

4 Explanation of Logical Theorems

The main focus of this chapter is to explore how logical theorems may be explained. In section 4.1 I argue that their ordinary grounding explanations do not appear to be completely satisfactory and identify desiderata for more satisfactory explanations. In section 4.2 I consider and criticize two proposals for extraordinary grounding explanations: grounding explanation by status and explanation by zero-ground. In section 4.3 I then offer several ways in which we might deal with the apparent failure of grounding to provide satisfactory explanations of logical theorems, and argue that empty-base explanations that do not involve grounding could satisfy the desiderata without running into the problems that confront grounding explanations. In the remainder of the chapter I explore two ways to implement this idea: First, in section 4.4 I develop Yablo's (2014) account of reductive truthmaking to allow for a kind of empty-base explanation of logical theorems by reductive truthmaking.

Second, in section 4.5 I turn to the idea of explanation by essence, according to which logical theorems can somehow be explained using essences. I discuss some attempts to address the primary challenge here, namely to make sense of an essence-involving explanatory notion whose instances of do not entail corresponding grounding statements. While I focus on finding kinds of explanations that afford more satisfactory explanations for logical theorems than grounding does, we will see that the applications of the alternative explanatory notions to be discussed may not be confined to the explanation of logical theorems; for example, I will argue that the existence of a certain (non-grounding-entailing) kind of explanation by essence is likely to have significant upshots for other areas of philosophy, including philosophy of mind and the mind-body problem.

Furthermore, in chapter 3 I argued that proposals for explanations by essence (i.e. proposals that try to explain why P in terms of it being part of some essence that P) should best be understood as empty-base explanations featuring essential conditionals or some other kind of essence-involving propositions as their explanatory links. The discussion of explanation by essence in this chapter attempts to better characterize these essence-involving explanatory links.¹⁵⁵

Finally, the chapter concludes with section 4.6 and a look at some remaining options for non-grounding-explanations of logical theorems.

¹⁵⁵ For the sake of convenience, unless stated otherwise, schematic letters and formulae like ' P ' and ' $P \vee \neg P$ ' will be used both in sentence position and to (schematically) refer to the corresponding propositions in this chapter.

4.1 Ordinary grounding explanations and why they might be unsatisfactory

Our question is how logical theorems such as $P \vee \neg P$, which will be our schematic example, can be explained. More specifically, our goal is to answer why P for every logical theorem P – hence, to answer why $P \vee \neg P$ (for instance, why the sun is shining or it is not the case that the sun is shining). Since, as we may assume, the desired explanation is not a causal one, it is natural to turn to grounding explanation. Indeed, a standard kind of grounding explanation for logical theorems is readily available: For example, the logic of ‘because’ in Schnieder (2011) and the logic of ground in Fine (2012) specify grounds for logical theorems of classical first-order logic.

According to these proposals, the grounds of a logical theorem are propositions that correspond to some of the atomic formulae (or negations thereof) into which the formula that expresses the logical theorem can be decomposed, namely those that make the logical theorem true – ground it – on a given occasion. For example, since in general a disjunction is grounded in its true disjunct, a logical theorem of the form of $P \vee \neg P$ is grounded in and hence can be explained by its true disjunct, with the corresponding because-claim being ‘ $P \vee \neg P$ because P ’ or ‘ $P \vee \neg P$ because $\neg P$ ’, depending on whether P or $\neg P$ is true.¹⁵⁶ Call this kind of explanation the ‘ordinary grounding explanation’. To give a concrete example, given that the sun is shining, [the sun is shining or it is not the case that the sun is shining] is fully grounded in [the sun is shining].¹⁵⁷ Correspondingly, given that the sun is shining, the sun is shining or it is not the case that the sun is shining because the sun is shining.

As for instance Schnieder (2011, 457f.) observes, these ordinary grounding explanations may not seem completely satisfactory. For example, one might have thought that logical theorems are a good candidate for truths that possess some sort of special, perhaps somehow particularly good, explanation. Some desiderata that may be the source of this idea are that a satisfactory explanation of logical theorems should

¹⁵⁶ I will assume here, as I have elsewhere, that the truth of the grounding claim is sufficient for the truth of the corresponding because-claim.

¹⁵⁷ As always, I use square brackets to refer to the proposition expressed by the sentence within the brackets. For the sake of convenience, I sometimes use a predicational idiom of grounding and assume grounding to relate propositions; no commitment as to the nature of grounding’s relata, if any, and concerning operatorational versus predicational views of grounding is intended by this. Cf. Correia and Schnieder (2012, sec. 3.1) for the distinction between these two views.

1. somehow also account for their necessity or their status as logical theorems,
2. be modally stable in that it holds with necessity,
3. give rise to no, or just very few, or not especially pressing further why-questions, or
4. be compatible with certain non-classical logics in which e.g. $P \vee \neg P$ can be true without either of its disjuncts being true.

It is clear that ordinary grounding explanations do not satisfy these desiderata: An ordinary grounding explanation of, say, $P \vee \neg P$ in terms of its true disjunct gives rise to the question why this disjunct obtains and hence to all the why-questions that an explanation of that disjunct gives rise to. Since the disjunct is arbitrary, no special explanatory status with respect to what further why-questions arise in that fashion seems available for $P \vee \neg P$, if it only has an ordinary grounding explanation. There also appears to be no sense in which the ordinary grounding explanation could account for the necessity of $P \vee \neg P$ or its status as a logical theorem. After all, $P \vee \neg P$ has the same kind of ordinary grounding explanation as any true disjunction. Since the ordinary grounding explanation of a disjunction proceeds through its disjuncts, it also fails to be modally stable: If P is contingent, $P \vee \neg P$ will be grounded in P if P is true, and in $\neg P$ otherwise. For the same reason the fourth desideratum fails for ordinary grounding explanations: If $P \vee \neg P$ is to be explained in a setting in which it can be true without either P or $\neg P$ being true, then ordinary grounding explanations will not do.¹⁵⁸

Before we continue, let me note that it is not clear that there *must* be explanations for logical theorems that satisfy (one or more of) the desiderata – for example, perhaps the status of logical theorems as logical theorems can be accounted for by explaining not the theorems themselves, but rather explaining why they are logical theorems. It is also not clear that necessary truths should require necessary explanations if it is necessary that they do have an explanation, just not the same in every possible circumstance. Nevertheless, I will take the dissatisfaction with the ordinary grounding explanations as a datum and attempt to find (additional) alternative explanations for logical theorems that satisfy the desiderata just identified.

¹⁵⁸ I consider the fourth desideratum to be weaker than the first three, but I take these to be compelling on their own in any case.

4.2 On extraordinary grounding explanations

On reflection, two proposals for alternative grounding explanations of logical theorems readily come to mind: First, there is the idea that logical theorems might somehow be grounded in (and thereby explained by) propositions expressing their status as logical theorems, their being logical or metaphysical laws, or their being part of certain essences. Following the discussion in chapter 3, call these proposed explanations ‘explanations by status’ and the mentioned status-expressing propositions ‘status propositions’.¹⁵⁹ The corresponding because-claims would then have the form ‘ $(P \vee \neg P)$ because $\blacksquare(P \vee \neg P)$ ’, where ‘ \blacksquare ’ is a placeholder for the respective status-expressing operator.

According to the second proposal that comes to mind, logical theorems are zero-grounded. For readers who have skipped chapter 2, let me say a little bit about the notion of zero-ground: Normally, metaphysical grounding is taken to be a relation (or at least something approximately like a relation) between a plurality of propositions or facts, the *grounds*, and a single proposition or fact, the *grounded* proposition/fact or *groundee*. Zero-grounding is a limiting case of grounding in which the set of grounds is empty. A zero-grounded proposition or fact is grounded and not ungrounded, but it does not require any propositions or facts to ground it – it is grounded in zero propositions/facts.¹⁶⁰

More precisely, if we assume grounding statements to have the form ‘ $T < \phi$ ’, then since in the case of zero-grounding statements, the ‘ T ’ stands for an empty plurality of grounds, statements of zero-grounding have the form ‘ ϕ ’. Alternatively we might express zero-grounding using sentences of the form ‘ $\emptyset < \phi$ ’. As for the corresponding because-statements, we can adopt a similar convention and use ‘ \emptyset ’ to stand for the empty set of grounds, which gives us ‘ $P \vee \neg P$ because \emptyset ’. Somewhat tongue-in-cheek, we will take and adapt the natural language expression ‘just because’, giving us ‘ $P \vee \neg P$ just because’.¹⁶¹

Intuitively, at least, both proposals promise to scratch an explanatory itch that the ordinary grounding explanations do not address: They do, in some sense, account for the special status of logical theorems, they are necessary, they satisfy the alternative-logics desideratum, and (as we have seen in chapter 3) at least

¹⁵⁹ A conceivable related option that I will not address is that while logical theorems cannot in general be explained by propositions expressing *their* status, they can be explained by other status propositions.

¹⁶⁰ The notion of zero-ground has been introduced by Fine (2012, 47f.). One prominent application of the notion is Litland’s (2017) account of the grounds of ground, but see also Donaldson (2017), Muñoz (2020), De Rizzo (2020), and Litland (2022).

¹⁶¹ I discuss the nature of this explanatory proposal further in section 4.3.

explanation by status has been considered by some to be an – in some way or other – especially good kind of explanation.¹⁶²

One way to spell out the latter point is to focus on the idea that logical theorems are grounded in propositions that express their status as essential truths and to adopt Dasgupta's (2014b) idea that such propositions are explanatorily autonomous, i.e. not in need of any explanation. The grounding explanation of logical theorems in question would then be particularly good because according to it, logical theorems are grounded in propositions which themselves do not require any further explanation. Quite similarly, the zero-grounding proposal promises particularly good explanations of logical theorems in that the relevant explanatory candidates do not involve any grounds of logical theorems at all for which further explanations could be demanded.

That the proposals promise to satisfy the other desiderata can be seen as follows. First, the proposal for explanation by status accounts for the special status of logical theorems by employing that very status in explaining logical theorems. This status can then, so to speak, be read off these explanations of logical theorems. The desideratum for modal stability is satisfied by the proposal for explanation by status because the status propositions are necessary and the (at least in this context) eminently plausible principle that grounding is non-contingent, according to which if propositions Γ together ground Q , then necessarily, if all propositions Γ are the case, then the Γ together ground Q .¹⁶³

According to the zero-grounding proposal on the other hand, logical theorems are grounded in the empty plurality of grounds. Since all propositions in the empty plurality of grounds are necessarily the case, this, together with the principle that grounding is non-contingent, also results in logical theorems being necessarily zero-grounded. In the same way, the necessary status of logical theorems can be read off their proposed zero-grounding explanations, whereby this proposal also satisfies the first desideratum. Moreover, the special status of being zero-grounded itself can be read off the proposed zero-grounding explanations: According to the proposal, logical theorems do not only logically follow from zero premises, they are also grounded in and hence explained by zero premises. Finally, the alternative-logics desideratum can be satisfied by both proposals simply because they offer grounds for logical theorems that obtain even if we assume that, e.g., $P \vee \neg P$ obtains without either P or $\neg P$ obtaining.

¹⁶² This idea is common in the literature on why there is anything at all, in which explanation by necessary status is often taken to be of particular explanatory value. Cf., e.g., Goldschmidt (2013).

¹⁶³ For the principle, cf. Correia and Schnieder (2012, 21ff.).

A shared drawback of these two proposals for extraordinary grounds of logical theorems is that they both conflict with Fine's (2012, 63f.) attractive account of the logic of ground, according to which conjunctions can only be grounded via their conjuncts and disjunctions can only be grounded via their (true) disjuncts.¹⁶⁴ According to this assumption, our example $P \vee \neg P$ can also *only* be grounded via its true disjunct. Since the alternative grounds proposed above are not in general either the true disjunct of $P \vee \neg P$, nor do they ground it, these proposals are ruled out by the present assumption. Note in particular that this is also true for the zero-grounding proposal: According to the assumption, $P \vee \neg P$ can only be zero-grounded if one of its disjuncts is zero-grounded. But of course, only in very specific instances will the true disjunct of a disjunction be zero-grounded (or grounded in the relevant status-expressing proposition).

As I have argued in chapter 3 though, understanding explanations by status as advancing the status propositions as *grounds* of their explananda is problematic for a number of further reasons. Instead, I proposed that explanations by status should best be understood as hinting at empty-base explanations. Given that we are currently looking for a more satisfactory, extraordinary *grounding* explanation of logical theorems, this would mean endorsing the view that logical theorems are empty-base grounding explained and hence zero-grounded, and hence still incur the above problem from the logic of ground.

4.3 What to do?

Let us consider some possible reactions to this difficulty:

1. Accept that despite intuitive appearance to the contrary, an explanation of logical theorems that does not proceed via the ordinary grounding explanation cannot be had. Additionally, it might be argued against the need for any additional explanation.¹⁶⁵
2. Change the target: Perhaps a more satisfactory explanation can only be had for a proposition in the vicinity of $P \vee \neg P$. One salient candidate would be to

¹⁶⁴ More specifically, the logic of Fine (2012) captures this idea by postulating elimination rules for the impure logic of ground, for instance the rule E. But the insight is more general than this implementation, it is for example also contained in Fine's (2017b) account of grounding in terms of truthmaking.

¹⁶⁵ A notable variant of this reaction would be to suggest that while no more satisfactory explanations *why* of logical theorems can be had, perhaps other kinds of explanations *wh-* such as explanations *what* can be had. For a recent application of the distinction between explanation *why* and explanation *what* in metaphysics see Skiles (2019).

explain why certain status propositions obtain, such as the propositions that it is a logical theorem, a necessary truth or a metaphysical law that $P \vee \neg P$ (rather than explaining why $P \vee \neg P$).

3. Revise the logic of ground to allow for more diverse – extraordinary – grounds for logical theorems.
4. Find a different explanatory notion that allows for a more satisfactory explanation of logical theorems than grounding does.

I have some reservations with respect to the first three options: First, it seems that we should only, despite appearance, accept that no more satisfactory explanation can be had and try to explain away the need for a better explanation, if indeed no alternative candidate is available. As a matter of fact, such a candidate may be available, as I will argue below. With respect to the second option, I have a similar reservation: While it is an interesting question what, if anything, explains truths expressible by sentences of the form ‘It is a logical theorem that . . .’, I first want to investigate whether a more satisfactory explanation of logical theorems themselves can be found.

With respect to a revision of the logic of ground, I have the following reservations: First, the logic as it is is neat and somewhat intuitively motivated. Second, there is some reason to suspect that if we try to change the principles of the logic of ground, we end up talking about different propositions involving different operators and nothing has been won with respect to our original question. The thought is this: According to Fine (2017a), propositions can be defined in terms of their exact truthmakers. But to postulate an extraordinary ground of a logical theorem P in addition to its ordinary grounds is, in effect, to change its set of exact truthmakers.¹⁶⁶ So it seems that we would be dealing with two propositions: The proposition P_1 that only has the ordinary grounds and associated exact truthmakers, and the proposition P_2 that additionally has the extraordinary ground and associated exact truthmakers. But what we were interested in was not an explanation of P_2 , but an explanation of P_1 .

Here, it could be objected that our goal was to find satisfactory explanations for propositions such as [the sun is shining or it is not the case that the sun is shining] and that Fine’s theory is simply mistaken about what truthmakers this proposition has. Nevertheless, the following problem remains even if we admit that [the sun is shining or it is not the case that the sun is shining] has extraordinary grounds and associated truthmakers. For what about the proposition, call it

¹⁶⁶ See Fine (2017b) on the definition of grounding in terms of exact truthmakers, which, like Fine (2012), captures the idea that disjunctions can only be grounded via their true disjuncts.

P_1 , that according to the objection Fine mistakenly identified with [the sun is shining or it is not the case that the sun is shining], and which shares all truthmakers with this latter proposition except those required for its having extraordinary grounds? Plausibly, this proposition is also a logical truth for which we would like to have a satisfactory explanation, yet by assumption it cannot have extraordinary grounds. Of course, this argument could be resisted by denying that propositions like P_1 exist, but it is not clear to me on which basis.¹⁶⁷

Third, if we revise the logic of ground to be more permissive, logical theorems will have ordinary grounds (those they had all along) and extraordinary grounds (those that are required for the more satisfactory explanations of logical theorems). Then the question arises how extraordinary grounds can be characterized and how the difference between ordinary and extraordinary grounds can be accounted for.

In the remainder of this chapter, I will primarily pursue the fourth option. To approach the idea, let us take a step back and consider the zero-grounding proposal once more. As I have argued in chapter 2, explanations by zero-ground are instances of the more general phenomenon of empty-base explanation. And as I have argued in section 4.2, the structure of zero-grounding explanations is suitable to satisfy the desiderata for explanations of logical theorems. But it is now clear that it is more generally the case that the structure of empty-base explanation allows for the satisfaction of the desiderata:

Just like the zero-grounding proposal, empty-base explanations more generally promise particularly good explanations of logical theorems in that the relevant explanatory candidates do not involve any reasons why logical theorems obtain for which further explanations could be demanded. According to the empty-base proposals in general, logical theorems are explained in an empty plurality of propositions (i.e. reasons why the relevant logical theorem obtains). Since all propositions in the empty plurality are necessarily the case, this, together with an assumption to the effect that the relevant explanatory notion is non-contingent (understood in analogy to the principle of non-contingency of grounding assumed above), also results in logical theorems being necessarily empty-base explained.

Likewise, the necessary status of logical theorems can be read off their proposed empty-base explanations, whereby this proposal also satisfies the first desideratum. Moreover, the special status of being empty-base explained itself can be read off the proposed empty-base explanations: According to the proposal, logical theorems do not only logically follow from the empty set of premises, but

¹⁶⁷ See also footnote 168.

they are also explained by this empty set of reasons. Lastly, the alternative-logics desideratum can be satisfied by empty-base explanations in general, because such explanations can provide their reasons for logical theorems (i.e. none) even if we assume that, say, $P \vee \neg P$ can be true without either disjunct being true.

So, logical theorems seem to be suitable candidates for empty-base explainability, but given what we have said before, not for zero-groundability. Thus, our question is whether we can find an alternative explanatory relation that provides us with an empty-base explanation of logical theorems. Here, the most salient idea is perhaps to look again at the proposal that logical theorems can be explained by propositions that express their having a certain status. Indeed, as I have argued in chapter 3, proposals for explanation by status can be understood as empty-base explanations in which the status proposition plays the role of an explanatory link (rather than ground) that can explain the corresponding explanandum on its own, without requiring help from anything in the explanatory base.

This is the plan for the remainder of the chapter: In the next section, I try to characterize an explanatory relation on the basis of Yablo's (2014) thoughts about *reductive truthmaking* that allows for a corresponding empty-base explanation of logical theorems. As will become clearer later, most of what I am going to say can alternatively be understood as realizing the third option above (i.e. revising the logic of ground) by conceiving of the newly characterized explanatory notion as a special case of grounding. Then, in section 4.5, I will turn to the idea of (non-grounding-involving) explanation by essential or law-like status. The two sections can be read independently of each other.

4.4 Explanation by reductive truthmakers

Yablo (2014, ch. 4) distinguishes two conceptions of truthmakers: the recursive conception and the reductive conception. Here, recursive truthmaking approximately corresponds to our notion of grounding. In particular, disjunctions like $P \vee \neg P$ are recursively made true by the fact that corresponds to its true disjunct. As an alternative to recursive truthmaking, Yablo proposes a notion of reductive truthmaking. Here he is motivated by intuitions like the following:

A disjunction is true [. . .] because of a fact that verifies one disjunct, or a fact that verifies the other. This does not seem to exhaust the options. Why not a fact that ensures that one disjunct or the other is true, without taking sides? (Yablo 2014, 60)

Consider next a conditional $P \wedge Q \rightarrow P \wedge Q \wedge R$. It owes its truth, on the recursive conception, either to a fact that falsifies P , or a fact that falsifies Q , or a fact that verifies $P \wedge Q \wedge R$. Why not a fact [like the fact that R] that blocks the *combination* of $P \wedge Q$ true, $P \wedge Q \wedge R$ false, without pronouncing on the components taken separately? (Yablo 2014, 60)

To capture these intuitions, Yablo (2014, 61) proposes the following notion of reductive truthmakers, defined via his notion of a minimal model.¹⁶⁸

(Minimal model)

m is a minimal model of ϕ iff_{def.} m is a partial valuation of the language of ϕ that verifies ϕ and no proper subvaluation of m verifies ϕ .

(Reductive truthmakers)

ϕ 's reductive truthmakers (falsemakers) are its minimal models (countermodels), or the associated facts.

This idea needs some amendment and explication: First, the definition of a minimal model has to be fixed: According to Yablo (2014, 61) the formula $P \rightarrow (P \wedge Q)$ has as a minimal model the partial valuation that assigns truth to Q . But it is not clear how such a valuation verifies $P \rightarrow (P \wedge Q)$, since the truth-conditions for this formula require P to be false or $P \wedge Q$ to be true. But the truth-conditions for these in turn are not satisfied in the proposed model. This problem can be solved by adopting the following definitions:

(Minimal model*)

m is a minimal model of ϕ iff_{def.} m is a partial valuation of the language of ϕ such that all its *supplementations* verify ϕ and no proper subvaluation of m is such that all its supplementations verify ϕ .

(Supplementation)

A supplementation m^* of a partial valuation m of a language is a (full) valuation of the language such that m is a subvaluation of m^* .

Second, we need to clarify what the facts that are associated with a minimal model are. Here, we only look at a propositional language, so a minimal model

¹⁶⁸ An interesting alternative option to treat Yablo's cases would be to determine how Fine's truthmaker semantics would have to be revised to capture these cases. It probably is possible to capture the first case by allowing certain additional truthmakers for disjunctions. Consider for example the disjunction $(P \wedge Q) \vee (\neg P \wedge Q)$. For this particular case, the additional truthmakers of $(P \wedge Q) \vee (\neg P \wedge Q)$ would be the truthmakers of Q , and these are part of the truthmakers of both disjuncts. Interestingly, the second case seems to differ from the first in this respect: If we conceive of the conditional as the disjunction $\neg(P \wedge Q) \vee (P \wedge Q \wedge R)$, we can see that the truthmakers of R that would have to be added to capture Yablo's idea need not be part of the truthmakers of the first disjunct. Additionally, as mentioned in section 4.3, a rationale would have to be found why this does not leave the original propositions defined by Fine without satisfactory explanations.

(viz. partial valuation of the language) is a partial truth-value assignment to atomic formulae. I will further assume that every atomic formula ϕ expresses exactly one state of affairs, and I shall say that such a state of affairs *obtains according to a model* iff the model assigns truth to ϕ . We can then stipulate that the facts that are associated with a minimal model are the states of affairs that (1) obtain according to the model, and (2) that do in fact obtain.¹⁶⁹ An analogous definition can be given for falsemakers and countermodels.

Furthermore, we will use Yablo's convention to refer to states of affairs and facts: p is the state of affairs or fact associated with the formula P and \bar{p} is the state of affairs or fact associated with the formula $\neg P$. Yablo further refers to models by the set of simple states of affairs that obtain according to the model. For example, a model that only assigns truth to P can be referred to using ' $\{p\}$ '.¹⁷⁰

Third, Yablo sometimes talks as if all the minimal models of a formula themselves are the truthmakers of that formula. Alternatively, we can give a corresponding (perhaps more perspicuous) definition, according to which the states of affairs that obtain according to a minimal model are the reductive truthmakers:

(Reductive truthmakers_{NF})

ϕ is *reductively made true_{NF}* by states of affairs Γ , iff there is a minimal model m of ϕ such that the Γ are the states of affairs that obtain according to m .

This notion of reductive truthmaking is non-factive: it defines a relation between states of affairs and formulae irrespective of whether or not the states of affairs obtain or the formulae are true. In addition to this non-factive notion, we need a factive notion of reductive truthmaking: According to an intuitive understanding of 'making true', only facts can make anything true. For example, $P \rightarrow (P \wedge Q)$ has both $\{\bar{p}\}$ and $\{q\}$ as minimal models, but of course it might be true without either Q being true (namely if $\neg P$ is true) or $\neg P$ being true (namely if Q is true). While both minimal models contain reductive truthmakers in the non-factive sense, we also want a notion to express what *actually* makes the formula in question true. Moreover, the explanatory relation that we want to define using reductive truthmaking, and the notion of 'because' are factive: If Q is not even true, it surely cannot explain why

¹⁶⁹ If we want to assume that facts are distinct from states of affairs that obtain, then we can say that the facts that are associated with a minimal model are the facts that correspond to the states of affairs that obtain according to the model, and that do in fact obtain.

¹⁷⁰ Note that we could alternatively omit reference to truth from the definition of a model and let the model assign states of affairs and specify whether they obtain according to the model or not.

$P \rightarrow (P \wedge Q)$. Therefore, we define a factive notion of reductive truthmaking (to be used in the following unless stated otherwise) like this:

(Reductive truthmakers_F)

ϕ is *reductively made true_F* by facts Γ , iff there is a minimal model m of ϕ such that the Γ are the facts associated with m .

So far, we have followed Yablo in defining a notion of truthmaking for *formulae* or *sentences*. To obtain a corresponding notion for *propositions*, we assume that a proposition P is associated with a minimal model m iff there is a sentence S that expresses P and m is a minimal model of S , as defined above. Accordingly, we define that P is reductively made true by states of affairs Γ , iff there is a sentence S that expresses P and S is reductively made true by the states of affairs Γ .

With respect to Yablo's motivating examples, the above definitions yield the following results:

- $(P \wedge Q) \vee (P \wedge \neg Q)$ has $\{p\}$ as a minimal model. If p obtains, then $(P \wedge Q) \vee (P \wedge \neg Q)$ is reductively made true by p .
- One of the minimal models of $P \wedge Q \rightarrow P \wedge Q \wedge R$ is $\{r\}$. If r obtains, then $P \wedge Q \rightarrow P \wedge Q \wedge R$ is reductively made true by r .

We can now look at what the proposal says about logical theorems, for example $P \vee \neg P$:

- $P \vee \neg P$ has $\{\}$ as a minimal model. This holds for every logical theorem.

Here, ' $\{\}$ ' refers to the empty model which makes no truth-value assignment. Above we said that the reductive truthmakers of a proposition are the facts that are associated with its minimal models. We can correspondingly say that for a proposition P and a minimal model m of P , P is reductively made true by the facts that are associated with its minimal model m . Consequently, since no facts are associated with the empty minimal model $\{\}$, logical theorems such as $P \vee \neg P$ are reductively made true by zero facts, i.e. the empty plurality of facts.

We have now already arrived at a situation and instance of reductive truthmaking that is clearly reminiscent of zero-grounding – namely reductive truthmaking by zero facts. Some more work needs to be done to arrive at a corresponding kind of empty-base explanation of logical theorems. We do this as follows:

First, we assume that for every state of affairs that obtains according to a minimal model, there is a corresponding proposition that has this state of affairs and no other as a (non-factive) reductive truthmaker, and we say that such a proposition *expresses* its (non-factive) reductive truthmaker. We then define explanation by reductive truthmaking:

(Explanation by reductive truthmaking)

For every true proposition P with associated minimal model m , propositions Γ explain P by reductive truthmaking iff the Γ express the reductive truthmakers associated with m , and P does not itself express one of its reductive truthmakers.

For the limiting case in which P is made true by zero facts, we can then say that the empty plurality Γ ‘expresses’ the reductive truthmakers of P , i.e. none. Now since P is made true by zero facts, there is no reductive truthmaker of P that P could express, thus we can say that P is explained (via reductive truthmaking) by the propositions Γ , viz. zero propositions. Under the assumption that explanation via reductive truthmaking so construed corresponds to because-claims, we can state this more succinctly in terms of ‘because’: P is empty-base explained and P holds just because.

Now, the proposal yields the following because-claims:

- $(P \wedge Q) \vee (P \wedge \neg Q)$ because P , if P is true.¹⁷¹
- $P \vee \neg P$ just because.¹⁷²

$P \vee \neg P$ and logical theorems in general can be empty-base-explained in this fashion because they are reductively made true by zero facts. As explained in section 4.3, we can use ‘just because’ to express empty-base-explanations, so for every logical theorem ϕ , we obtain the result that ϕ just because.

At this point, one might perhaps worry whether what we have characterized so far is really an *explanatory* relation that underwrites because-claims and affords explanations why. Note at the outset that it is not quite clear what would constitute a satisfactory response to this worry. I will simply provide some considerations in support of our relation being explanatory.

First, let us see whether the relation satisfies some formal features that explanatory relations are often assumed to possess: The relation satisfies irreflexivity because of the requirement that a proposition P can only be explained (via reductive truthmaking) by the propositions Γ that express the reductive truthmakers corresponding to a minimal model m of P , if P does not itself express one of its reductive truthmakers. The relation satisfies asymmetry for similar reasons: Suppose P explains Q by reductive truthmaking. Then P expresses a reductive truthmaker of Q , say p . According to our assumptions, for Q to in turn explain P by reductive truthmaking, Q must express a reductive truthmaker of P , say q . But

¹⁷¹ Note that $(P \wedge Q) \vee (P \wedge \neg Q)$ is partially grounded in P , if P is true. Therefore, the ordinary grounding account already allows that $(P \wedge Q) \vee (P \wedge \neg Q)$ partially because P . The present proposal on the other hand allows that $(P \wedge Q) \vee (P \wedge \neg Q)$ (fully) because P .

¹⁷² Or, alternatively, ‘ $P \vee \neg P$ because \emptyset ’.

by our definition of what it is to express a reductive truthmaker, P has just the single reductive truthmaker that it expresses, so $p = q$. But then Q expresses p , which is its own reductive truthmaker, so according to (Explanation by reductive truthmaking), if P explains Q by reductive truthmaking, then Q does not explain P by reductive truthmaking.

The requirement of transitivity is satisfied because the explanatory structure that results from the proposal is somewhat flat: Propositions corresponding to complex logical formulae are directly explained by propositions corresponding to atomic formulae (or their negations), and in the case of logical theorems, they are empty-base explained. Thus, the situation does not arise in which, for example, an atomic formula P explains a complex logical formula Q , which in turn explains a further complex logical formula R , such that the question could arise whether P explains R .¹⁷³ While this can cover the logical cases we are considering here, it is in general a question for further investigation whether and if so, how the proposal extends to non-logical cases.

A third, broadly formal feature that explanatory relations are sometimes argued to have is what Yablo (2014, 47f.) calls proportionality. But, as Yablo shows, this observation may even identify a particular strength of the reductive truthmaking proposal, since reductive truthmakers seem to have an especially good claim to proportionality compared to ordinary grounds:

Truthmakers, like causes, should not be overladen with extra detail. [. . .] [Truthmakers] should [. . .] not incorporate irrelevant extras, in whose absence we'd still have a guarantee of truth. (Yablo 2014, 48)

There thus appears to be a kind of explanatory relevance that is captured by the new notion that is not captured by grounding.

¹⁷³ One might wonder if the flatness of the explanatory structure is not implausible. For instance, given that P fully explains $(P \wedge Q) \vee (P \wedge \neg Q)$, one might think that also $P \vee \neg P$ fully explains $((P \vee \neg P) \wedge Q) \vee ((P \vee \neg P) \wedge \neg Q)$. But as it stands, the proposal does not deliver this result. As a reviewer for the paper on which this chapter is based has pointed out, the present approach also has trouble handling the generalization to infinitary non-modal propositional logic, for it relies on the assumption that any formula with models has minimal models (i.e. minimal partial valuations): Consider a countably infinite set S of semantically independent atomic formulas $\{P_0, P_1, P_2, \dots\}$ and a formula INF that in effect says that S has infinitely many true members, e.g. an infinite disjunction of infinite conjunctions of each infinite subset of S . Then INF has models but no minimal models, since any model of INF can be reduced by dropping its assignment of a truth-value to one member of S . I leave to future research the questions of how forceful these objections are, and whether the proposal can be amended in such a way as to meet them.

Finally, our proposal captures intuitively appropriate explanatory proposals that otherwise would remain uncaptured; we should not forget that with respect to logical theorems, the proposal from reductive truthmaking is supposed to deliver the desired alternatives to grounding explanations. So, let us make explicit how explanation by reductive truthmaking indeed provides more satisfactory explanations of logical theorems than grounding explanation. As we have seen, it is not completely straightforward to spell out how in what respect the ordinary grounding explanations seem to be lacking. Yet, explanation by reductive truthmaking provides logical theorems with empty-base explanations with all their special explanatory features that have been mentioned above.

Here, recall once more the four desiderata for explanations of logical theorems identified in section 4.2: accounting for the status as necessary truths or logical theorems, modal stability, not giving rise to further (or just very few or not very pressing) why-questions, and compatibility with certain non-standard logics. Satisfaction of the first desideratum might be witnessed by the following reasoning: According to the proposal, logical theorems are explained in the empty set of facts. Necessarily, all facts in this set obtain. Under the assumption that explanation by reductive truthmaking transmits necessity, the necessity of logical theorems follows. Likewise, the explanation is modally stable: Whatever may be the case, logical theorems can be explained in the empty set of facts. Like every empty-base explanation, the explanatory proposal at hand does not involve reasons why its explanandum obtains and hence does not give rise to corresponding demands for further explanations. The empty-base proposal is moreover (given small adjustments) compatible with at least some logical settings in which $P \vee \neg P$ can be true without either of its disjuncts being true: In such a case, an ordinary grounding explanation is unavailable, but $P \vee \neg P$ can still be empty-base explained by reductive truthmaking. For example, in a supervaluationist setting, we can define minimal models as follows:

(Minimal model_{sv}^{*})

m is a minimal model of ϕ iff_{def.} m is a partial *supervaluation* of the language of ϕ such that all its *supplementations* verify ϕ and no proper *subsupervaluation* of m is such that all its *supplementations* verify ϕ .

(Supplementation_{sv})

A supplementation m^* of a partial *supervaluation* m of a language is a (full) *supervaluation* of the language such that m is a *subsupervaluation* of m^* .

Here, a (full) *supervaluation* is a set of (full) classical valuations and a partial *supervaluation* is a set of partial classical valuations. Moreover, we define that m

is a subsupervaluation of m^* iff every classical valuation in m is a subvaluation of a classical valuation in m^* . According to these definitions, $P \vee \neg P$ has an empty minimal model. In the supervaluationist setting, $P \vee \neg P$ can be (super-)true without either P or $\neg P$ being (super-)true. Because it has an empty minimal model, $P \vee \neg P$ is (reductively) made true by zero facts in this case as well.

Given these considerations, explanation by reductive truthmaking appears to be promising with respect to our goal of finding more satisfactory explanations of logical theorems. While I am inclined to treat the developed notion as distinct from grounding, we could (as mentioned in section 4.3) alternatively conceive of it as a special case of grounding and revise the logic of ground accordingly such that, for instance, a disjunction may be grounded via its disjuncts, or it may be grounded in propositions that express its reductive truthmakers.¹⁷⁴

To conclude my discussion of the empty-base explanation of logical theorems via reductive truthmaking, let me anticipate one objection: According to the proposal, some explanatory claims arise that, in a certain light, may seem problematic: For example, suppose that $\neg P$ and Q are the case. Then, according to the above proposal, it is the case that (1) Q explains why $P \rightarrow (P \wedge Q)$ and (2) that Q explains why $\neg P \vee (P \wedge Q)$ (and analogously for the corresponding because-claims). This can appear intuitively problematic: It can seem that in some sense for Q to explain why, e.g., $\neg P \vee (P \wedge Q)$, Q has to *ensure* that $\neg P \vee (P \wedge Q)$ is being the case. But one may wonder how Q can achieve this, if not together with P . Yet, as stipulated, P is not the case and thus Q cannot ensure that $\neg P \vee (P \wedge Q)$ is the case.¹⁷⁵

I propose to respond to this worry by taking a closer look at the notion of ensurance involved in the objection: Apparently, it is closely tied up with grounding, or perhaps it is indeed the notion of grounding. But then the objection appears to miss its mark: Presently, we are trying to find and characterize a different kind of explanatory relation that is distinct from grounding and hence must not assess the explanatory proposals it occurs in in the same way in which we assess grounding explanations. In response to the objection, we can then claim that the intuitive doubts arise because of an assessment of the explanatory proposals as grounding explanations, while in fact they are a different kind of explanation that does not involve grounding.

For this defense to be successful, we should be able to show that the explanatory proposals in question need not appear to be intuitively dubious. Here, talk of ensurance can actually help: While there is a sense of ‘ensurance’ in which the

¹⁷⁴ As mentioned in footnote 164, one rule that would have to be changed is the elimination rule for disjunction.

¹⁷⁵ An analogous problem arises for Yablo’s example $P \wedge Q \rightarrow P \wedge Q \wedge R$.

above ensurance-claims hold, there surely is another (not merely modal) sense in which Q alone *does* ensure that $\neg P \vee (P \wedge Q)$: After all, given Q , whether P or $\neg P$ turns out to be the case can appear, in a sense, irrelevant to whether $\neg P \vee (P \wedge Q)$ obtains or not – Q alone already does the job. From this point of view, the intuitive doubts should dissolve. Here, recall also Yablo from above:

Consider next a conditional $P \wedge Q \rightarrow P \wedge Q \wedge R$. It owes its truth, on the recursive conception, either to a fact that falsifies P , or a fact that falsifies Q , or a fact that verifies $P \wedge Q \wedge R$. Why not a fact that blocks the *combination* of $P \wedge Q$ true, $P \wedge Q \wedge R$ false, without pronouncing on the components taken separately? (Yablo 2014, 60)

The rhetorical question here invokes the intuition that there is indeed a sense of making true (or ensuring the truth) according to which a fact r (corresponding to R) makes true (ensures the truth of) $P \wedge Q \rightarrow P \wedge Q \wedge R$, even if $P \wedge Q$ is false. This is the sense we set out to capture above.

4.5 Explanation by essence and metaphysical law

The purpose of this section is to gain a better understanding of explanation by essence and metaphysical law and to see whether explaining logical theorems by their essential or metaphysically law-like status is viable. As we have seen in the previous chapter, explanatory proposals of the form ‘ P because it is part of the essence of . . . that P ’ and ‘ P because it is a law of metaphysics that P ’ face several worries, although they are not without proponents, such as Glazier (2017b), who proposes that they are not grounding explanations but a distinct kind of ‘essentialist explanation’. In this section I will explore another option, namely that explanations by essential status are at least sometimes to be understood as empty-base explanations whose link is a proposition that expresses the essential status of the explanandum (or a closely related proposition, see the relevant discussion on the form of links of empty-base explanations in chapter 2 and chapter 3). To begin, consider the following two proposals for the explanation of logical theorems:

(Metaphysical law)

It is a metaphysical law that $P \vee \neg P$. Metaphysical laws of unconditional form can serve as links of empty-base explanations and metaphysical laws of conditional form can serve as links of ordinary explanations.

(Essential conditionals)

It is some kind of essential fact that $P \vee \neg P$. Essential facts of unconditional form can serve as links of empty-base explanations and essential facts of conditional form can serve as links of ordinary explanations.¹⁷⁶

Using ‘■’ as a placeholder for the essence or metaphysical law operator again, these proposals amount to ordinary explanations of the following form:

Base: P

Link: $\blacksquare(P \rightarrow Q)$

Result: Q

Empty-base explanations of logical theorems would have the following form according to the two proposals:

Base: /

Link: $\blacksquare(P \vee \neg P)$

Result: $P \vee \neg P$

The next step in the development of this proposal is to provide a characterization of the corresponding explanatory relation (involving essence or metaphysical laws) that meets the following desiderata:

For the case of metaphysical laws, an account is needed according to which they are sufficiently distinct from grounding – namely such that there are explanations involving metaphysical laws as links that do not correspond to grounding explanations, otherwise the metaphysical law of the form ‘ $\blacksquare(P \vee \neg P)$ ’ would threaten to have a corresponding zero-grounding fact, which is the very thing that we set out to avoid. As far as I know, an account of this type of metaphysical law has not yet been given. One could attempt to characterize these laws negatively as those metaphysical laws which are not grounding laws, but this would at least require a sufficiently informative account of metaphysical laws. In what follows, I will instead focus on the notion of essence.

For the case of essence, one might think that certain essential conditionals can play the role of explanatory links, but this proposal would have to be properly developed; additionally, just as with metaphysical laws, the resulting explanatory notion would have to be sufficiently distinct from grounding. For example, Kment (2014, 164) can be understood as claiming that for every explanation e with a

¹⁷⁶ Again, if links of empty-base explanations are to have a slightly different form, namely that of a conditional with an empty antecedent, make the relevant substitutions here.

metaphysical law or essential conditional as link, there is a corresponding grounding fact that holds between the elements of the base and the result of e . Given this assumption, it would be plausible that a metaphysical law or essential truth of the form ‘ $\blacksquare(P \vee \neg P)$ ’ possesses a corresponding zero-grounding statement.¹⁷⁷

Here, one could also reason as follows: At least some essence conditionals appear to correspond to grounding facts. For example, consider the essence of the fact $[P \vee \neg P]$ or the alethic essence which we can express using the operator ‘ $\Box_{P \vee \neg P}$ ’.¹⁷⁸ One might now think that it is part of at least one of these essences that if P , then $P \vee \neg P$. But these essential conditionals correspond to the non-factive grounding fact $[P \Rightarrow (P \vee \neg P)]$. If there are essential conditionals that can serve as explanatory links but do not correspond to grounding facts in this fashion, the friend of explanation by essence should give an account of what distinguishes them from grounding-corresponding essential conditionals and give an account of the kind of explanation that they allow, in contrast to grounding explanation.

Importantly, this problem of corresponding grounding claims only arises if we want to find an explanatory notion that is truly distinct from grounding. If we on the other hand aim to characterize a range of special, extraordinary cases of (zero-)grounding, no such problem arises.

A further desideratum concerns the question of whether a kind of essence conditional can be found that is structurally adequate to serve as explanatory links. We may for example identify a certain kind of essential conditional as an explanatory link, but then this conditional might still fail to exhibit the structural features of explanatory links. For instance, it is presumably true for any proposition $[P]$ that it is part of the essence of $[P]$ that $P \rightarrow P$, i.e. $\Box_{[P]}(P \rightarrow P)$.¹⁷⁹ But if so, the essential conditional expressed by ‘ $\Box_{[P]}(\dots \rightarrow \dots)$ ’ is reflexive. This specific problem seems fixable by treating this essential conditional as a weak ‘explanatory’ notion which can be used to define a strict explanatory notion (in analogy to weak and strict ground, see Fine 2012).¹⁸⁰

But in general it is not clear that essential conditionals will play nice and exhibit the structural features of explanatory links: For example, there might be

177 Some discussion on principles connecting grounding and essence is available in the literature, see for example Correia (2013b) and Correia and Skiles (2019), but note that it is at least not obvious that these proposals indeed lead to the problem just described.

178 For the notion of alethic essence see Correia (2013b).

179 For simplicity’s sake this example uses individual essence, but other notions of essence can be considered as well.

180 A further option might be to invoke the distinction between constitutive and consequential essence, thanks to Jonas Werner here.

cases of symmetric essential dependence, e.g. a case of *a* and *b* such that first it is part of the essence of *a* that if *b* exists, then so does *a*, and second it is part of the essence of *b* that if *a* exists, then so does *b*. If there are such cases and all essential conditionals are explanatory links, it follows that the existence of *a* explains the existence of *b* and vice versa, violating the asymmetry of explanation.¹⁸¹

Let us now consider how these challenges could be met: First, given the unclear epistemology of essence, the widely varying theories about essential properties of things there are, and given that our intuitions concerning many essential claims are (if they exist at all) often weak or easily turned over, it is difficult to justifiably maintain strong opinions about essential facts. The issue is exacerbated by the variety of notions of essence – for example, for a given intuition about essence the question arises whether it should be cashed out as a claim concerning the individual essence (e.g. of a property) or as a generic essential claim.¹⁸² For our proposal, this predicament is both boon and bane. Bane, because we cannot sufficiently rely on our intuitions to simply check whether essential conditionals satisfy the formal properties of explanatory links. Boon, because it may allow us to treat the formal properties of explanatory links as a constraint on essential conditionals – at least to some extent. Finally, if it should nevertheless turn out that essential conditionals do not, so to speak, cooperate – or as long as we do not know whether they do – we might still be able to use them to hint at a related explanatory notion, just like we might do in the case of grounding. I will now sketch a way in which this might be done which relies on two ideas: First, essential conditionals are differently ‘localized’. Second, differently localized essential conditionals correspond to different explanatory notions.

In what follows, I suggest that we can take some steps towards satisfying the desiderata by differentiating essential conditionals that are differently ‘essentially localized’, in a sense to be clarified momentarily. The basic idea is this: Essential conditionals that correspond to grounding claims are localized in the essences of the corresponding groundees (we might call these *downwards essential conditionals*), but there may also be essential conditionals that are differently localized (e.g. *upwards essential conditionals*), that can serve as explanatory links of metaphysical explanations distinct from grounding (or at least serve to characterize such a notion).

It has been argued in the literature that there exists a close connection between essence and ground: While the details differ, it has emerged that in some sense, grounding facts correspond to facts that are essential to the corresponding

¹⁸¹ One way one could attempt to solve this specific problem is by claiming that the two essential conditionals are instances of different explanatory relations (one corresponding to the essence of *a* and the other corresponding to the essence of *b*) that are not in harmony.

¹⁸² For the distinction between individual and generic essence see Correia (2006).

groundee (or perhaps some of its constituents).¹⁸³ Thus, variants of the following simple link between ground and essence are proposed:

E1: For all Γ, p : $((\Gamma < p) \rightarrow \Box_{[p]}(\Gamma \rightarrow p))$

Here, the essence in question is the essence of the grounded fact, viz. the groundee. Variations on this idea could adduce the essence of a constituent of the groundee or the alethic essence of the groundee. As an example of an instance of the above principle, consider the grounding claim that the existence of Socrates grounds the existence of singleton Socrates. At least if we take this grounding claim for granted, it is quite plausible to assume that it is part of the essence of the proposition or fact that singleton Socrates exists that if Socrates exists, then singleton Socrates does exist as well. Similar and likewise plausible essences can be found if the proposals involve essences of constituents of the groundee or the groundee's alethic essence.

We might say that grounding claims give rise to corresponding essential conditionals which are 'localized' in or 'flow from' their groundees – meaning that it is part of the essence of the groundee (or constituents of the groundee) that the corresponding conditional obtains. Now the idea that I want to develop here is that we might be able to characterize other metaphysical explanatory relations by what essential conditionals they entail and how they are localized. Something stronger may be possible, namely that the different essential conditionals themselves are explanatory links, but note that even something weaker may do: To characterize an explanatory notion it may also already be sufficient (or at least useful) to know that corresponding explanatory links *normally*, *often*, or even just in special cases give rise to certain essential facts.¹⁸⁴

For example, the essential conditional entailed by such an explanatory relation might belong to the essence of the base-relatum or some of its constituents (let ' $< *$ ' stand for the explanatory notion that is to be characterized):

E2: For all Γ, p : $((\Gamma < * p) \rightarrow \Box_{[\Gamma]}(\Gamma \rightarrow p))$

Here, it is harder to give clear examples, but we can look at the case of colors and their determinates (which incidentally could be troublesome for E1): Consider the case of an object a 's being scarlet and its being red. The object's being scarlet

¹⁸³ For proposals like this and discussion thereof see e.g. Fine (2012, 74ff.), Correia (2013b), Correia and Skiles (2019), and Goff (2017, sec. 2.2.2).

¹⁸⁴ With 'characterization' I do not mean definition but rather a weaker notion of elucidation along the lines of hinting at, describing and explicating the notion such as to allow for a decent grasp of it.

metaphysically explains its being red, and ordinarily, this is taken to be a grounding explanation. But, as Fine (2012, 74ff.) in effect points out, there is some doubt that the essence obtains that is required if we assume a grounding-essence link like E1. According to E1, it would have to be part of the essence of the fact that the object is red, that if the object is scarlet, then it is red. But, using Fine's (2012, 75) idiom, that fact's (or alternatively redness's) essence does not 'know anything about scarlet'.¹⁸⁵ Nevertheless, it is plausible to assume that there is an essence that underlies the metaphysical explanatory relation at play here, namely the essence of being scarlet or the fact that the object *a* is scarlet. According to this proposal, it is part of the essence of the fact that the object *a* is scarlet that if the object is scarlet, then it is red.

To give a final option for a link between a metaphysical explanatory notion and essence, we could consider the collective essence of the base- and result-relata or their constituents (let ' $<_{**}$ ' stand for the explanatory notion to be characterized):

E3: For all Γ, p : $((\Gamma <_{**} p) \rightarrow \Box_{[\Gamma, p]}(\Gamma \rightarrow p))$

Again, considerations concerning color can be used to motivate this idea: As I have argued elsewhere (Kappes 2020b), we can maintain the idea that the essence of, for example, greenness does not mention blueness and yellowness, and at the same time assume that the three colors are essentially linked, namely by using the notion of collective essence. Given this idea, it is plausible to assume that is part of the collective essence of blueness, yellowness, and greenness that if something is the color of a subtractive mixture of blue and yellow, then it is green. But this essential conditional seems to correspond to an explanation of the thing's being green in terms of it having the color of a subtractive mixture of blue and yellow. Moreover, a variant of this idea might serve as a further example for upwards essence à la E2: Perhaps it is not the collective essence of blueness, yellowness, and greenness in which the relevant conditional is located, but it is rather part of the collective essence of blueness, yellowness, and subtractive color mixing that if something is the color of a subtractive mixture of blue and yellow, then it is green.

There are several further options that may come to mind: For example, the relevant conditionals might belong to the essence of constituents of base- or result-relatum, they might belong to the essence of facts in general or being the

¹⁸⁵ For responses to this that defend something like E1 see Correia (2013b) and Correia and Skiles (2019).

case in general, and another candidate is perhaps the essence of the explanatory notion to be elucidated (although this would perhaps threaten the elucidatory potential of the connection between essence and the explanatory notion that we are trying to get at).

The existence of explanatory relations characterized by E2 or E3 would have significant upshots for topics beyond the explanation of logical theorems. For example, take the following example from the philosophy of mind: According to a number of authors, phenomenal introspection provides us with some grasp of the essence of the relevant phenomenal properties. According to some (e.g. Goff 2017), this ‘revelation’ of essential properties of consciousness amounts to a challenge for physicalism and the metaphysical explanation of the mental in terms of the underlying physical reality. In a nutshell, Goff argues for a strong form of revelation according to which so much of the essence of phenomenal consciousness is revealed that were it grounded in underlying physical properties (or even Russellian (proto-)phenomenal for that matter), and given a suitable grounding-essence link, we would be able to discern these grounds in introspection. But since we are not able to do so, phenomenal consciousness is not so grounded.

Here, an explanatory relation characterized by E2 could in principle afford an alternative kind of metaphysical explanation of phenomenal consciousness in terms of some underlying physical (or Russellian (proto-)phenomenal) reality that is compatible with both a strong principle of revelation and what we actually learn from introspection: In this case we would not have to learn about essential conditionals’ connection to the underlying reality to facts about our phenomenal states through introspection. Rather, the relevant kind of metaphysical explanation would correspond to essential conditionals that flow from the essence of the underlying reality (rather than the essence of the explanandum constituted by our phenomenal states). For example, a physicalist could claim that it is part of the essence of certain fundamental physical entities, properties or processes that if they are present, then certain phenomenal states will be present. Likewise, a Russellian monist or panpsychist could claim that it is part of the essence of certain fundamental (proto-)phenomenal properties that if they are present, then certain of our ordinary phenomenal states will be present. Thereby they could even maintain their hope of being (in principle) able to close the explanatory gap: If we could only grasp the essences of the (proto-)phenomenal fundamental properties, we would understand how they give rise to our ordinary phenomenal states.

4.5.1 Considering downwards essence

Let us now come back to our original topic and consider how the above ideas may be applied to the topic of explaining logical theorems. Let us first think about whether the essences of the relevant logical operators or the essence of the relevant explananda can help characterize the explanatory notion that we are after. We will consider $[P \vee \neg P]$ as an example and ask part of which essence this fact plausibly is. Note first that it is somewhat intuitively plausible that $[P \vee \neg P]$ is indeed part of some essence or other and that if all necessary truths (i.e. truths expressed by sentences with a \Box -operator in front) are grounded in (or otherwise depend on) essential truths as Fine (1994) proposes, then there arguably has to be an essence part of which is the fact $[P \vee \neg P]$.¹⁸⁶ So, let us make this assumption and consider the following options:

1. It is part of the essence of the relevant logical operators (in our case presumably negation and disjunction) that $P \vee \neg P$.
2. It is part of the essence of the proposition or fact $[P \vee \neg P]$ that $P \vee \neg P$.
3. It is part of the alethic essence of $[P \vee \neg P]$ that $P \vee \neg P$.

If explanatory links of empty-base explanations can have an unconditional form like 1., 2., and 3., then these might be candidates for explanatory links of an empty-base explanation of $[P \vee \neg P]$, or they might serve to characterize such links. But recall the discussion of the conditional form of explanatory links in chapters 2 and 3: If the links of empty-base explanations have a conditional form with an empty antecedent, then we must consider other candidates for our desired explanatory links or their characterization (again, ' Γ ' stands for the empty plurality of facts):

1. It is part of the essence of the relevant logical operators (in our case presumably negation and disjunction) that $\Gamma \rightarrow (P \vee \neg P)$.
2. It is part of the essence of the proposition or fact $[P \vee \neg P]$ that $\Gamma \rightarrow (P \vee \neg P)$.
3. It is part of the alethic essence of $[P \vee \neg P]$ that $\Gamma \rightarrow (P \vee \neg P)$.

Let us comment on these proposals in turn: Concerning 1. and 4. we can observe that essential conditionals that concern the essence of logical operators arguably cannot (in general) serve as links of explanatory theorems, because they violate asymmetry: For example, it is plausibly part of the essence of conjunction that if

¹⁸⁶ In fact, matters may be less straightforward. $[\Box(P \vee \neg P)]$ could perhaps be zero-grounded in the empty set of essences. Or it could be grounded in other essential facts – but which would these be?

P and Q , then $P \wedge Q$. But then equally, it seems that it is part of the essence of conjunction that if $P \wedge Q$, then P and Q . Furthermore, it is unclear how the idea of hinting at a corresponding explanatory relation alluded to above should be spelled out in this case, but perhaps the idea might be salvaged by a complexity criterion that states that the explanatory relation in question runs from the less complex to the more complex formulae.¹⁸⁷

Concerning 5., the following problem arises: Suppose we assume that $[P \vee \neg P]$ is not zero-grounded and we assume the converse of the grounding-essence link E1 (substituting non-factive grounding for factive grounding in the consequent):

E1*: For all Γ, p : $(\Box_{[\Gamma]}(\Gamma \rightarrow p) \rightarrow (\Gamma \Rightarrow p))$

Then it follows that it is not part of the essence of $[P \vee \neg P]$ that $\Gamma \rightarrow (P \vee \neg P)$, where ' Γ ' stands for an empty plurality of facts. This is because from it being part of the essence of $[P \vee \neg P]$ that $\Gamma \rightarrow (P \vee \neg P)$, and principle E1* it would follow that $[P \vee \neg P]$ is non-factively zero-grounded (and hence factively as well). Therefore, if we make the above plausible assumptions about the relationship between essence and grounding, the fact that it is part of the essence of $[P \vee \neg P]$ that $\Gamma \rightarrow (P \vee \neg P)$ (that is fact 5.) cannot be the explanatory link (or help characterize the explanatory link) of a non-grounding empty-base explanation of $[P \vee \neg P]$.

While this argument does not go through like this for the remaining options 2., 3., and 6., I suspect that plausible alternatives to the grounding-essence links E1 and E1* can be found such that corresponding arguments can be formulated. In any case, the proposals are all uncomfortably close to the fifth proposal which is confronted with the problem that I have just laid out: All formulate essential claims that, given plausible assumptions about the relation between grounding and essence, either entail that $[P \vee \neg P]$ is zero-grounded (which we wanted to avoid), or come close to such essential claims.

4.5.2 Considering upwards essence and other localizations

So, what about the other options? Above I suggested the idea that an explanatory connection can, as happens in the case of grounding, be 'localized in' or 'flow from' the essence of the explanandum (or something involved therein). This idea was exemplified by E1. Alternatively, other explanatory connections might be thought to flow from the essence of the explanans (or something involved therein), or even

¹⁸⁷ Thanks to Jonas Werner here.

from some other essence that we have not considered yet. So perhaps the essence of facts in general or being the case in general (as quickly mentioned above), or perhaps the essence of the proposition $[P]$ could be candidates here, but I rather want to address the idea that the explanatory connection can flow from the essence of the explanans, as exemplified by E2.

Now for our case, if we want to stick with the idea that logical theorems are empty-base explained, the idea of the explanatory connection flowing from the essence of the explanans runs into an obvious problem: There are no explanans-propositions or -facts from whose essence (or from whose corresponding alethic essence) the relevant explanatory links could flow. Thus, this idea appears to be a non-starter. That is, unless we consider – well – the essence of the *empty* plurality of propositions or facts, or the alethic essence of the propositions that make up the *empty* plurality of propositions. For example, for the case of $[P \vee \neg P]$, one way to spell this idea out would be the claim that it is part of the essence of the empty plurality of propositions that if the empty plurality of propositions obtains, then $P \vee \neg P$. More precisely, the claim would be: $\Box_{[I]}(I \rightarrow (P \vee \neg P))$, where ' I ' stands for the empty plurality of propositions. This essential conditional could then be used to characterize the explanatory connection between the empty plurality of propositions and $[P \vee \neg P]$ as suggested above.

Are there any obvious problems for this idea? Well, setting aside the question of whether idea of the essence of the empty plurality of propositions or facts is indeed sensible, there is the worry that essences in some sense have to be relevant to what they are essences of. But why should essences involving arbitrary disjunctions of the form ' $P \vee \neg P$ ' be relevant to the empty set of propositions? Moreover, it is natural to assume that the proposal can be generalized to a non-empty plurality $[I]$, which would explain $[(I \wedge P) \vee (I \wedge \neg P)]$, while the link from $[I]$ to $[(I \wedge P) \vee (I \wedge \neg P)]$ flows from the essence of $[I]$. But in that case, the essence of the arbitrary truths $[I]$ would involve the arbitrarily chosen proposition $[P]$ – the essence of every truth would then in some way involve every proposition, which could be considered to be somewhat in tension with the relevance idea connected to essence.

What could, on the other hand, be said in favor of this idea? Well, first, it is not obvious that the problem runs into worries regarding asymmetry. Second, it captures somewhat neatly both the idea that logical theorems are empty-base explained, but that this is not a case of grounding, and the idea that the kind of explanation in play flows from the essence of the explanans as opposed to flowing from the essence of the explanandum (that the explanation in question flows from the essence of the explanans is supposed to mean either that the essence of the explanans contains the explanatory links in question, or that it can be used to hint at those links, as sketched above). Of course, it would be nice if we could,

thirdly, claim some intuitive plausibility of the proposal (beyond its capturing the idea that logical theorems are empty-base explained), but at least my intuitions are either silent or somewhat divided on the matter.

4.6 Remaining options and conclusion

Are there any other candidates for explanatory notions that could constitute links of explanations of logical theorems besides those discussed above? To mind come perhaps some varieties of grounding that have been suggested in the literature. Here, one might think about Fine's (2012) varieties of grounding: Perhaps one could think about differentiating logical grounding as defined in Fine (2017a, 2017b) from metaphysical grounding and suggest that the ordinary grounding explanations of logical theorems concern their logical grounds, while other grounds (perhaps zero-ground) are available when metaphysical grounding is considered. Alas, one obstacle to this is that Fine's varieties of grounding have not yet been particularly well clarified.

Another suggestion would be to take a cue from a recent proposal by Jason Turner (2017), according to which we should differentiate two kinds of grounding, namely metaphysical specification and metaphysical causation. The thought would be that the ordinary grounding explanations of logical theorems involve grounding-as-metaphysical-specification, while grounding-as-metaphysical-causation can afford the more satisfactory explanations that we are looking for. So, the question is whether $[P \vee \neg P]$ (as our placeholder for logical theorems in general) might be metaphysically caused by something in addition to being metaphysically specified by its true disjunct.

Several *prima facie* issues arise to confront an affirmative answer: First, ordinary causation works together with grounding (understood as metaphysical realization) to explain disjunctions via their true disjuncts. Thus, at least in this respect, metaphysical causation would have to differ from ordinary causation. Second, something fit to metaphysically cause the disjunction would have to be found. Perhaps facts expressing the special status of the disjunction could play the role, but this potentially runs into the problems from chapter 3. So alternatively, maybe grounding-as-metaphysical causation allows for a corresponding kind of zero-ground as well? I will not attempt to answer this question, but rather use it to transition to the next chapter: *If* grounding-as-metaphysical causation allows for its own kind of zero-grounding and *if* metaphysical causation and causation share many of their features, should we admit the possibility of a phenomenon that we might dub *zero-causation* (or *causation ex nihilo*)? Approaching this latter question is part of what will occupy us in the following.

Let us end this chapter by drawing a little on what I will eventually say in the next chapter. There is an interesting connection between the idea floated at the end of section 4.5 and the idea of logical theorems being empty-base explained via metaphysical causation that I have just considered: In section 5.2.2, I will use the idea that instances of causation involve causal powers of constituents of the associated causes. This raises the question whether metaphysical causation stands in a similar relation to an analogue of causal powers. Here, a conjecture that may be worth thinking about further is the following: Analogous to causal powers for metaphysical causation are essences of the corresponding metaphysical causes (or essences of these causes). But given this conjecture, metaphysical causation starts to look like the explanatory relation that I have tried to characterize at the end of the last section, namely like a kind of explanation whose explanatory links are part of (or at least closely related to a part of) the essence of the corresponding explanatory base or sources.