Contents

Preface —— VII

1 Basic principles of tomography —— 1
1.1 Tomography —— **1**
1.2 Projection —— **3**
1.3 Image reconstruction —— **6**
1.4 Backprojection —— **8**
1.5 Mathematical expressions —— **9**
1.5.1 Projection —— **9**
1.5.2 Backprojection —— **10**
1.5.3 The Dirac δ-function —— **12**
1.6 Worked examples —— **14**
1.7 Summary —— **18**
Problems —— **18**
Bibliography —— **19**

2 Parallel-beam image reconstruction —— 20
2.1 Fourier transform —— **20**
2.2 Central slice theorem —— **21**
2.3 Reconstruction algorithms —— **23**
2.3.1 Method 1 —— **24**
2.3.2 Method 2 —— **24**
2.3.3 Method 3 —— **26**
2.3.4 Method 4 —— **26**
2.3.5 Method 5 —— **26**
2.3.6 Method 6 —— **27**
2.4 A computer simulation —— **28**
2.5 ROI reconstruction with truncated projections —— **29**
2.6 Mathematical expressions —— **34**
2.6.1 The Fourier transform and convolution —— **34**
2.6.2 The Hilbert transform and the finite Hilbert transform —— **35**
2.6.3 Proof of the central slice theorem —— **37**
2.6.4 Derivation of the FBP algorithm —— **38**
2.6.5 Expression of the convolution backprojection algorithm —— **39**
2.6.6 Expression of the Radon inversion formula —— **40**
2.6.7 Derivation of the backprojection-then-filtering algorithm —— **40**
2.6.8 Expression of the derivative–backprojection–Hilbert transform algorithm —— **41**
2.6.9 Derivation of the backprojection–derivative–Hilbert transform algorithm —— **42**

2.7 Worked examples — **42**
2.8 Summary — **49**
Problems — **50**
Bibliography — **50**

3 Fan-beam image reconstruction — 52
3.1 Fan-beam geometry and the point spread function — **52**
3.2 Parallel-beam to fan-beam algorithm conversion — **55**
3.3 Short scan — **57**
3.4 Mathematical expressions — **59**
3.4.1 Derivation of a filtered backprojection fan-beam algorithm — **60**
3.4.2 A fan-beam algorithm using the derivative and the Hilbert transform — **61**
3.4.3 Expression for the Parker weights — **63**
3.4.4 Errors caused by finite bandwidth implementation — **64**
3.5 Worked examples — **65**
3.6 Summary — **68**
Problems — **69**
Bibliography — **69**

4 Transmission and emission tomography — 71
4.1 X-ray computed tomography — **71**
4.2 Positron emission tomography and single-photon emission computed tomography — **75**
4.3 Noise propagation in reconstruction — **78**
4.3.1 Noise variance of emission data — **78**
4.3.2 Noise variance of transmission data — **79**
4.3.3 Noise propagation in an FBP algorithm — **79**
4.4 Attenuation correction for emission tomography — **80**
4.4.1 PET — **80**
4.4.2 SPECT: Tretiak–Metz FBP algorithm for uniform attenuation — **81**
4.4.3 SPECT: Inouye's algorithm for uniform attenuation — **83**
4.5 Mathematical expressions — **85**
4.5.1 Expression for Tretiak–Metz FBP algorithm — **85**
4.5.2 Derivation for Inouye's algorithm — **85**
4.5.3 Rullgård's derivative-then-backprojection algorithm for uniform attenuation — **87**
4.5.4 Novikov–Natterer FBP algorithm for nonuniform attenuation SPECT — **88**
4.6 Worked examples — **89**

4.7 Summary — **91**
Problems — **91**
Bibliography — **92**

5 Three-dimensional image reconstruction — 94
5.1 Parallel line-integral data — **94**
5.1.1 Backprojection-then-filtering — **96**
5.1.2 Filtered backprojection — **98**
5.2 Parallel plane-integral data — **98**
5.3 Cone-beam data — **100**
5.3.1 Feldkamp's algorithm — **101**
5.3.2 Grangeat's algorithm — **102**
5.3.3 Katsevich's algorithm — **104**
5.4 Mathematical expressions — **108**
5.4.1 Backprojection-then-filtering for parallel line-integral data — **108**
5.4.2 FBP algorithm for parallel line-integral data — **109**
5.4.3 Three-dimensional Radon inversion formula (FBP algorithm) — **112**
5.4.4 Three-dimensional backprojection-then-filtering algorithm for Radon data — **113**
5.4.5 Feldkamp's algorithm — **113**
5.4.6 Tuy's relationship — **114**
5.4.7 Grangeat's relationship — **117**
5.4.8 Katsevich's algorithm — **119**
5.5 Radon transform and ray transform in n dimensions — **125**
5.6 Worked examples — **126**
5.7 Summary — **129**
Problems — **130**
Bibliography — **130**

6 Iterative reconstruction — 133
6.1 Solving a system of linear equations — **133**
6.2 Algebraic reconstruction technique — **138**
6.3 Gradient descent algorithms — **139**
6.3.1 The gradient descent algorithm — **139**
6.3.2 The Landweber algorithm — **141**
6.3.3 The conjugate gradient algorithm — **142**
6.4 ML-EM algorithms — **143**
6.5 OS-EM algorithm — **144**
6.6 Noise handling — **144**
6.6.1 Analytical methods: windowing — **145**
6.6.2 Iterative methods: stopping early — **145**
6.6.3 Iterative methods: choosing pixels — **146**

6.6.4 Iterative methods: accurate modeling — **148**
6.7 Noise modeling as a likelihood function — **149**
6.8 Including prior knowledge (Bayesian) — **152**
6.9 Mathematical expressions — **153**
6.9.1 ART — **153**
6.9.2 The Landweber algorithm — **154**
6.9.3 CG algorithm — **155**
6.9.4 ML-EM — **157**
6.9.5 OS-EM — **159**
6.9.6 MAP (Green's one-step-late algorithm) — **160**
6.9.7 Matched and unmatched projector/backprojector pairs — **162**
6.10 Reconstruction using highly undersampled data — **164**
6.11 Worked examples — **167**
6.12 Summary — **179**
Problems — **180**
Bibliography — **182**

7 MRI reconstruction — 185
7.1 The "M" — **185**
7.2 The "R" — **187**
7.3 The "I" — **190**
7.3.1 To obtain *z*-information: slice selection — **190**
7.3.2 To obtain *x*-information: frequency encoding — **191**
7.3.3 To obtain *y*-information: phase encoding — **192**
7.4 Mathematical expressions — **195**
7.5 Image reconstruction for MRI — **198**
7.5.1 Fourier reconstruction — **198**
7.5.2 Iterative reconstruction — **199**
7.6 Worked examples — **200**
7.7 Summary — **201**
Problems — **202**
Bibliography — **202**

8 Using FBP to perform iterative reconstruction — 204
8.1 The Landweber algorithm: from recursive form to nonrecursive form — **204**
8.2 The Landweber algorithm: from nonrecursive form to closed form — **205**
8.3 The Landweber algorithm: from closed form to backprojection-then-filtering algorithm — **206**
8.3.1 Implementation of $(A^T A)^{-1}$ in the Fourier domain — **206**

8.3.2 Implementation of $I - (I - \alpha A^T A)^k$ in the Fourier domain — **207**
8.3.3 Landweber algorithm: backprojection-then-filtering algorithm — **207**
8.3.4 Numerical examples of the window function — **207**
8.4 The Landweber algorithm: the weighted FBP algorithm — **209**
8.4.1 Landweber algorithm: FBP without noise weighting — **209**
8.4.2 Landweber algorithm: FBP with view-based noise weighting — **210**
8.4.3 Landweber algorithm: FBP with ray-based noise weighting — **211**
8.5 FBP algorithm with quadratic constraints — **213**
8.5.1 Example of minimum-norm constrained FBP — **214**
8.5.2 Example of reference-image constrained FBP — **216**
8.6 Convolution backprojection — **217**
8.7 Nonquadratic constraints — **220**
8.8 A viewpoint from calculus of variations — **223**
8.9 Noise-weighted FBP algorithm for uniformly attenuated SPECT projections — **226**
8.10 Other applications of the Landwever-FBP algorithm — **227**
8.10.1 The Landweber-FBP version of the iterative FBP algorithm — **227**
8.10.2 Estimation of the initial image's contributions to the iterative Landweber reconstruction — **228**
8.10.3 FBP implementation of the immediately after-backprojection filtering — **229**
8.10.4 Real-time selection of iteration number — **230**
8.11 Summary — **230**
Problems — **231**
Bibliography — **231**

9 Machine learning — 233
9.1 Analytic algorithms versus iterative algorithms versus machine learning algorithms — **233**
9.2 Basic principles of machine learning — **234**
9.2.1 Adaptive linear neuron (ADALINE) — **235**
9.2.2 Universal approximation — **236**
9.2.3 A deep model for a continuous univariate function — **242**
9.2.4 When the input is an *n*-tuple — **245**
9.2.5 An *n*-tuple to *m*-tuple mapping — **246**
9.3 Image reconstruction as an *n*-tuple to *m*-tuple mapping — **249**
9.4 Convolutional neural network (CNN) as a matched filter for identification — **250**
9.5 Convolutional neural network (CNN) as a nonlinear denoise filter — **255**
9.6 Super-resolution — **258**
9.7 Training — **260**

9.8 Attention — **261**
9.9 Perspectives — **262**
9.10 Summary — **265**
Problem — **266**
Bibliography — **267**

Index — 269