Foundations of Communication
Library Edition

Editor
Roland Posner

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From Axiom to Dialogue

A philosophical study of logics and argumentation

Walter de Gruyter · Berlin · New York
1982

Library of Congress Cataloging in Publication Data

Barth, E. M.
From axiom to dialogue.
(Foundations of communication)
Bibliography: p.
Includes indexes.
1. Logic. I. Krabbe, E. C. W. (Erik C. W.), 1943–
II. Title. III. Series.
BC61.B37 1982 160 82-16223
ISBN 3-11-008489-9 (lib. bdg.)

CIP-Kurztitelaufnahme der Deutschen Bibliothek

Barth, Else M.:
From axiom to dialogue : a philosoph. study of logics and argumentation / E. M. Barth ; E. C. W. Krabbe. – Berlin ; New York : de Gruyter, 1982.
(Foundations of communication : library ed.)
ISBN 3-11-008489-9
NE: Krabbe, Erik C. W.:

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Satz und Druck: Arthur Collignon GmbH, Berlin.
Buchbinder: Lüderitz & Bauer, Berlin.

To the memory of Evert Willem Beth

Preface

Most books on modern logic, whether textbooks or otherwise, fall into one of two classes. The first is that large group of texts aimed exclusively at readers whose main interest lie in the foundations of mathematics, and who maintain that all other problems concerning logics and their uses are derivative, and should be treated, if at all, as secondary. The other class contains elementary textbooks that are obviously written for persons who have not yet acquired even the beginnings of philosophical, historical or scientific sophistication, or who are at least willing to forget about it for the period of their instruction in logic.

It seemed to us that yet a third approach to the relationship between logic, philosophy and our culture at large is possible and valuable, and in fact that a new attitude is long overdue. The present book is therefore written specifically with the following audience in mind: philosophers and students of general philosophy, as well as scholars and students in neighboring fields such as linguistics, physics, communication, political science, law, sociology, economics, cognitive psychology, theology, and the humanities. We hope that it will in some measure satisfy the basic needs for information about and instruction in logic, as felt by persons from these fields of contemplation and research, and that it will show or suggest to them something of the relationship between logic and their own academic or practical pursuits.

The book is not really meant for students of mathematics, although we hope that some of them, too, will be interested in parts of it. They have their own books, containing considerable material that we do not even mention in passing here. On the other hand, we believe that it would be a mistake to try to avoid "mathematical" symbolism entirely, especially as this may help to eliminate misinterpretations (admittedly after an initial period of learner's confusion). Nor should one abstain from using mathematical methods, wherever there are potential readers – those who are mathematically trained – who will demand such a treatment before they are able to agree that our conclusions are sound. It is not, however, merely to obtain this agreement, important though it is, nor is it merely to conform to academic fashion, that we have made some use of mathematical methods. We consider that everyone, from engineers to theologians (not necessarily very different people, by the way), should be shown with what care modern logical theories are formulated and evaluated. The metalogical problem of the "completeness" (relative to something else) of a logic system is therefore discussed throughout. Philosophical readers, on the other hand, should be taught this without being led to assume that mathematics is in all respects metaphysically fundamental and verbal intercourse in all respects derivative. We have tried consistently to treat the verb "to prove" as, basically, a ternary relative verb: someone proves (or fails to prove) something *to someone*. To drop consistently the words "to someone", even in basic uses of the verb

"to prove" – and this is still a habit among logicians – is to enshroud the topic in mystery and invite accusations of "elitism" or "authoritarianism" which can in fact be avoided.

A second aim of our book is to contribute to the development of a theory of rational and critical argumentation by offering a complete formulation of *Lorenzen's dialogue method* in "formal" logic, with forays into "material" logic in the form of material dialogues; this we hope to have brought within the reach of non-mathematicians.

Third, we intend to show – for propositional logics – the *equivalence of the various contemporary approaches to elementary formal logic*: the dialogue method and dialogical tableaux, axiomatic derivations, natural deductions and deductive tableaux, model theory and semantic tableaux. This is an extension of the programme of: E. W. Beth, *Formal Methods* (1962), where the dialogue method is missing (for chronological reasons), and where natural deduction is not well distinguished from the method of deductive tableaux. In *Formal Methods*, Beth spoke of the unity of the *three* methods. His book, moreover, was hastily written and as a result it has never been much read, despite its contents, which were eminently suited (at that time, 1962) to the purpose of explaining logic, and what logic is, to the general philosopher. We believe that no text is available in any language that incorporates all these approaches and demonstrates their "unity". This degree of comprehensiveness explains the length of our book.

We shall show this (i.e., the equivalence of several different approaches) for three logics, namely "classical" (two-valued) logic, constructive (or, intuitionistic) logic and minimal logic; and this we shall do for *three different language forms*, viz. for purely implicational languages and for languages with, as well as for languages without, a "rigid" sentential descriptor for falsity or absurdity.

Our aims are not only didactic in the narrower sense. We should like to think that the following features of the book have a certain originality:

– Sections on the philosophy of logic and on the historical development of the general outlook on what logic is all about (Chapters I, II, XII, and interwoven with other topics at numerous places);

– A normative foundation for Lorenzen's dialogue method, and thereby for elementary logics in any style, presented in a way suited to the goal of establishing a social contract about verbal behavior (Chapter III);

– A conceptual basis in, among other things, the notion of a *conflict of avowed opinions*, bringing to the fore the thesis that logical (dialectical) activity has *conflict resolution* as its intrinsic goal;

– Dialectical rules for minimal logic, with and without rigid sentential descriptors for falsity (Chapter IV);

– Particularly simple algorithms for transformation of (closed) dialogical strategy tableaux into (closed) deductive tableaux, which are clearly distinguished from the former; and also for the transformation of (closed) deductive tableaux into natural deductions (Chapter VII); also for the transformation of (closed) classical, constructive and minimal semantic tableaux into winning strategies (Chapter XI);

– Discussion of the ways in which material (e.g., ostensive) methods and moves can be incorporated into dialectic systems (Chapter IV);

– Reinterpretation of Kripke's constructive and of Kripke-styled minimal epistemic models of growth of knowledge into *dialectical models of growth of agreement*; in other words, *the models* for the various dialectic systems – classical, constructive, minimal – *are taken from the field of argumentational activity itself* (Chapter IX);

– Completeness/Soundness proofs for the dialogical tableau method in formal logic *via* the deductive and semantic tableau methods, together with dialectical models. Thereby we have circumvented Gentzen's cut-elimination theorem and its analogues, which are conceptually inaccessible to the majority of general philosophers.

More important still, these new philosophical models are of immediate interest in themselves, offering a new perspective for logic and general philosophy. The budding Theory of Argumentation may, given these models, be regarded as a self-contained field and a self-contained philosophical programme, i.e., as a philosophical and a practical pursuit which is independent of remnants of classical, "ontology"-based, epistemology. *Social (dialectical) models are offered corresponding to dialectic systems* (as shown in Chapter XI).

Perspectives such as these are sketched briefly in Chapter XII, where we discuss the possibility that "ontology", even in its analytic form, was constructed as an ("the") absolute theoretical state – a notion without which processes of growth of agreement could not formerly be conceived, and without which structures of models for such processes could not be formulated. Two-valuedness belongs with ontology construed as "the" final or absolute state.

In order to present all this material to as many philosophers as possible, and in a single volume, *we have refrained from a discussion of the treatment of quantifiers or of modal operators.* The book contains examples and exercises, and is also intended for use as a textbook. It has become almost standard behavior to announce in logic texts that "no previous knowledge of logic is required"; and there have been some bold attempts at teaching the theories of Gödel within the first academic year. We do not join this chorus. An extensive introductory course is, for practically everybody, a prerequisite for the second and third parts, but not for the first part (and perhaps not for Chapter XII), which can be read – and in large measure understood – by anyone with some philosophical experience. In our courses for philosophy students (which of course also contained other material) we have found that material such as that in Part One can be profitably treated in the second year of the curriculum (at university level), and the second and third parts in the third year. Courses based on this book can be envisaged which do not follow the order of presentation of topics that we have chosen here. Thus a course may very well start with natural deduction (Chapter VI), or with dialectical models (Chapter IX).

The cultural and practical importance of logic (logics) is often mentioned but rarely demonstrated. It is usual to find the whole subject, including its history, treated in isolation from practically all philosophy, to say nothing of (other) public affairs. We wanted to write a text which would meet a number of culturally hot philosophical topics head-on. (What are we to understand by dialec-

tics? Are formal logics reactionary?, and so on.) This may sound pretentious, but someone has to break the ice surrounding modern logic, and to run the risks that this implies.

We have been much stimulated by the response of our students in Utrecht and, later, in Groningen. We are grateful to them as well as to many others – to colleagues and students in the social sciences, for example – for encouragement and tokens of interest in the connections and perspectives discussed here. We received helpful criticism or other valuable comment from F. H. van Eemeren, R. Grootendorst, J. A. Hoeben, M. Henket, T. Kruiger, K. Lorenz, J. D. North, V. Sánchez Valencia, F. H. H. Schaeffer, E. Thijssen, F. J. M. M. Veltman and R. de Vrijer. Their comments led to a number of improvements in the text.

Warm thanks are due to those who undertook the arduous task of typing our manuscript. Large parts of the text were meticulously handled by F. A. Mulder at the Centrale Interfaculteit, Utrecht, and by C. A. M. Roy. F. C. van der Louw and P. M. Vooijs-Treffers did their utmost to help complete the typing as we became pressed for time. We wish to thank C. A. M. Roy and P. van Ulsen for their assistance in proofreading. The book would not have contained any indexes but for the efforts of P. van Ulsen.

Our especial gratitude is to C. A. M. Roy, of the University of St. Andrews, who also improved our English. Being a trained psycho-linguist she understood what she read, offered enlightened comments on grammar, idiom and style, and with the greatest of tact even pointed out inconsistencies in our presentation.

There are others whom we do not mention by name but to whom we are grateful for their constant support.

Groningen and Utrecht
February 1981/March 1982

E. M. Barth and E. C. W. Krabbe

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definitions, rules, lemmas, theorems and important terms

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A List of Variables

	A value of the variable	*is a (n)*
problem situation	γ	company, dialectical subject
	π	logico-intellectual language problem discerned by a company γ
older philsophical notions, analysis of –	d	dialectical situation of a company/dialectical subject γ
	D	set of dialectical situations d
	R	development relation, to be defined on D
	R^{as}	asymmetric relation obtained from R
	ω_R, $\omega_{R^{as}}$	conceptual result of applying Aristotle's Principle of the Absolute to R (to R^{as})
	T	type of sentence tag for indicating *objective truth* [ontological value, truth value]
	F	type of sentence tag for indicating *objective falsity* [ontological value, truth value]
	$\mathfrak{S}$ [see below]	ontological structure ⟨**T**, **F**⟩
	σ	language system or principle, (system of) rule(s) for uses of language; – considered by γ in pursuit of/ as/ a solution to π
purely descriptive languages	$\mathfrak{L}$	language (lexicon – here a set of atomic sentences –, logical constants, and syntactic rules), purely descriptive (declarative) –
	Λ	veljunction of all absurd/decidedly false sentences
	$\mathfrak{L}^{\Lambda}$	language containing at least one absurd/decidedly false sentence
	T, . . ., Z	sentence

classical (ontological) semantics	I [see below]	classical (ontological) interpretation of a language $\mathfrak{L}$ (here, function from atomic sentences to the elements of {**T**, **F**})
	M [see below]	classical (ontological) model $\langle \mathfrak{S}, I \rangle = \langle \mathbf{T}, \mathbf{F}, I \rangle$ with respect to a language $\mathfrak{L}$
	v_M [see below]	classical valuation (function) of (the sentences of) a language $\mathfrak{L}$, induced by a classical (ontological) model M
formal$_3$ dialectics	N	dialectical role, e.g., Proponent or Opponent
	$\mathfrak{L}_D$, $\mathfrak{L}_D^{\wedge}$	dialectically augmented language
	$T, \ldots, Z$	statement
	T, T_1, T_n	thesis, initial thesis, nth local thesis
	Con, Con_1, Con_n	set of concessions, set of initial concessions, nth set of local concessions
	s	stage in a dialogue/discussion
	S	dialogue situation (constellation of rights, obligations and attitudes, at the beginning of a chain of arguments or at the completion of some stage s)
	aU	verbal attack on U
	dU	move in defense of U against the attack aU
	ca	counteractive/indirect defense (counter-attack)
	pU	protective/direct defense of U
	a_iU	sentence used in an attack on U of the i'th kind
	$p_{ij}U$	sentence used in a protective defense of U of the j'th kind, against an attack on U of the i'th kind
	$\mathbb{T}$ $\mathbb{T}_0$	set of {implicitly / explicitly} accepted sentences (based on some material truth procedure adopted by γ)
	$\mathbb{F}$ $\mathbb{F}_0$	set of {implicitly / explicitly} rejected sentences (based on some material falsity procedure adopted by γ)

dialectical semantics	**A**	type of sentence tag for indicating *positive agreement* on a sentence (in some dialectical situation d and by some dialectical subject γ) [dialectical value]
	N	type of sentence tag for indicating *lack of positive agreement* on a sentence [dialectical value]
	Abs	set of absurd dialectical situations
	$\mathcal{S}$	dialectical structure, normal: $\langle \mathbf{A}, \mathbf{N}, D, R\rangle$, or minimal: $\langle \mathbf{A}, \mathbf{N}, D, Abs, R\rangle$
	I	interpretation of a language $\mathfrak{L}$ or $\mathfrak{L}^{\Lambda}$ on a dialectical structure (here, function from atomic sentences and dialectical situations to the elements of $\{\mathbf{A}, \mathbf{N}\}$)
	M	dialectical model $\langle \mathcal{S}, I\rangle = \langle \mathbf{A}, \mathbf{N}, D, R, I\rangle$ (or = $\langle \mathbf{A}, \mathbf{N}, D, Abs, R, I\rangle$) of a language $\mathfrak{L}$ or $\mathfrak{L}^{\Lambda}$
	v_M	constructive or minimal valuation (function) of (the sentences of) a language $\mathfrak{L}$ or $\mathfrak{L}^{\Lambda}$, induced by a dialectical model M
	τ	tableau, tree diagram
Beth-tableaux	Π [see below]	set of sentences: premises in a deduction problem, or sentences to be given the value **T** (or the value **A**) in a solution of a valuation problem
	Γ [see below]	set of sentences: those which in a solution of a valuation problem are to be given the value **F** (or the value **N**)
	Z	concludendum (a sentence)
	S [see below]	sequent Π/Z or Π/Γ – characterizes a deduction/valuation problem; partial description of a dialectical situation d
	Σ	set of sequents; partial description of a dialectical model

dialogical strategy tableaux

Π	set of sentences: those stated as concessions
Δ	set of sentences: those representing structural protective defense rights on the Opponent's side
Φ	set containing at most one sentence, viz., the one uttered by the Proponent in the preceding stage
Γ	set of sentences: those representing structural protective defense rights on the Proponent's side
$\vec{\Pi}, \vec{\Delta}, \vec{\Phi}, \vec{\Gamma}$	sequences of sentences (compare Π, Δ, Φ, Γ)
T	sentence stated in the local thesis *T* set of sentences, viz., the set of *all* sentences that are stated in local theses up to a certain stage
S	dialogue sequent – characterizes a strategy problem; partial description of a dialogue situation
	a. (non-material) $\Pi; \Delta/\mathrm{T}/_{\mathrm{N}} \Phi; \Gamma$
	b. (material, MatDial) $\vec{\Pi}; \vec{\Delta}/\mathbb{T}/_{\mathrm{N}} \vec{\Phi}; \vec{\Gamma}$