that the resulting architectural elements would integrate seamlessly with the existing site conditions. Once the model is created, we can store all the necessary parameters in the voxels, which can be queried at any level.

To store attributes in the voxel model, each voxel can be associated with a set of data values or properties. For example, a voxel might store information about its color, transparency, texture, or other relevant properties. The data can be stored directly in the voxel, or it can be stored in an external data structure that is indexed by the voxel's position. Each voxel in the grid is assigned a specific position in 3D space, typically represented by its *XYZ* coordinates. There are two approaches possible for storing data in a voxel model: to use a regular grid or a sparse representation, where only the voxels that contain relevant data are stored. While this approach can be useful for models that contain a large amount of empty space or for cases where high levels of detail are required, we have used the first approach. This storage method is combined with a coarsening and refining approach to adjust the level of detail in the model as needed (Fig. 3).

Grid coarsening or refinement can be applied to optimize the model for specific purposes, such as reducing memory requirements, increasing processing speed, or improving the visual fidelity of the model. To coarsen our volumetric model, the level of detail in the model is reduced by decreasing the number of voxels used to represent the object or environment. This is achieved by merging adjacent voxels together or by removing voxels that do not contribute significantly to the overall structure of the model. To refine the model, the level of detail in a grid is locally increased by adding more voxels or by subdividing existing voxels into smaller units. This can be done to improve the visual fidelity of the model, increase the level of detail in specific areas of interest, or to provide more accurate representations of complex shapes or structures. While it is possible to refine the computational model indefinitely to achieve a higher level of detail and precision, practical considerations related to the fabrication process must be considered. In our case, the smallest size of the 3D printing nozzle was 3 mm, which determined the minimum resolution achievable in the final printed structure.

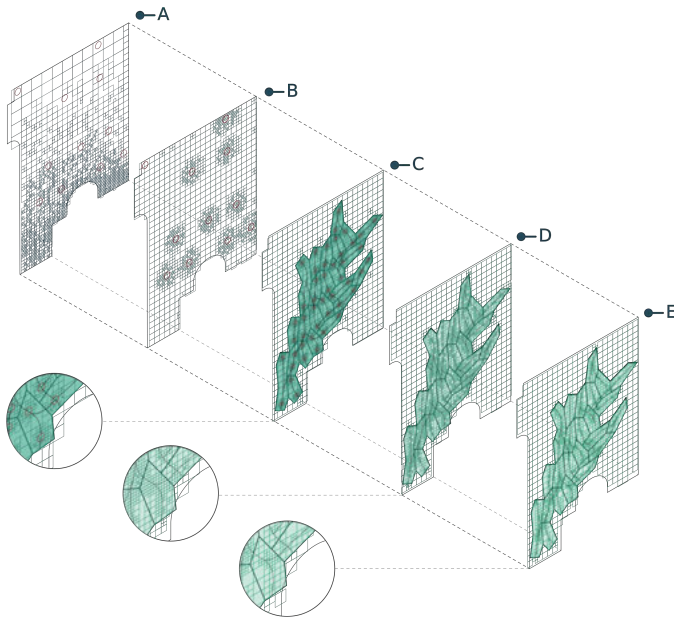


Fig. 3: Illustration of the different information layers of the volumetric in variable resolution.

A: Height; B: Distance to existing features on the wall; C: Distance to attachment points; D: Base panel geometry (layers 4–8); E: Base panel geometry (layers 8–12).

By incorporating this limitation and using 3mm as the smallest value in our model, we strike a balance between achieving intricate detail and ensuring the feasibility of the fabrication process.

4.2 Panelization

We utilized the information layers within the volumetric model to discretize the space into multiple panels of varying sizes. These layers provided essential data about the height, structural components, and elements of interest in the environment. We considered the constraints imposed by the existing site, such as connections to the wall and immovable elements, using them as drivers to inform the segmentation process. Additionally, we employed a dynamic relaxation model to determine the optimal wall connections for each panel.

By considering these factors, we were able to generate a suitable arrangement of panels that accommodated the site's unique conditions while maintaining the desired design intent. The discretization resulted in a total of 24 panels (Fig. 4), each tailored to their specific location and function while taking into consideration its neighbors to allow a small overlap for visual continuity.

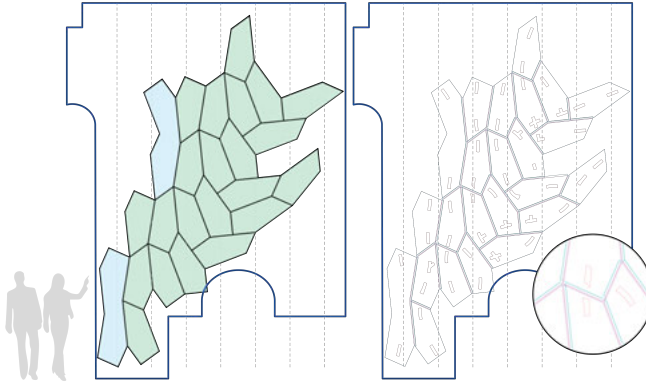


Fig. 4: Panelization strategy outcome.

4.3 Informed Print Path Generation

Our modeling approach for the panels was based on two distinct strategies. First, we focused on the “trunk”, a strong and continuous design element that runs through all panels, serving as the backbone of the structure and providing a coherent visual narrative across the entire installation. The trunk connects the panels and reinforces the overall unity of the project, giving it a consistent aesthetic and structural identity. We utilized a natural growth algorithm called Space Colonization (Runions et al. 2007). This algorithm mimics the way that trees and other plants grow in nature, by simulating the growth of branches from a central trunk. In our case, we used a modified version of the Space Colonization algorithm to generate a set of points on each panel, representing the locations where branches would grow. Typically, the growth direction is calculated based on the distance to the points within a configurable field of view. However, for our purpose, the points were weighted based on the volumetric model, ensuring that the branching structure would be informed by the underlying contextual data and adapt to the specific requirements (Fig. 5).

We have also considered the smaller “branches” that extend from the trunk and exhibit unique characteristics based on their specific context within the installation. These branches were designed with varying parameters, such as thickness, length, and material behavior, allowing them to adapt to different functional requirements. To generate these branches, we employed an agent-based modeling algorithm, which simulates the growth of branches from the trunk while ensuring that they avoid collisions with each other. This algorithm considers the spatial constraints and specific characteristics of each panel and the different material recipes, allowing the branches to grow in a controlled and organized manner.

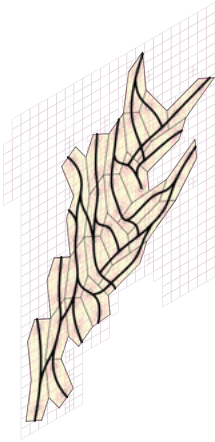


Fig. 5: “Trunk” on the design composition.

4.4 Material Specification

In the above sections, we have detailed how we used our modeling approach allows for a responsive toolpath geometry generation. Another benefit is its ability to carry information that drives the material aspects of fabrication process, therefore interfacing geometric description, material specification and robotic control code. The material specification of the print needs to respond to the geometry and structural considerations of the design, while incorporating aesthetical considerations. For example, the toolpath density and structural performance of a panel dictate its load-bearing capacity, while the choice of material, will affect its stiffness and weight.

In Radicant we have worked with a collagen-glue material mixed with waste-stream cellulose fillers: Bark flour, Wood flour, Cotton, and Seagrass. The choice of fibers affects not only the structural properties of the mix, but also its texture, color, and smell. For example, glue matrix mixed with 30 % wood flour produces a material with 0.7 g/cm^3 density, 1.35 GPa Youngs Modulus and a yellow sandy finish, whereas mixing the glue matrix with 30 % cotton results in a satin blue material with 3.15 GPa Youngs modulus, and 1.1 g/cm^3 density. A large number of permutations and interactions between different fillers and their percentages offers a customizable recipe space that can be leveraged by the print to respond to its functional and aesthetic requirements at different regions. This can be further empowered using a Machine Learning model that would return required material composition for a multi-objective specification.

By leveraging the suggested framework, we are able to assign specific material values to the individual voxels, therefore resulting in a comprehensive representation of the spatial distribution and characteristics of the installation. For example, we employ the stiffest and lightest materials in the base layers of the panels using a vertical gradient based on height blending from seagrass at the bottom to wood flour at the

top. The top layer employs materials that are stronger in tension, creating a blending gradient from cotton to bark flour symmetrical to the trunk. This combined information is then translated into a vector field, where the material choices are discretized to the panel (Fig. 6), and which serves as the foundation for generating the final printing path with the correct layering sequence.

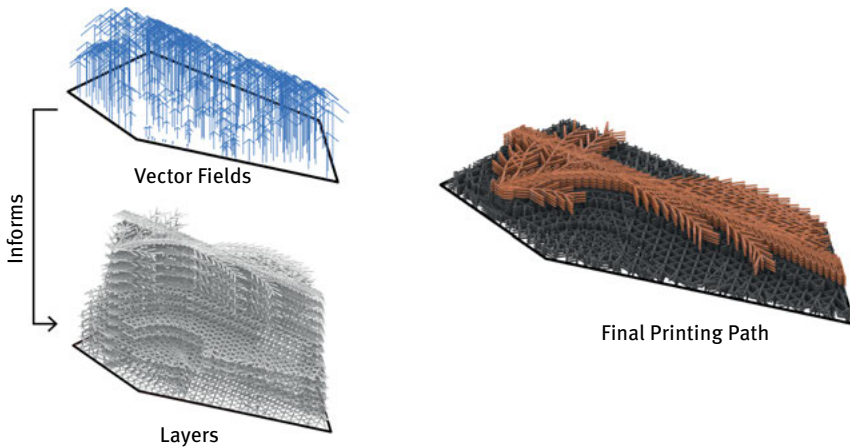


Fig. 6: Density, height, and material properties are used to generate the final print path.

4.5 Generation of Fabrication Information

The generation of fabrication data for the robotic printing process was an essential aspect of our design pipeline. This step involved transforming the geometric information and material properties derived from the volumetric model into a set of commands that the robotic printer could execute. To ensure a smooth and efficient printing process, we also considered the robot's reachability, making certain that the designed elements were within the operational range of the robot. In order to achieve a continuous and seamless print, we utilized an informed path generation algorithm that processed the curves and aligned their start and end points. This approach optimized the print path, minimizing the need for the robot to reposition itself, thus reducing overall printing time.

This fabrication information is input to two Universal Robots (UR16e) equipped with custom print heads. 24 panels with dimensions 2100×600 mm and 1200×600 mm were printed over a period of 3 weeks. In this setup, each Universal Robot printed with one material recipe, and multiple robots were used to print each panel (Fig. 7). Subsequent to this project we have developed a single multi-material print head that allows a panel to be printed by a single robot receiving multiple material feeds.

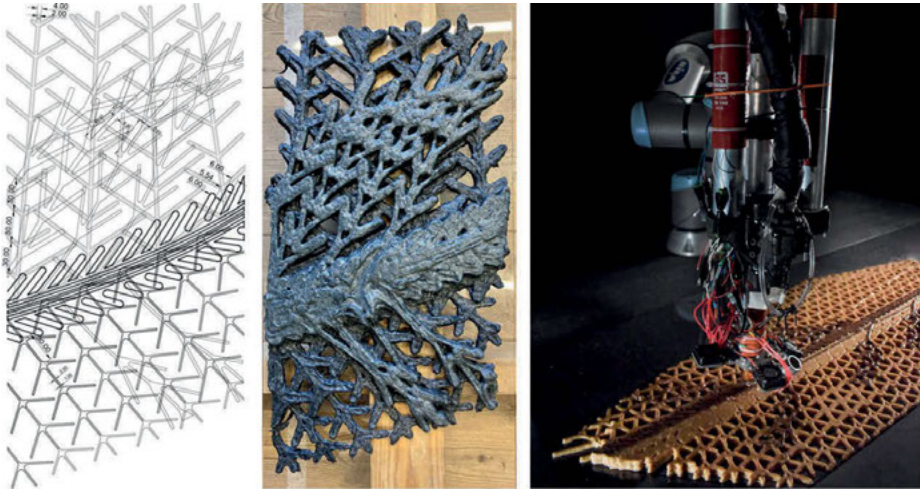


Fig. 7: 3D printing – Generated print path, test of Biomaterial with recycled cotton, and final print with seagrass and bark fibrous filler with multimaterial Biopolymer printhead.

5 Process Evaluations and Results

The development of the design to fabrication pipeline took place in parallel to the development of the protocols and hardware for large scale printing, drying and post-processing of collagen-based materials, the adaptation and optimization of the material for 3D printing with a range of fillers as well as the development and refinement of design concept and interfaces between the different parts of the structure. This process enforced a high level of modularity and adaptability into the developed framework, as feedback on conceptual design level, from small- and large-scale test prints and finally full-scale mockups needed to be integrated at any moment. The development of the framework followed the same linear order as the development of the design, from conceptual to full scale material production level. It was however the ability of the pipeline to connect these very different layers and their model representation, that was tested constantly and with positive results. Changes on any model could and have been executed at any moment until the end of the production phase. Here a measurement of the printed collagen panels was undertaken and showed as expected an uneven shrinkage of every panel of up to 20 % in area. The geometric model of the pipeline could be updated, and the model of the timber substructure was automatically updated – just in time for production (Lharchi et al. 2023).

The project allows us to reflect on the initial four aims for the framework:

- Efficient data storage and manipulation: The framework has been able to connect models of various datatypes such as point clouds from 3D scanning, voxel-based volumetric data for material properties, mesh models for structural analysis, and

curve-based data for fabrication paths with the aim to produce finally information that steers the printing of 24 large panels, with over 18000 lines of robotic code for small panels, and the double for the larger ones. However, the performance could be improved, as each regeneration took several hours.

- Integration with digital design workflows: Our framework was integrated seamlessly into a Rhinoceros 3D/Grasshopper workflow and could ingest other data, as from point clouds, structure analysis, and material properties. The framework's potential for incorporating additional information layers, such as material aging and degradation or acoustic properties, highlights its versatility in addressing various design considerations. Furthermore, by selectively increasing the resolution where needed, the framework can provide a more accurate representation of the design, while maintaining computational efficiency.
- Advanced volumetric operations: The framework range of operations for processing volumetric data, such as filtering, smoothing, and resampling have been crucial for providing information on site, and other high- and low-level information in the pipeline.
- Scalability: The flexible data structure could indeed accommodate different scales of volumetric data, from micro-scale material properties to macro-scale architectural elements, and the transfer of information between these scales. It was especially important that this adaptation could take place at any time of the process, so that incrementally gained insights gained in process could be accommodated, as well more radical updates, as the required adaptation to the large shrinkage of the panels.

6 Conclusion

In this paper we have presented a grid-based workflow that integrates contextual data, generative operations, and print path generation across multiple scales of resolution. The workflow has been motivated by the architectural and sustainable potentials of multi-material 3D printed biopolymers. We have described the components and application of this workflow to the design and fabrication of an interior wall paneling system. Evaluating this specific case indicates the potential for a larger set of applications and extensions, based on incorporating more of the specificity of the printing process itself, the possibility for grading material more continuously and at a finer resolution, and through generalization of the pipeline to materials beyond biopolymers. More fundamentally, this research points towards the need to move from boundary to volumetric representations to better enable the full potential of design and fabrication with emerging biomaterials.

Author Contributions

The manuscript was written with the contribution of all authors. All authors have approved the final version of the manuscript. Nicholas, P. project conceptualization, methodology, design concept, writing – original draft, reviewing and editing, supervision, Lharchi, A., design concept, computational modeling framework development, 3D print strategy, fabrication, installation, writing – original draft, reviewing and editing, Tamke, M., project conceptualization, funding acquisition, methodology, writing – original draft, reviewing and editing, installation, supervision, funding acquisition, Valipour Goudarzi H., design concepts, visualization, prototyping, fabrication, installation, Eppinger, C., design concept, 3D print hardware development, prototyping, fabrication, installation, Sonne, K., prototyping, fabrication, installation, Rossi, G., material specification strategy, Ramsgaard Thomsen, M. project conceptualization, methodology, design concept, writing – review and editing, lab infrastructure, supervision, funding acquisition.

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