Farran Mackay, Jantien Smit, Arthur Bakker, and Ronald Keijzer

Supporting teachers to scaffold students' language for mathematical learning

1 Introduction

Teachers often are unaware of language issues and avoid linguistic challenges in their classrooms to focus on mathematics (e.g., Van Eerde et al., 2008). Specifically, teachers typically do not attend to the language students need for mathematical learning, and rarely know how to support the development of subject-specific language required for mathematical learning (e.g., Hajer & Norén, 2017). Yet students, especially those with low language proficiency, require support from teachers within this subject because shortcomings in subject-specific language can impede their development of mathematical understanding (Moschkovich, 2010). Despite the importance of improving language-responsive teaching, there is a profound lack of opportunity for teachers to develop the required teaching practices, especially in mathematics (Essien et al., 2016). The required teaching practices integrate language learning and mathematics in a domain-specific way (Van Eerde & Hajer, 2009). Although there are some insights into the professional development of secondary school teachers (e.g., Prediger, 2019), relatively little is known about how to support primary school teachers in realizing language-responsive teaching. This chapter provides insights into how teachers can be supported within a professional development program (PDP), focusing on genre awareness and scaffolding students' language for mathematical learning.

2 Theoretical background

To specify our approach to supporting teachers in a PDP, we first address what the literature considers essential learning goals for teachers (2.1) before we characterize our approach to designing and evaluating our PDP (2.2).

https://doi.org/10.1515/9783110661941-020

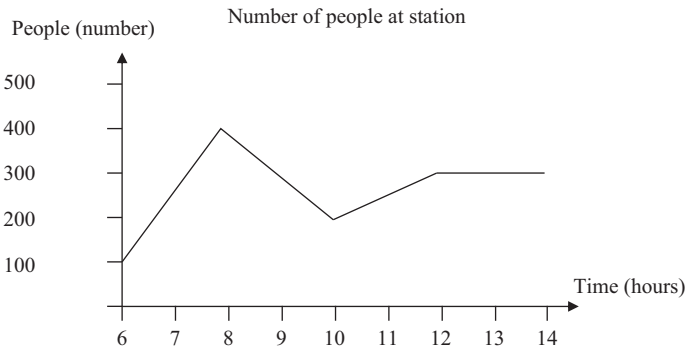
2.1 Learning goals for teachers: Genre awareness and scaffolding language

Key to participating in mathematical discourse is access to the subject-specific language required for learning mathematics (e.g., Prediger, 2019). Each domain has not only its own vocabulary but also phrases which learners need to recognize and adequately use to participate successfully (Moschkovich, 2010; Schleppegrell, 2007). Consequently, a PDP should help teachers pay attention to the required language and establish an environment that allows learners to interact and communicate at a mathematical level (e.g., Lampert & Cobb, 2003). Genre pedagogy is a promising approach to explicitly address the language required for learning in that it provides learners with metalinguistic knowledge about how (both spoken and written) language is structured and used to achieve particular goals (e.g., describing or persuading) (Martin, 2009). The notion of genre is typically associated with certain literary forms, such as poem or novel. In genre pedagogy, the notion of genre is mainly used for academic text types used throughout the curriculum. Six key genres have been distinguished (e.g., narratives, reports). However, for mathematics education, a more domain-specific investigation and identification of genres for mathematical learning are needed to centralize linguistic competency explicitly (Moschkovich, 2010).

Using genre pedagogy in mathematics education, Smit et al. (2016) formulated linguistic and structure features needed to identify the mathematical language to describe and interpret line graphs (see Fig. 1). The linguistic features included, for example, subject-specific vocabulary and phrases, and also the use of an expression of gradation steepness (as in "the graph descends *gradually*"), as well as general academic language to be employed when interpreting the graph (e.g., *the number* of people *increases*). The structure features comprised the stages of students' reasoning about graphs. For example, students are expected to interpret each part of the graph (e.g., "it was less busy") with a description related to the course of the graph ("you can tell as the graph shows a steep fall").

Linguistic and structure features of genres can provide teachers with a lens through which they can identify the language for mathematical learning (Smit et al., 2016). Through this identification, teachers can become more aware of the language required for mathematical discourse, and they can then better support their students express their thoughts in a mathematically accepted way (e.g. Schleppegrell, 2007).

Next to awareness of genres, it is considered useful for teachers to learn how to scaffold their students' language (Gibbons, 2002). Scaffolding, in short, is temporary, adaptive support, fostering students' independence regarding a



At 06:00 there are 100 people at the station. Between 06:00 and 08:00 the number of people increases, as the graph ascends. Between 08:00 and 10:00 the number of people decreases; the graph descends. Between 10:00 and 12:00 the number of people slowly increases as the graph gradually ascends again. From 12:00 the number of people remains the same. The graph is constant.

Fig. 1: Line graph and exemplary text from a domain-specific genre of interpreting line graphs.

particular topic (Maybin et al., 1992). For whole-class scaffolding, Smit et al. (2013) formulated three key characteristics. The first, *diagnosis*, is the assessment of the learners' level and needs. Second, *responsiveness* is the adaptation of support to learner's needs based on a diagnosis. The third characteristic is *handover to independence* which is the fading of support as the learner's independence increases.

The literature suggests that teachers can enact several scaffolding strategies to offer adaptive support (e.g., Gibbons, 2002). Our PDP's goal was that teachers would learn to use the scaffolding strategies in Tab. 1 (Smit et al., 2016) in response to the diagnoses of students' language proficiency as part of a language-responsive approach informed by genre pedagogy.

2.2 Teacher learning within adaptive professional development

For conceptualizing and analyzing teacher learning in our study, we used a framework developed by Bakkenes et al. (2010). It distinguishes four main categories of teacher learning: changes in knowledge and beliefs, intentions for practice, changes in practice, and emotions.

Tab. 1: Strategies for scaffolding language for mathematical learning.

Strategies	Example
Reformulating students' utterances into more academic wording	[In response to *the graph goes higher*] *Indeed, the graph rises steeply*
Ask students to be more precise or improve their spoken language	*What do you mean by "it"?*
Repeat correct student utterances	*Indeed, the graph descends slowly*
Refer to features of the text type	*Into how many segments can we split the graph?*
Use gestures or drawings to support verbal reasoning	Gesturing a horizontal axis when discussing a topic
Remind students (by gesturing or verbally) to use a designed scaffold (i.e., word list or writing plan) as a supporting material	*The word you are looking for is written down for you here*
Ask students how written text can be produced or improved	*How can we rewrite this using mathematical wording?*

To design a PDP based on previous research on genre pedagogy and scaffolding language, we capitalized on several sources. First, we drew on four learning activities identified by Bakkenes et al. (2010) as key elements for developing teacher expertise in the context of educational renewal: (1) learning by experimenting; (2) learning in interaction with others; (3) using external sources (e.g., publications); and (4) consciously reflecting on one's teaching practices. Second, we drew on insights concerning the promotion of language-responsive teaching (e.g., Hajer & Norén, 2017). Prediger (2019) gained insights into the learning pathways and obstacles of secondary school teachers in their development of language-responsive teaching practice concerning the identification of language for mathematical language. Despite these insights, there are still ongoing challenges in guiding teachers in the identification and therefore support of language for mathematical learning. One such challenge, as noted by Lyon (2013), is that teachers need to view language as inherent to one's classroom culture and not merely a technical issue to control. As such, the guidance of teachers in the identification of language for mathematical learning is required to be adaptive to teachers' contexts and needs.

A third source used for developing our PDP formed insights on adaptivity – generally assumed to be an essential characteristic of PDP as teacher learning depends on numerous factors (Putnam & Borko, 2000). Individual teacher learning

may be different, and variations across settings need to be acknowledged (e.g., Goldsmith et al., 2014). If a PDP is adaptive, it provides the opportunity for participant teachers to take ownership of the content (e.g., Davis, 2002). To date, little is known on how adaptivity to teachers' individual needs can be achieved, which specific steps must be made in the design of the PDP to allow for this, and what teacher learning takes place in the context of these efforts.

The aim of the study reported here was to gain insights into how primary school teachers in an adaptive professional development program (2.2) can be supported in developing genre awareness and the scaffolding of students' mathematical language (2.1). We ask two questions:

1. What adaptations to the program were necessary to support teacher learning?
2. What did a case-study teacher learn from participating in the program?

3 Methods

We used design research to design and evaluate the PDP. Design research can be characterized as an interventionist approach to which prediction of and reflection on learning processes are central, and where the design of an intervention and the actual research are intertwined (McKenney & Reeves, 2018; Prediger et al., 2015). The design and redesign of sessions of the PDP were carried out by the second author. She acted as both researcher and teacher educator within the PDP, with a larger team acting as a soundboard. The PDP comprised seven monthly group sessions of 2.5 h each.

To achieve adaptivity to the teachers' needs, several steps were taken. Before the PDP, the participants completed questionnaires, and a mathematics lesson was observed to determine the starting point concerning language-responsive teaching. Throughout the PDP participants completed electronic logs, which the researcher-educator could access. The role of the participant logs was twofold. First, the participant logs consisted of open questions designed to promote teachers' reflection on their learning and classroom practice (Bakkenes et al., 2010) in the context of the PDP. Second, the participant logs gave insights into teachers' self-reported learning. Inspired by hypothetical learning trajectories (Simon, 1995), a researcher log was written before each session with the intentions and expectations for the session. After each session, a reflection document was created where the researcher log was compared with what actually happened in the session. The reflection document plus the most recent participant logs formed the basis for the researcher log of the next session. As such, the goals of the PDP were addressed while being adaptive to participants' needs and levels of understanding.

3.1 The five participants

One participating teacher (female, 25 years of experience), who taught grade 3 (students aged 8–9), agreed to be a case study to allow for more in-depth insight into the learning processes and development of the participants occurring within the PDP. The second participant (female, 13 years of experience) taught grade 4. A third participant (female, 27 years of experience) taught mathematics in a one-on-one setting supporting special education in a mainstream primary school. The final two participants came from the same school where one (female, 29 years of experience) and the other (male, 10 years of experience) taught grade 6.

3.2 Instruments and data collection

Apart from the aforementioned participant and researcher logs, and reflection documents for each session, the data collection consisted of completed exercises by the participants and verbatim transcription of the interaction between the researcher-educator and participants from video recordings of each group session.

To gain more in-depth insight into the responses in the participants' logs and PD sessions, two semi-structured interviews of the case-study teacher were conducted: one between the fourth and fifth group session and the second after the final session. The interview timing was chosen to allow for the characterization of the teacher's learning throughout the whole PDP. In the interviews, we asked the teacher to elaborate on points of interest mentioned in her logs. Audio recordings of both semi-structured interviews were transcribed verbatim.

3.3 Data analysis

3.3.1 Enactment and adaptation of the PDP

Participants' progress in terms of the learning goals mentioned in Section 2.1 was analyzed to evaluate the need for adaptation of the PDP. To detect these progressions within the PDP, the researcher logs and reflection documents were analyzed in the following way. We compared our expectations with what happened and traced how decisions were made based on new insights. For constructing the narrative presented in 4.1, we focused on the most apparent discrepancies

between expectations and actual teacher learning. For example, we used Smit et al. (2016) genre of interpreting line graphs to illustrate what a domain-specific genre with its linguistic and structure features could look like. However, we quickly noted that teachers had little affinity with that domain. Hence we needed to adapt our initial plans and find another example that was closer to teachers' own practices. For more details, see Bakker et al. (2019). The first author created the theme-oriented narratives, after which the researcher-educator reviewed them; minor updates for clarification were made.

3.3.2 Teacher's perceived learning within the PDP

To gain insight into teachers' learning during the adaptive PDP, the semi-structured interviews with Mary (pseudonym) were analyzed based on the aforementioned main categories of Bakkenes et al. (2010) framework for teachers' self-reported learning.

Tab. 2: Coding scheme for reported learning outcomes.

Code	Global description	Example
CKB	Change in knowledge/beliefs: The teacher reports on growing awareness acquired knowledge, or the teacher reports on the confirmation of already existing beliefs	I am more aware of the role language plays in the mathematics classroom.
CP	Change in practice: The teacher reports that things have changed in his/her way of teaching or students' participation in the mathematics lessons.	I now prepare my lessons with a language goal in mind.
IP	Intention for practice: The teacher reports that he/she wants to teach differently in the future or reports that he/she wants to hold on to certain teaching practices	I want to spend more attention to reasoning steps in the future.
E	Emotion: Teacher reports on emotions related to using the knowledge from the PDP in the classroom, or reports on being surprised	I was surprised to see the level of interaction increased in the class.

All utterances in the transcripts of the interviews, in which Mary explicitly reports on a learning outcome, were identified by the observing researcher. Two

independent raters then coded the 126 utterances to measure the inter-rater reliability of the coding process. The coding by the first and second coders resulted in an agreement of 95.2% and a Cohen's kappa of .91, meaning the four categories could be reliably distinguished.

The utterances were placed in chronological order by category to gain insight into the nature of the reported learning outcome. The utterances were analyzed, and the observing researcher generated a summary of changes in the four categories. The researcher-educator read the ordered data to validate the conclusions drawn from the analysis; minor updates for clarification were made.

4 Results

4.1 Enactment and adaptation of the PD

Analysis of the enactment of the PDP through the researcher and participant logs, and reflection documents, yielded two separate chronological narratives focused on the two main themes of the program: genre awareness and scaffolding language. For the latter theme, relatively minor adaptations were made to the PDP. Therefore we focus in this chapter on the adaptations made for the theme of genre awareness sharing the chronological narrative.

4.1.1 Narrative of genre awareness

In the first session, the participants were introduced to the identification of language required for mathematical learning through the concept of genre. The researcher-educator presented the concept of genre by using an example from the domain of line graphs (Smit et al., 2016). However, based on the reactions of the participants, the researcher-educator diagnosed that since line graphs are not regularly taught in primary school, this example was not close enough to the teachers' own practices. As such, the line graph example did not provide sufficient support for the teachers to understand the idea of genres in the domain of mathematics.

In the second session, the researcher-educator responded by characterizing two other genres for domains that were closer to the participants' teaching: estimation and expanded column method for subtraction. During the analysis of these genres, she drew the attention of the participants not only to the linguistic features (i.e., general academic language and subject-specific language) but also

to the structure features (i.e., the required ordering of the steps by students to give mathematical meaning). While reviewing the in-session completed tasks, the researcher-educator diagnosed that the participants were still struggling with the concept of genre. This diagnosis was corroborated when two participants contacted the researcher-educator to report that they could not grasp how to complete the homework assignment related to the estimation genre. The researcher-educator concluded that the notions of linguistic and structure features of genres were not as fully understood as the researcher-educator had anticipated they would be at this stage in the PDP and that the term "genre" was a stumbling block for the participants. During this conversation, the researcher-educator explained the concept of genre in the context of language for mathematical learning as the text that includes the specific language and reasoning that is particular to that domain. It was in this conversation that the term "domain text" was first coined as a concretization to the more technical concept of a genre, where domain text was considered to be a prototypical text for a particular domain.

In the third session, the researcher-educator made three fundamental adaptations to the PDP. The first was to replace genre by *domain text*. Rather than using the more abstract and unfamiliar notion of genre, the researcher-educator thus responsively reframed the language for mathematical learning as *domain texts* that represent typical language usage in the different mathematical domains (deploying particular words and phrases). The second was to shift focus from identifying the structure features of the spoken or written mathematical text to identify the *reasoning steps* needed by students to solve mathematical problems. This new focus was better aligned with participating teachers' existing views on how students can solve mathematical problems. The third adaptation was to shift identifying the language for mathematical learning for a particular mathematical *domain*, to that for solving a particular mathematical *problem* within a domain. This adaptation was regarded as crucial by the participants and researcher-educator, as each mathematical problem, even within the same domain, requires its language to be identified in association with the particular reasoning steps.

In the fourth session, the participants used a domain text preparation template that was developed as a response to the diagnosis that the participants needed a scaffolding device for the identification of domain texts based on the completed homework assignments of the participants. The preparation template included the identification of reasoning steps and the required language components for solving the mathematical problem. The participants first had to identify the reasoning steps needed to solve the problem and then identify the language components of the domain text. Finally, the participants had to write how a student should

articulate the solution. While the participants were analyzing videos of language-focused mathematics classes and the accompanying domain text, the participants reported that the dual analysis of the video and the domain text helped with their understanding of the identification of language for mathematical learning. In response, the researcher-educator and participants agreed that in the following sessions, every participant should present for peer feedback at least one video of their language-focused mathematics classes and the associated domain text.

In the fifth session, the teacher-educator led a discussion with the participants on the differences between a teacher's instruction (procedural steps) and students' thinking (reasoning steps) required to solve a mathematical problem. The addition of the discussion was based on the analysis of the participants' homework assignment to develop a domain text. The researcher-educator diagnosed an increase in independence as the participants were beginning to identify words and formulations for their chosen domain. However, as part of their development of domain texts, three of the five participants were still unable to centralize student thinking in their identification of reasoning steps and instead focused on procedural steps. To initiate the discussion and emphasize the importance of centralizing students' thinking in a lesson, the researcher-educator asked one participant who had already perceived this difference between procedural steps and reasoning steps, to explain the difference to the other participants. The discussion clarified the concept of reasoning steps for the other participants. The participants made comments such as "reasoning steps stimulate thinking," "maybe we give too little attention to reasoning steps," and "normally language in the mathematics lesson is focused on the mathematical procedures, not the reasoning steps of a student." By the end of the sixth session, most of the participants showed some form of independence concerning reasoning steps during the session: "You get closer to the thinking of the children" and, concerning language and reasoning steps, "[language and reasoning steps] support each other. You can see the thought process in the children."

In sum, responsive adaptations to genre pedagogy could be characterized as from a more global to a more local orientation on the one hand, and from more abstract (i.e., genre with language and structure features for a domain) to more concrete notions (i.e., language and reasoning steps for a problem) on the other hand.

4.2 Teacher's perceived learning within the PDP

The distribution of utterances among the categories of reported learning is shown in Tab. 3. With 125 utterances in total, the majority of the reported learning outcomes fell into the category changes in practice.

Tab. 3: Distribution of reported learning outcomes among three categories.

	Mid-interview (Mid)	Post-interview (Post)
Changes in knowledge and beliefs	15	16
Changes in practice	30	31
Intentions for practice	13	20

In the following subsections, quotes from the interviews (from which interview is cited in parentheses) and observations are presented to illustrate Mary's development from the mid- to post-interview for each category of reported learning.

4.2.1 Changes in knowledge and beliefs

Based on the analysis of the semi-structured interviews, the changes in knowledge and beliefs primarily concerned the overarching intention of the PDP for scaffolding language for students' mathematical learning. The most noticeable change in Mary's beliefs was the importance of language in the mathematics classroom and that "language in a mathematics lesson is not separate, mathematics and language belong together" (Mid). For Mary, the concept of the scaffolding language seemed easy to understand as early in the PDP she had formulated for herself that "they are strategies I can use to help the students use more precise language" (Mid).

However, with the theme of identification of genres or language required for mathematical learning, Mary had some conceptual issues. Mary struggled in shifting her view from considering that the language required for mathematical learning consists only of vocabulary to viewing it as a means to allow a student to articulate their reasoning steps. Mary suggested that her difficulties in comprehending the concept of reasoning steps may have been due to her choice of domain. The domain of measuring – at the age of Mary's class – does not involve a large amount of reasoning knowledge but does involve substantial procedural knowledge of how to measure. For Mary, this was highlighted when she reflected on another participant's domain text where she could immediately identify the reasoning steps.

Mary reported that the learning structure within the PDP where participants spent time analyzing their assignments and video recordings of their classroom interaction as a group was for her valuable and crucial to her learning as "You cannot do that alone. And also not by reading a book" (Post).

4.2.2 Changes in practice

Mary reported several changes in practice as a result of the PDP. She changed the preparation of her mathematics lessons to include a language goal. Specifically, she now considers "what language is important to focus on in the lesson and how they (learners) can use it" (Post).

By the end of the PDP, Mary reported how several scaffolding language strategies had become embedded in her class: "how can we say that using mathematical language? That is a standard sentence I use often" (Mid) and "discussing with each other how to word it more precisely" (Post). She also recognized the benefits of not only confirming that her student had used the correct articulation but asking another student to also try as "it may help the student who had not yet understood it" (Post).

The focus in Mary's mathematics class shifted from getting the answer to allowing the students to articulate their reasoning steps:

> The students articulate their reasoning steps. You then know: the answer is not correct but in how you tried to get there one small step is missing. If you can explain something, then you understand it. They previously had not been able to articulate that. (Mid)

Mary also observed changes in the form of interaction she had with the students: "I noticed I have a lot more dialogue with the students" (Mid); "they use the required terms and often say 'oh we need to say that using mathematical language' " (Post).

4.2.3 Intentions for practice

In the mid-interview, Mary's intentions for practice were focused on scaffolding students' language required for mathematical learning. In the post-interview, Mary reported the intention of continuing to work on components of the domain text, the identification of the articulation of the reasoning steps. Mary also intended to use the knowledge she gained in the PDP, not only in the mathematics class but to extend scaffolding of language to other subjects: "I think that I will try it in other subjects" (Post).

In the post-interview, Mary's reported intentions were not limited to her learning and practice but also included the dissemination of knowledge to her colleagues. Mary also noted that the dissemination of the knowledge related to the scaffolding of language required for mathematical learning is not something that can be achieved in the short term "but slowly as this is something that

must be absorbed. This cannot be done in a couple of months. This is something where you need a year to become more confident about it" (Post).

5 Discussion

The presented study aimed to gain insights into how a professional development program (PDP) can adaptively support teachers to gain awareness of genres relevant to learning mathematics and scaffold language required for mathematical learning within their lessons. To fulfil this aim, two aspects of the PDP have been analyzed. The first was the adaptations made to the PDP and the observations that triggered these adaptations. The second aspect was the learning reported by a participating teacher.

The researcher-educator had the opportunity to adapt the contents of the sessions to the needs of the group and individual participants by analyzing the development of the teachers in between PDP sessions through reflection documents and participant logs. Similar to findings in other studies (e.g. Prediger, 2019), the identification of language for mathematical language proved to be challenging. For example, the notion of "genre" proved too abstract for the participants to identify the language for mathematical learning. This led us to use the notion of "domain text" instead – a move that can be considered practicalizing principled theoretical knowledge (Bakker et al., 2019; Janssen et al., 2015). The shift from the abstract to concrete was achieved through using a narrower focus of language needed for reasoning about and solving a particular mathematical problem (called "reasoning steps" – "denkstappen" in Dutch). In line with the approach taken with secondary school teachers (Prediger, 2019), focusing on a mathematical problem, as opposed to an entire mathematical domain, was closer to teachers' own practical experiences. This practicalization of principled theoretical knowledge was enacted both in the design of the course activities (e.g., the preparation template) and in interaction with the participants during sessions.

We give this as an example to underpin a broader call to action for researchers and educators. Practicalization requires flexibility on the side of the researchers and educators, a willingness to adapt original plans where the input and knowledge from the participants are employed to judge what works in the context of their teaching practice. By doing so, researchers and educators have the opportunity to concretize the methods and strategies, aligned with the theory in question, that the teachers can successfully employ in their classrooms. However, such adaptations should not lead to "lethal mutations" (Brown & Campione, 1996).

Second, we analyzed what a teacher perceived to have learned from participating in the PDP. The case-study teacher reported an increase in awareness in the connection between mathematics and language. She also reported that the most noticeable impact of the PDP was the change in social norms concerning mathematical language during the interaction that occurred within her class, both between herself and the students and between the students themselves, in line with often reported observations (e.g., Yackel & Cobb, 1996). She reported how a number of the language scaffolding strategies had become embedded in her class with the effect that students became accustomed to using the language for mathematical learning as well as verbalizing their reasoning steps.

A limitation of our study is that we only analyzed the participants within the PDP itself and relied on self-report for their teaching in the classroom. The impact of the PDP on the participating teachers' classroom enactment and students' mathematical performance was not observed or analyzed, and the extent of the handover to independence could thus not be fully monitored.

What we as authors have learned about our national case is that when promoting a particular vision on mathematics education, professional development has to be taken much more seriously. For example, the ideas from Realistic Mathematics Education have been influential both nationally and internationally (Treffers & van den Heuvel-Panhuizen, 2020) but proved hard to realize in practice in the Netherlands. The general ideas and curricular materials were insufficient for implementing Realistic Mathematics Education in education (Gravemeijer et al., 2016), pointing to a lack of investment in professional development. Thus there should be a focus not only on the education characteristic in question (in our case language-responsiveness) but also on continued investment in the professional development that is adaptive to the local context.

References

Bakkenes, Inge, Vermunt, Jan D. & Wubbels, Theo (2010): Teacher learning in the context of educational innovation: Learning activities and learning outcomes of experienced teachers. *Learning and Instruction* 20 (6), 533–548.

Bakker, Arthur, Mackay, Farran, Smit, Jantien & Keijzer, Ronald (2019): Practicalizing principled knowledge with teachers to design language-oriented mathematics lessons: A design study. In Graven, M, Venkat, H., Essien, A, Vale, P. (eds.): *43rd Conference of the International Group for the Psychology of Mathematics Education*. 2, Pretoria, South Africa: PME, 57–64.

Brown, Ann L. & Campione, Joseph C. (1996): Psychological theory and the design of innovative learning environments: On procedures, principles, and systems. In Schauble,

L., Glaser, R. (eds.): *Innovations in learning: New environments for education*. Mahwah, NJ: Lawrence Erlbaum, 289–325.
Davis, Kathleen S. (2002): "Change is hard": What science teachers are telling us about reform and teacher learning of innovative practices. *Science Education* 87, 3–30.
Essien, Anthony A., Chitera, Nancy & Planas, Núria (2016): Language diversity in mathematics teacher education: Challenges across three countries. In Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N., Setati-Phakeng, M., Valero, P., Villavicencio Ubillús, M. (eds.): *Mathematics Education and Language Diversity*. Cham: Springer, 103–119.
Gibbons, Pauline (2002): *Scaffolding Language, Scaffolding Learning: Teaching Second Language Learners in the Mainstream Classroom*. Portsmouth, NH: Heinemann.
Goldsmith, Lynn T., Doerr, Helen M. & Lewis, Catherine C. (2014): Mathematics teachers' learning: A conceptual framework and synthesis of research. *Journal of Mathematics Teacher Education* 17 (1), 5–36.
Gravemeijer, Koeno, Bruin-Muurling, Geeke, Kraemer, Jean-Marie & Van Stiphout, Irene (2016): Shortcomings of mathematics education reform in the Netherlands: A paradigm case? *Mathematical Thinking and Learning* 18 (1), 25–44. doi:10.1080/10986065.2016.1107821.
Hajer, Maaike & Norén, Eva (2017): Teachers' knowledge about language in mathematics professional development courses: From an intended curriculum to a curriculum in action. *EURASIA Journal of Mathematics Science and Technology Education* 7b (13), 4087–4114.
Janssen, Fred, Westbroek, Hanna & Doyle, Walter (2015): Practicality studies: How to move from what works in principle to what works in practice. *Journal of the Learning Sciences* 24 (1), 176–186.
Lampert, Magdalena & Cobb, Paul (2003): Communication and language. In Kilpatrick, J., Martin, W. G., Schifter, D. (eds.): *A Research Companion to Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 237–249.
Lyon, Edward G. (2013): What about language while equitably assessing science?: Case studies of preservice teachers' evolving expertise. *Teaching and Teacher Education* 32, 1–11.
Martin, James R. (2009): Genre and language learning: A social semiotic perspective. *Linguistics and Education* 20 (1), 10–21.
Maybin, Janet, Mercer, Neil & Stierer, Barry (1992): 'Scaffolding' learning in the classroom. In Norman, K. (ed.): *Thinking Voices: The work of the National Oracy Project*. London: Hodder and Stoughton, 185–195.
McKenney, Susan & Reeves, Thomas C. (2018): *Conducting Educational Design Research (2nd Ed.)*. London: Routledge.
Moschkovich, Judit N. (2010): *Language and Mathematics Education: Multiple Perspectives and Directions for Research*. Charlotte, NC: IAP.
Prediger, Susanne (2019): Investigating and promoting teachers' expertise for language-responsive mathematics teaching. *Mathematics Education Research Journal* 31 (4), 367–392.
Prediger, Susanne, Gravemeijer, Koeno & Confrey, Jere (2015): Design research with a focus on learning processes: An overview on achievements and challenges. *ZDM* 47 (6), 877–891.
Putnam, Ralph T. & Borko, Hilda (2000): What do new views of knowledge and thinking have to say about research on teacher learning? Educational Researcher 29 (1), 4–15.

Schleppegrell, Mary J. (2007): The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly* 23 (2), 139–159.

Simon, Martin A. (1995): Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26 (2), 114–145.

Smit, Jantien, Bakker, Arthur, Van Eerde, Dolly & Kuijpers, Maggie (2016): Using genre pedagogy to promote student proficiency in the language required for interpreting line graphs. *Mathematics Education Research Journal* 28 (3), 457–478.

Smit, Jantien, Van Eerde, Henriëtte A. A & Bakker, Arthur (2013): A conceptualization of whole-class scaffolding. *British Educational Research Journal* 39 (5), 817–834.

Treffers, Adri & Van den Heuvel-panhuizen, Marja (2020): Dutch Didactical Approaches in Primary School Mathematics as Reflected in Two Centuries of Textbooks. In van den Heuvel-panhuizen, M. (ed.): *National Reflections on the Netherlands Didactics of Mathematics*. Cham: Springer, 77–103.

Van Eerde, Dolly & Hajer, Maaike (2009): The integration of mathematics and language learning in multiethnic schools. In César, M., Kumpulainen, K. (eds.): *Social Interactions in Multicultural Settings*. Rotterdam/Taipei: Sense, 269–296.

Van Eerde, Dolly, Hajer, Maaike & Prenger, Joanneke (2008): Promoting mathematics and language learning in interaction. In Dean, Jeannie, Haijer, Maaike, Koole, Tom (eds.): *Interaction in Two Multicultural Mathematics Classrooms. Processes of Inclusion and Exclusion*. Amsterdam: Aksant, 31–69.

Yackel, Erna & Cobb, Paul (1996): Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education* 27, 458–477.