Preface

This collection of papers originates from plenary lectures presented during the First Congress of Greek Mathematicians (FCGM–2018), which was held at the Mathematics Department of the National and Kapodistrian University of Athens, Greece, from 25 to 30 June 2018. The goal of this landmark event for Greek Mathematics and Greek mathematicians was to celebrate the Centennial Year of the Hellenic Mathematical Society and the Year of Mathematics for Greece 2018, making Athens a city of reunion of all Greek mathematicians living in Greece and abroad.

The Hellenic Mathematical Society (HMS) was founded in 1918 with the main goal to promote and enhance the study and research in mathematics and its numerous applications, as well as the evolution of Mathematical Education in Greece. The HMS today has more than 15,000 members and 39 regional branches within Greece, with an outstanding presence in scientific and cultural areas. Among other activities, it holds mathematical competitions, participates in international mathematical competitions and olympiads and organizes annual meetings, seminars and summer schools.

The First Congress of Greek Mathematicians was held under the auspices of the President of the Hellenic Republic Mr. H. E. Prokopios Pavlopoulos and was organized by the Hellenic Mathematical Society, the Departments of Mathematics of: the National and Kapodistrian University of Athens (NKUA), National Technical University of Athens (NTUA), the Aristotle University of Thessaloniki, the University of Patras, the University of Ioannina and the University of the Aegean, the Department of Mathematics and Applied Mathematics of the University of Crete, the Department of Statistics of the Athens University of Economics and Business, and the School of Science and Technology of the Hellenic Open University, in cooperation with the Cyprus Mathematical Society (CMS), the Mathematical Society of Southeastern Europe (MASSEE), the General Secretariat for Greeks Abroad, and the Municipality of Zografou.

The program of the Congress covered practically all areas of pure and applied mathematics. It hosted plenary lectures, invited lectures and 14 sessions and minisymposia in: Algebra and Combinatorics; Analysis; Control Theory, Operations Research and Optimization; Differential Equations and Dynamical Systems; Geometry; History and Philosophy of Mathematics; Logic and Foundations of Mathematics; Mathematical Physics; Mathematical Finance; Mathematics Education; Number Theory, Arithmetic Geometry and Cryptography; Numerical Analysis and Scientific Computing; Probability and Statistics; Topology and Applications.

The plenary speakers of the Congress with the titles of their talks were:

Spyridon Argyros (NTUA): The space $\mathcal{L}(\mathfrak{X})$ for certain Banach spaces
Demetrios Christodoulou (ETH Zürich): Geometric analysis in the development of shocks in compressible fluids
Mihalis Dafermos (Princeton U.): The mathematics of black holes

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Constantinos Daskalakis (MIT): From von Neumann's Minimax Theorem to generating beautiful images
Panagiota Daskalopoulos (Columbia U.): Ancient solutions to geometric flows
Athanasios Fokas (Cambridge U.): From the Wiener-Hopf technique to the Lindelöf hypothesis
Stavros Garoufalidis (Georgia Inst. Tech.): Classical and quantum topology
Loukas Grafakos (U. Missouri): A short glimpse of the giant footprint of Fourier analysis and recent multilinear advances
Nicos Kapouleas (Brown U.): Recent gluing constructions in geometry and the gluing methodology
George Karniadakis (Brown U.): Fractional partial differential equations: theory, algorithms and applications
Michael Katehakis (Rutgers U.): Reinforcement learning: connections between MDPs and MAB problems
Alexander Kechris (CalTech): Dynamics of non-Archimedean groups, logic and Ramsey theory
Phokion Kolaitis (U. California, Santa Cruz): Logic, constraint Satisfaction and quantum information
Charalambos Makridakis (FORTH): Mathematics of computational modeling: the challenge of computing nonlinear phenomena
Maryanthe Malliaris (U. Chicago): Model theory and ultraproducts
Antonios Melas (NKUA): Sharp estimates for dyadic type maximal operators and stability
George Papanicolaou (Stanford U.): Imaging with correlations
Georgios Pappas (Michigan State U.): Shimura varieties over the integers
Georges Skandalis (U. Paris Diderot): Lie groupoids and index theory and
Harry Tamvakis (U. Maryland): Theta polynomials in geometry, Lie theory and combinatorics.

Further, a roundtable discussion was organized on the theme: *"The leakage of Greek scientific potential in the years of the crisis. Which factors could help to contain it?"* It was coordinated by Vassilios Dougalis (NKUA), while interventions were made by Constantinos Daskalakis (MIT), George Papanicolaou (Stanford U.), Ioannis Sakellaridis (Rutgers U.), Georges Hourdakis (Greek Ministry of Education) and Ioanna Sapountzi (National Bank of Greece).

The Central Organizing Committee consisted of: Anargyros Fellouris (NTUA, President of the HMS); Ioannis Emmanouil (NKUA, Treasurer of the HMS); Sofia Lambropoulou (NTUA, Council Member of the HMS); and Apostolos Giannopoulos (NKUA, Member of the HMS). The idea for organizing such an event came from the third listed member of this Committee, who proposed it to the Council of the HMS, and the idea was adopted unanimously.

The website for the conference is: http://www.hms.gr/fcgm2018/index.html.

The link includes detailed information on the organization of the Congress, such as: the members of the Organizing Committee (chaired by Anargyros Fellouris, NTUA and President of the HMS) and the Scientific Programme Committee (President: Vassilios Dougalis, NKUA), the Session Organizers and the full program of talks. More than 350 mathematicians attended the events of the Congress.

We are indebted to the staff of the HMS, to Apostolos Bournetas, Chairman of the Mathematics Department of the NKUA, and to all staff members and student volunteers from the Department of Mathematics of NKUA for their organizational work toward the realization and the success of the Congress.

This volume features cutting-edge research papers written by Congress plenary speakers. The authors were asked to include illuminating state-of-the-art surveys and overviews of their research fields and of the topics they presented in the Congress. The book is expected to be most useful for researchers who wish to expand their research in new directions, to learn about new tools and methods in various areas of mathematics and need to find relevant and recent bibliography.

We proceed now to give summaries of the chapters.

The paper *Geometric analysis in the development of shocks in compressible fluids*, by Demetrios Christodoulou, surveys tools and techniques for the shock development problem in fluid mechanics. In this contribution, the author outlines the mathematical methods of his most recent monograph "The Shock Development Problem". The subject of this monograph is the shock development problem in fluid mechanics. This problem is formulated in the framework of the Eulerian equations of a perfect compressible fluid as completed by the laws of thermodynamics. These equations express the differential conservation laws of mass, momentum and energy and constitute a quasilinear hyperbolic first order system for the physical variables, that is, the fluid velocity and the two positive quantities corresponding to a local thermodynamic equilibrium state. Smooth initial data for this system of equations leads to the formation of a surface in spacetime where the derivatives of the physical quantities with respect to the standard rectangular coordinates blow up. In this way, one obtains a mathematical notion of maximal development of the initial data.

The paper *Ancient solutions to geometric flows* by Panagiota Daskalopoulos studies ancient solutions that play an important role in studying singularities. These are special solutions to an evolution equation that exists for all time: $-\infty < t \leq T$, with $T \leq +\infty$. They typically appear as blow-up limits near a singularity. The paper discusses some of the recent developments regarding the classification of ancient solutions to geometric flows, in particular the mean curvature flow and the Ricci flow.

The paper *Boundary value problems, medical imaging and the asymptotics of Riemann's zeta function* by Athanassios S. Fokas discusses the use of the Wiener–Hopf technique in acoustics and other physical problems. For many years, it was the only manifestation in applications of the Riemann–Hilbert formalism. However, during the last 50 years, this formalism and its natural generalization called the ∂-bar formalism have appeared in a large number of problems in mathematics and mathematical

physics. In this paper, the impact of the mentioned formalisms in three separate areas is reviewed. The first one concerns the development of a novel, hybrid numerical–analytical method for solving boundary value problems for linear and integrable nonlinear PDEs, known as the "unified transform" or the "Fokas method". Second, the author discusses the introduction of a new algorithm in nuclear medical imaging called the "attenuated spline reconstruction technique" (aSRT). Third, the author presents a novel approach to the study of the Lindelöf hypothesis, which is a close relative of the Riemann hypothesis.

The paper *A short glimpse of the giant footprint of Fourier analysis and recent multilinear advances* by Loukas Grafakos provides a concise overview of the genesis and impact of Fourier analysis in mathematics. It provides a review of some important results that have driven research during the last 50 years and a discussion of recent advances in multilinear aspects of the theory.

The paper *Fractional calculus and numerical methods for fractional PDEs* by George Karniadakis and Ehsan Kharazmi, Zhiping Mao, Guofei Pang and Mohsen Zayernouri reviews the recent relevant literature and development in numerical methods for solving fractional differential equations (FDEs) with a focus on high-order schemes. After an overview of the classification criteria of derivatives and their definitions according to various physical backgrounds, the authors discuss the extension to fractional Laplacian and fractional vector calculus for the case of higher-dimensional problems in a physical domain. They present a brief summary of existing numerical methods and explain the recently developed spectral/spectral element methods with several numerical examples. They also discuss some of the recent techniques of parameter estimation in the context of fractional models.

In the paper *Reinforcement learning: a comparison of UCB versus alternative adaptive policies* by Wesley Cowan, Michael Katehakis and Daniel Pirutinsky, the authors consider the basic version of reinforcement learning that involves computing optimal data-driven (adaptive) policies for Markovian decision processes with unknown transition probabilities. They provide a brief survey of the state of the art of the area and compare the performance of the classic UCB policy of Burnetas and Katehakis with a new policy developed herein which they call Markovian decision pdrocesseterministic minimum empirical divergence (MDP-DMED), and a method based on posterior sampling (MDP-PS).

The paper *Mathematics of computational modelling: some challenges of computing nonlinear phenomena* by Charalampos Makridakis and I. Gkanis, G. Grekas and E. Karnessis considers examples of singular phenomena that arise in complex systems and are modelled by nonlinear PDEs and discrete microscopic systems. Such phenomena are interesting for their applicability and pose challenges in mathematical research. Since natural computational methods may predict physically irrelevant solutions, techniques from mathematical analysis are necessary to analyze the sensitivity of the computational schemes, in order to assure convergence into a physically relevant solution. The authors describe in detail three nonlinear problems. The first

one is related to the design and analysis of approximate atomistic-continuum energies related to models arising in crystalline materials, and the second one concerns cell interactions within a fibrin medium. As far as this second problem is concerned, it is shown how mathematical modeling and numerical analysis may lead to reliable predictions and clarify the mechanism of the interactions. Finally, in order to study a problem arising in statistical inference of solutions to nonlinear hyperbolic systems, a new discrete kinetic model is proposed, and the corresponding kinetic formulation of the relevant conservation laws is studied, leading to the computation of measured-valued solutions.

The paper *Sharp estimates for dyadic type maximal operators and stability* by Antonios Melas is a survey on various results that are related to generalizations of dyadic maximal operators. The Bellman functions that are associated with the dyadic maximal operators in Euclidean spaces comprise a very useful tool in the study of these operators. These notions may be generalized in the context of tree-like families of measurable subsets in a non-atomic probability space. Certain sharp inequalities for these Bellman functions are presented, whose proof uses an effective linearization method developed by the author. Analogous estimates can be obtained for the Bellman functions that are associated with local inequalities as well as for Lorentz-type Bellman functions. Finally, certain stability results for extremal or approximate extremal functions are presented; such properties of extremal functions are needed for a deeper analysis of Bellman function estimates.

The paper *Data structures for robust multifrequency imaging* by George Papanicolaou, Miguel Moscoso, Alexei Novikov and Chrysoula Tsogka discusses the problem of imaging. Even though this problem is in general nonlinear, it is often formulated as an under-determined system of linear equations with a sparse unknown vector. A theoretical linear algebra framework is presented that allows the formulation of conditions under which two standard algorithms that are successfully used in signal processing and imaging (namely, ℓ_1-minimization for a single measurement and MUSIC for multiple measurements) admit an exact solution. The authors also consider the phase retrieval problem in an optical microscopy regime and explore numerically the robustness of the imaging methods to noise when the background medium is weakly inhomogeneous. Finally, numerical simulations illustrate the relevance of the theoretical results obtained in the paper in the assessment of image resolution.

The paper *Theta and eta polynomials in geometry, Lie theory, and combinatorics* by Harry Tamvakis discusses the classical Schur polynomials, which form a natural basis for the ring of symmetric polynomials and have geometric significance since they represent the Schubert classes in the cohomology ring of Grassmannians. Moreover, these polynomials enjoy rich combinatorial properties. In the last decade, an exact analogue of this picture has emerged in the symplectic and orthogonal Lie types, with the Schur polynomials replaced by the theta and eta polynomials of Buch, Kresch and the author. This expository paper gives an overview of what is known to date about this correspondence, with examples.

We hope the reader finds in this small collection of excellent papers a sense of the spirit of the First Congress of Greek Mathematicians and that this Congress will be the beginning of a sequence of similar events for Greek mathematicians, to be held regularly.

The Editors