Preface to the first edition

> There are times in life when you discover what was really going on but too late.
> F. Forsyth, The outsider: My life in intrigue, Chapter: A day with the arrows

This book is a concise introduction to the stochastic calculus of variations (also known as Malliavin calculus) for processes with jumps. It is written for researchers and graduate students who are interested in Malliavin calculus for jump processes. In this book, 'processes with jumps' include both pure jump processes and jump-diffusions. The author has tried to provide many results on this topic in a self-contained way; this also applies to stochastic differential equations (SDEs) 'with jumps'. This book also contains some applications of the stochastic calculus for processes with jumps to control theory and mathematical finance.

The field of jump processes is quite wide-ranging nowadays, from the Lévy measure (jump measure) to SDEs with jumps. Recent developments in stochastic analysis, especially Malliavin calculus with jumps in the 1990s and 2000s, have enabled us to express various results in a compact form. Until now, these topics have been rarely discussed in a monograph. Among the few books on this topic, we would like to mention Bichteler–Gravereaux–Jacod (1987) and Bichteler (2002).

One objective of Malliavin calculus (of jump type) is to prove the existence of the density function $p_t(x,y)$ of the transition probability of a jump Markov process X_t probabilistically, especially the very important case where X_t is given by an (Itô, Marcus, Stratonovich, ...) SDE, cf. Léandre (1988). Furthermore, granting the existence of the density, one may apply various methods to obtain the asymptotic behavior of $p_t(x,y)$ as $t \to 0$ where x and y are fixed. The results are known to be different, according to whether $x \neq y$ or $x = y$. We also describe this topic.

The starting point for this book was July 2009, when Prof. R. Schilling invited me to the Technische Universität Dresden, Germany, to teach a short course on Malliavin's calculus for jump processes. He suggested that I expand the manuscript, thus creating a book. Prof. H. Kunita kindly read and commented on earlier drafts of the manuscript. The author is deeply indebted to Professors R. Schilling, M. Kanda, H. Kunita, J. Picard, R. Léandre, C. Geiss, F. Baumgartner, N. Privault, and K. Taira.

This book is dedicated to the memory of the late Professor Paul Malliavin.

Matsuyama, December 2012 Yasushi Ishikawa

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