

# Index

- $E_8$ 
  - lattice 235, 271–276, 278, 279, 282
  - sphere packing 235, 271
- $n$ th root function 60
- analytic continuation 43
- analytic function 12
- anti-derivative 26
- arc length integral 24
- argument principle 53
- Arzelá–Ascoli theorem 133, 134
- automorphism 121, 122
  - group 120–122
  - of the complex plane 122
  - of the Riemann sphere 123
  - of the unit disc 126–129
  - of the upper half plane 129–131
- Bernoulli numbers 74, 90, 97
- Bessel functions 74, 287
- beta function 110, 111
- biholomorphism 118
- Casorati–Weierstrass theorem 52, 122
- Cauchy inequalities 40
- Cauchy–Riemann equations 14, 15, 18, 19, 72, 73
- Cauchy’s integral formula 36, 38, 40, 41
- Cauchy’s theorem 28, 31–36, 44, 45, 76
- Chebyshev function  $\psi(x)$  100–104, 108, 109, 117
- Cohn–Elkies sphere packing bounds 276
- complex torus 172, 173, 178, 190, 191, 209
  - classification 173, 190, 191, 209
- conformal
  - automorphism 121, 122
  - equivalence 118, 121
  - map 7, 16–18, 118, 121, 165, 168, 208
- congruence subgroup  $\Gamma(2)$  228, 229
- contour integral 22–28
  - fundamental theorem of calculus 26
- convolution 112, 113
- cotangent function
  - partial fraction expansion 65, 79, 194, 223
- derivative 13
  - logarithmic 53
- digamma function 111, 112, 114
- Dirichlet eta function 113
- Dirichlet series 115
- discriminant
  - of a cubic 168–170
  - of a lattice 170
- Eisenstein series 158–161, 179, 180, 182, 194, 197, 198, 211–214, 216, 219, 221, 223, 224, 230, 231
  - Fourier expansion 194
  - recurrence relation 160
  - weight 2 219, 221, 223, 224, 230, 231
- elliptic curve 146–149, 178
- elliptic invariants 159
- essential singularity 52
- Euler gamma function 83–89, 110, 111, 113, 257, 269
  - duplication formula 110
  - functional equation 85, 86, 88
  - infinite product formula 84, 88
  - multiplication theorem 110
  - reflection formula 85
- Euler pentagonal number theorem 232
- Euler totient function 115
- Euler–Mascheroni constant 84, 111, 113, 114
- Fourier transform 93, 270, 281, 286
  - radial 281, 286, 287
- function
  - analytic 12
  - doubly periodic 149, 217
  - elliptic 149
  - entire 13
  - harmonic 19, 38
  - holomorphic 12
  - inverse 61
  - locally injective 61
  - magic 278
  - meromorphic 50
  - radial 279
  - Schwartz 92, 270
- fundamental domain 187, 190, 193
  - extended 193
- fundamental theorem of algebra 9, 71
- gamma density 112, 113
- Gauss–Lucas theorem 73
- Goursat’s theorem 28, 31, 33

- Hankel transform 287
- holomorphic function 12
- homothetic lattices 171
- homotopy 32–34
- Hurwitz’s theorem 136, 137, 140
- infinite product 63–65
  - formula for the cosine function 68
  - formula for the sine function 65, 78
- inverse function theorem 61
- Jacobi theta function 91, 94, 95, 114, 115, 215
- Jacobi theta null functions 215, 216, 226, 227, 231, 232
- Jacobi triple product identity 227, 232
- Jordan curve 36
- Jordan curve theorem 36
- Jordan–Schoenflies theorem 36
- Klein’s  $J$ -invariant 170, 182, 194, 197, 205–209, 214, 216
  - Fourier expansion 197
- Laguerre polynomials 287
- Laplace transform 238, 239, 242
- Laplace’s equation 19
- lattice 149, 268
  - dual 269
- Laurent series 68, 80
- Leech lattice 265, 278, 279, 282, 288
- line integral
  - fundamental theorem of calculus 24
  - of the first kind 23, 25
  - of the second kind 23, 24
- Liouville  $\lambda$ -function 115
- Liouville’s theorem 12, 41, 76, 77
- logarithm 58, 59, 78
  - principal branch 58, 78
- logarithmic derivative 53, 64, 68, 223
- von Mangoldt function  $\Lambda(n)$  100, 101, 115
- maximum modulus principle 57, 126, 127
- mean value property
  - for harmonic functions 38
  - for holomorphic functions 38
- meromorphic function 50
- Möbius  $\mu$ -function 115
- Möbius transformation 123–125, 127, 130, 145, 182, 185–187
- modular
  - discriminant 170, 182, 194, 195, 197, 198, 213, 214, 216, 219, 223, 224
    - Fourier expansion 195
  - form 183, 210–227
    - entire 210, 213, 214
    - weak 210
  - function 199–205
  - group  $\Gamma$  184, 186, 208, 214, 215, 218, 228, 230
  - lambda function 216, 224, 226
  - variable  $\tau$  171, 182
- modular surface 190
- Montel’s theorem 134, 140
- Morera’s theorem 28, 40, 45
- Newman’s Tauberian theorem 104, 108
- normal family 134, 140
- open mapping theorem 57, 58, 62
- Picard’s theorem 217
- Poisson summation formula 92
  - for lattices 270, 276, 277
- pole 52
  - of a holomorphic function 47
  - order 47
  - simple 47
- power function 60
- power series 20–22, 40
- pre-modular form 192, 194, 195, 197, 199
  - Fourier coefficients 192
  - Fourier expansion 192
  - weak 192, 199
- prime number theorem 5, 82, 83, 99–102, 104, 108, 109, 116
- prime-counting function  $\pi(x)$  82, 100
- primitive 26, 32, 36
- principal part 48
- principle of analytic continuation 43
- radius of convergence 20, 74
- Ramanujan’s tau function 196, 198
- region 1
  - doubly connected 141, 143
  - enclosed by a simple closed curve 36
  - simply connected 33, 36, 125, 131, 137, 139, 140
- removable singularity 44, 52
- residue 48
- residue theorem 48, 49, 66

- Riemann hypothesis 83, 109
- Riemann mapping theorem 125, 131–140
- Riemann sphere 50, 51, 119, 123–125
- Riemann surface 51, 172, 174, 176, 178, 190, 208
- Riemann zeta function 89–99, 113, 179, 180, 215
  - Euler product formula 90–92
  - functional equation 90, 92, 96, 113
  - trivial zeros 90, 97
- Riemann’s removable singularity theorem 44
- Rouché’s theorem 55, 56, 77
  
- Sendov’s conjecture 80
- singularity 52
  - essential 52
  - pole 52
  - removable 44, 52
- sphere packing 5, 233, 235, 267
  - density 267, 277
  - lattice 268
  - periodic 268
- sum of divisors function 115, 182, 197, 198
  
- Tauberian theorem 102
- Taylor’s formula 22
  
- uniform convergence on compact subsets 45
  
- vector field 23, 76
  - conservative 76
  - irrotational 76
- Viazovska’s modular form inequality 256
- Viazovska’s theorem 234
  
- Weierstrass  $\wp$ -function 154–159, 161–167, 180, 217, 225, 228
  - addition theorems 180
  - differential equation 159
  - duplication formula 180
- winding number 54, 55
  
- zero
  - of a holomorphic function 46
  - order 46
  - simple 46

