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Fifth-grade students' production of mathematical word problems

1 Introduction

Mathematical word problems challenge students significantly, as empirical studies have shown (e.g., Bush & Karp, 2013; Lewis & Mayer, 1987). Difficulties mostly arise from two aspects, mathematical characteristics, and linguistic structure. Mathematical characteristics of the word problem, such as number size, number and complexity of required operations, and applicable strategies, increase problem difficulties. While on the linguistic side, semantic as well as syntactical characteristics of word problems add to the difficulty (for an overview, see Daroczy et al., 2015). Besides these factors, it is building a mathematical model based on a situation described in a text that is a main difficulty to identify in empirical research (Jupri & Drijvers, 2016; Leiss et al., 2010; Maaß, 2010).

We use the term "situation" to refer to a context, which serves the purpose of exemplifying a concept or set of related concepts. As a situation is related to a specific mathematical conceptual field, it formulates a mathematical problem that requires a predictive response. Thus, situations go beyond stimuli, which cause a specific behavior, but are rather typical settings in which mathematical concepts become visible. Situations can be given by illustrations and also by contextual descriptions with mathematics concepts embedded. While research on word problems has focused on contextual descriptions of situations, this chapter aims at investigating how children produce word problems from engaging with illustrated situations.

Children encounter word problems that contextualize a more, or less, complex mathematical task in a real-world situation in different ways (Verschaffel et al., 2000). A typical, simple word problem is: "Alex has 3 packages of chocolate. In every package there are 5 pieces. How many pieces of chocolate does Alex have in total?" In this example, the encoded arithmetic task ($3*5=?$) is rather transparent in the word problem, as all numbers are given and the multiplicative structure is highlighted by cue words or phrases (here: "in every") (LeBlanc & Weber-Russell, 1996). Jupri and Drijvers (2016) report that finding all these cue words and phrases is a main obstacle for students while mathematizing a situation. In such tasks, the real-world context often appears to be designed for the task, thereby casting the word problem's authenticity into doubt

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(Palm, 2009). The main purpose of these so-called "dressed-up" problems is practicing basic operations in real-world contexts (Leiss et al., 2019; Verschaffel et al., 2000). Actual mathematical problem solving requires a significantly more complex mathematizing process (Leiss et al., 2010). Mathematical problem solving is thus a more authentic application of mathematics to real-world problems (Maaß, 2010). Therefore, mathematical problem solving is a necessary part of national curricula (Bush & Karp, 2013; Jupri & Drijvers, 2016; KMK, 2005).

Mathematical word problems are commonly investigated in a manner we will call *receptive mode*: children are given mathematical word problems which they then have to solve (Thevenot & Barrouillet, 2015). In contrast to the receptive mode, Frank and Gürsoy (2014) investigated how grade 5 students create word problem texts in response to a given illustration, which we will call word problem *production*. As both aspects – context and task-relevant structure – are important to design such a text, the visualizations used as stimuli had pictorial properties. As their study had a linguistic research focus on children's awareness of language and multilingualism in mathematics classrooms, the extent to which such tasks can provide information about children's word problem-solving processes is rather limited (Frank & Gürsoy, 2014). This chapter reports on subsequent research which investigated how fifth graders write mathematical word problems in response to given illustrations, pictorial representations of situations, and how that relates to both individual characteristics (mathematical skills and reading comprehension proficiency) and to the features of the illustrated mathematical situations (obviousness of appropriate mathematical model for the illustration).

2 Mathematical modeling

In general, word problems require a complex process of modeling that translates the real-world context into a solvable mathematical task (Borromeo Ferri, 2006). Blum and Leiss (2007) propose a cyclic model of mathematical modeling, comprising cognitive, linguistic, and math-specific processes (Leiss et al., 2019). Fig. 1 shows the phases of mathematical modeling following Blum and Leiss (2007) that are involved in word problem solving that differ in extent and focus, depending on the task (Leiss et al., 2010, 2019).

The problem-solving process of a word problem can generally be divided into two phases. On the modeling side, children need to construct a suitable representation of the situation through identifying the embedded mathematical problem. On the computational side, the mathematical problem has to be solved by applying appropriate strategies and operations (Mayer, 1999). Modeling processes bridge the

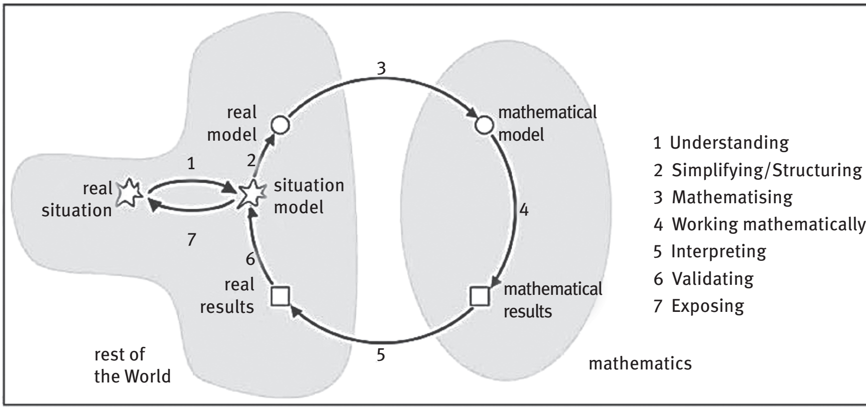

Fig. 1: Modeling cycle (Blum & Leiss, 2007).

gap between these phases and make mathematics applicable for real-world contexts (Leiss et al., 2019).

In the first step (1), children understand the real situation and translate it into a *situation model* of the context. This involves the student reading the text and sometimes providing an illustration containing the necessary information required to solve the task (Leiss et al., 2010). Thus, linguistic aspects are of great importance in this phase (Leiss et al., 2019). Empirical studies have shown that errors in this initial phase often lead to subsequent errors, which highlights the significance of understanding processes in word problems (Clements, 1980; Leiss et al., 2010). Recent research provided insights on linguistic difficulties in this phase (Daroczy et al., 2015; Leiss et al., 2019; Prediger et al., 2013). The information obtained from the word problem is organized and structured in the second step (2), leading to a *real model* (i.e., a model of the real-world problem). In this model, the context is reduced to (mathematically) relevant information and represented more precisely. Particularly in realistic word problems, this step might involve making hypotheses about information not given in the text or not obviously applicable (Leiss et al., 2010). For example, a problem such as “Linda bought 5 liters of juice for her café. How many glasses of 0.4 liters can she sell?” requires a child to keep in mind that you might not have partially filled glasses in some contexts (e.g., when selling them in a café).

At this point, the modeling process switches from real-world representations and considerations to the mathematical aspect of word problems (Mayer, 1999). While mathematizing (3), children transfer the real model into a *mathematical model* that can be solved by mathematical means (Leiss et al., 2010). As children need to know which mathematical model is appropriate, this step requires profound conceptual knowledge of the involved operations (Rittle-Johnson & Schneider, 2015; Daroczy et al., 2015; Leiss et al., 2010). In contrast, the purely mathematical solution process (4) that generates a *mathematical result* is more process-based and draws on strategy choice (e.g., counting, fact retrieval) as well as arithmetic proficiency (Daroczy et al., 2015).

In the next step (5), children interpret the mathematical result by relating the outcome to the real-world contexts. This process entails using conceptual knowledge and bridging the gap between mathematics and the real world again (Leiss et al., 2010; Mayer, 1999). This step leads to a *real result* that solves the mathematical part of the word problem under the given assumptions and with the numbers presented. However, this result has to be validated (6) within the context of the situation model (Leiss et al., 2010). Questions need to be asked: is the result realistic, are the assumptions adequate, and what other parameters besides the mathematical result could be taken into account? Based on these considerations in the situation model, children can finally present and explain (7) their solution to the word problem in terms of the real situation.

Obviously, the construction of a situation model is an integral part of the modeling process (Blum & Leiss, 2007), which can be underpinned by empirical research (Leiss et al., 2019, 2010). While building an adequate situation model of the real situation, children derive the necessary information such as numbers, operations, and their respective relation from the context. A mere combination and manipulation of numbers mentioned in the word problem is not sufficient to find a correct solution (Thevenot & Barrouillet, 2015; Verschaffel et al., 2000). Thus, the situation model allows transferring the not-yet-solved real-world problem into a solvable mathematical problem, if necessary under the constraint of simplification or additional reasonable assumptions; these restrictions are undone or at least discussed during the interpretation phase (Leiss et al., 2019, 2010).

3 Visualization and rewording in word problem solving

While constructing a situation model by understanding and organizing the real-world situation, children often make use of organizing illustrations. Hegarty and Kozhevnikov (1999) differentiate between pictorial illustrations that represent the context of a word problem and visual-schematic representations that organize the given information. For example, to a combinatorial word problem such as "Peter has two trousers and three shirts. How many different outfits can he wear?" a pictorial illustration might show Peter in front of his wardrobe, which does not give any hint of the problem structure or of an appropriate solving strategy. A visual-schematic illustration could show the trousers and shirts, enabling the imagining of combinations more easily. Empirical research revealed that the quality of the illustrations and in particular the use of visual-schematic illustrations is of significant relevance for solving word problems (Hegarty & Kozhevnikov, 1999; Vicente et al., 2008). More recently, Boonen et al. (2013) investigated the relation of the use of visual-schematic representations and reading comprehension, as both skills are discussed as crucial for solving word problems. Boonen et al. (2013) report significant effects of producing visual-schematic representations and reading comprehension. In contrast to visual-schematic visualizations, the use of pictorial illustrations seems to have negative effects on word problem performance (Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003). Against this background, some researchers suggest that pictorial representations rely on isolated information derived from single phrases or words, while schematic representations are more likely to integrate the information given in the text (Boonen et al., 2013; van Garderen & Montague, 2003).

As a consequence, a situation cannot be represented by only pictorial illustrations, but requires at least to some extent schematic information that indicates which mathematical concepts are involved. This applies in particular to dressed-up problems, in which the underlying relations are made as obvious as possible, because the intention of such problems is not teaching the modeling process, but practicing arithmetic within real-world contexts (Palm, 2009). Dressed-up problems do not contain an actual problem that has to be solved (e.g., "Linda bought 5 liters of juice for her café. How many glasses of 0.4 liters can she sell?"), but just contextualize a mathematical problem (e.g., "Linda has 2 glasses and gets 3 more. How many does she have now?"). Thus, a dressed-up problem refers to a specific and pre-defined solving strategy and a corresponding operation. In contrast, more complex problems allow for various solution strategies: In case of the given example of Linda's café above, several strategies are equally appropriate (divide 5 by 0.4, add up 0.4 until reaching 5, etc.). Illustrations of dressed-up and more complex problems differ in their obviousness. As dressed-up problems are meant to elicit the pre-defined operation, corresponding illustrations have to be as obvious as possible in order to guide children to that intended operation. A more complex problem, however, can be illustrated less obviously, as there is not one specific intended solution strategy. Naturally, less obvious illustrations of word problems are supposed to elicit more diverse strategies and operations than more obvious illustrations.

De Corte and Verschaffel (1987) highlight the relevance and benefits of rewording word problems as an instructional method in mathematics classrooms. Rewording means that children rephrase the situation of a given word problem in different words. Because children have to understand the structure and situation of the word problem to reword it correctly, it might contribute more to the “concept acquisition function” (De Corte & Verschaffel, 1987: 379) of arithmetic instruction. Therefore, rewording as instructional method might have positive effects on word problem performance (Thevenot & Barrouillet, 2015). As for visualizations, children also benefit from rewordings of the word problem structure. In contrast, situational rewordings that do not refer to the structure of a word problem but just slightly alter the context have no effect on word problem solving. These findings underpin the importance of children’s understanding of the situation given in a word problem.

4 Conceptual foundations of multiplication and division

Multiplication and division mark a change in conceptual thinking from addition and subtraction, in that the concepts are two-dimensional rather than one-dimensional. Whereas addition and subtraction can be conceptualized along a number line, multiplication and division require more complex metaphors. Early conceptions of multiplication or grouping can be understood as repeated addition; however, more complex conceptions of multiplication require two-dimensional models such as area models.

In addition and subtraction, quantities of the same type are added or subtracted, for example, 3 candies and 4 candies are added together; however, in multiplication the quantities involved are of different types, for example, 3 jars each with 4 candies per jar, means that there are 12 candies altogether. The division operation, explained as 12 candies divided equally between three jars, will result in four candies per jar (partitive division), or by placing four candies in each jar how many jars are needed will result in three jars (quotative division). From this distinction we see that in addition and subtraction the total number of candies is preserved, while in multiplication and division, the total number is transformed, the 4 candies are taken three times, which makes 12 candies (adapted from Schwartz, 1988: 41).

The developmental path from counting to additive reasoning to multiplicative reasoning is mapped below. We acknowledge that the multiplicative conceptual field incorporates many concepts and that proficiency may be incrementally gained through different pathways; nevertheless, there is a logical mathematical route for the teaching of these concepts that can be determined theoretically, and corroborated or challenged empirically.

5 From counting to additive reasoning to multiplicative reasoning

In the early years of schooling the students move from counting, to addition and subtraction, and to multiplication and division. The reasoning develops from additive reasoning to multiplicative reasoning and finally to this most important construct proportional reasoning, understood to be “the *capstone* of children’s elementary school arithmetic and the *cornerstone* of all that is to follow” (Lesh et al., 1988: 93–94).

Counting is the basic concept related to the creation of number concept (Desoete, 2015; Fritz et al., 2018). Counting meaningfully implies that students understand one-to-one correspondence, that is “the situation where there is one item to a set” (Bakker, van den Heuvel-Panhuizen & Robitzsch, 2014: 70). According to Bakker et al. (2014) one-to-many correspondence provides the student with “the awareness that a set has more than one item and the student can count the groups according to number in a set” (p. 70). This concept then leads to the understanding of multiplication and division, which are the base concepts for more complex mathematical concepts in the multiplicative conceptual field such as ratio, fractions, and linear functions (Vergnaud, 1983). Multiplicative reasoning relates to proportional reasoning (Lesh et al., 1988).

Vergnaud’s (1983) rationale for clustering the concepts involving multiplication and division as the multiplicative conceptual field is that both operations are related in many everyday situations. As multiplication and division are inverse operations, a problem containing a many-to-many relationship can often be solved by multiplication and by division. In the example task of Linda’s café above, multiplication (multiplying 0.4 with increasing numbers until reaching 5) or division (divide 5 by 0.4) is a suitable operation. This conceptualization of the many-to-many relationship from multiplicative concepts to situations supports the rationale for this study.

6 The current study

The aim of this study was to identify how children produce complete word problems for illustrations of given situations. Based on pictures of multiplication or division situations, we investigated children’s word problem production, a process which can be interpreted as an inversion within the modeling cycle (Leiss & Blum, 2007): Children start with a situation model (2). To write a mathematical word problem, they create an appropriate real situation (1), a possible real model

(3), and a mathematical model (4). The relationship between writing a full mathematical word problem, numeracy, and literacy, which might be new to many students, can provide insights into fifth-graders' production of word problems.

Characteristics of the given situation are likely to affect the mathematical word problems that children produce, in particular the choice of operations. For example, when a task contains numbers in a specific and salient relation (e.g., numbers that can be divided without remainder such as 12 and 4), it can be translated directly to a suitable mathematical problem. However, if there is no directly translatable operation, with no common factors, (e.g., 13 and 4), the given situation has to be transformed into a possible task by manipulating the given information, and an appropriate word problem might be harder to find.

Mathematical word problems form part of a specific text genre (Frank & Gürsoy, 2014; Hyland, 2007). This context implies that mathematical word problems have particular aspects such as the necessary numerical information, a clearly formulated relation between them, and a mathematical problem (Frank & Gürsoy, 2014). In addition, mathematical word problems can have typical structures, such as "dressed-up" problems. As children are used to such structures from a teaching environment, they might rely on patterns they encounter often in mathematics classrooms. The use of typical word problem structures while writing mathematical word problems might mirror students' approaches to word problem solving.

The scope of the study has been operationalized by the following research questions:

(RQ1) To what extent are fifth graders able to produce word problems from situations given in illustrations with varying level of obviousness regarding an intended strategy?

While reading and solving word problems has been addressed by several studies in the past, children's production of word problems is less well researched (Frank & Gürsoy, 2014). We expect that children have more difficulties with writing word problems that are less obvious in terms of less clearly suggesting a certain multiplicative or divisional relation between the numbers.

(RQ2) How can multiplication and division performance as well as reading fluency predict the successful production of word problems?

Arithmetic performance and reading fluency both were predictors for solving word problems in recent studies (e.g., Leiss et al., 2019; Stephany, this volume). This raises the question, how arithmetic performance and reading fluency can predict children's writing of word problems?

(RQ3) To which operations do students refer in their word problems and to what extent does their choice relate to the illustration?
This research question focusses how illustration type and intended operation in the word problems written by the children relate. Among other representations, the multiplicative conceptual field can be illustrated as distribution contexts, which refer more to division tasks (e.g., Tasks 1 and 2 in Fig. 2), or repeated addition contexts, which refer more to multiplication tasks (e.g., Tasks 3 and 4 in Fig. 2). We expect that more obvious illustrations (Tasks 2 and 3 in Fig. 2) mostly lead to the operations intended by the illustrations.

(RQ4) How do fifth graders verbalize the parts of a complete mathematical word problem (background story, mathematical problem, and mathematical task)?
Word problems consist of different parts, such as a background story contextualizing the problem, a mathematical problem that has to be solved, and a mathematical task that specifies which question has to be solved in the word problem (Frank & Gürsoy, 2014). In addition, these parts have to match (e.g., the mathematical task has to refer to the mathematical problem). This research question aims at investigating how children address the different parts of a word problem and to what extent they can match them.

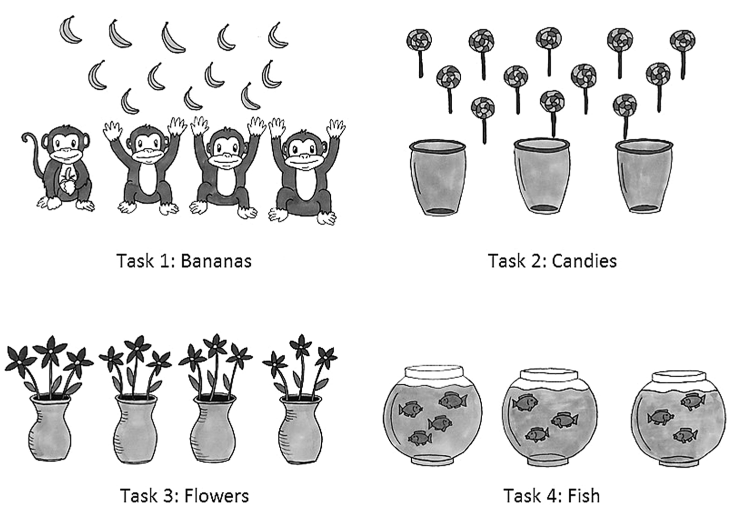

Fig. 2: Illustrations used in the word problem writing tasks (Frank & Gürsoy, 2014).

7 Method

7.1 Sample

Fifth graders (n = 368; 226 girls, 61.4%; 142 boys, 38,6%; m_{age} = 136.0 months, SD_{age} = 5.7 months) from Western Germany participated in the study. Students from 12 different schools were tested at the end of grade 5. In this part of Germany, students are separated based on their academic performance in primary school into three different school levels after grade 4. In this study, 234 (63.6%) students attended the highest school level ("Gymnasium"; preparing for university), 62 (16.8%) the medium school level ("Realschule"), and 72 (19.6%) the lowest school level ("Gesamtschule" and "Sekundarschule").

7.2 Instruments

Writing word problems: In line with Frank and Gürsoy (2014), children were asked to write a mathematical word problem for four given illustrations (Fig. 2). The illustrations included schematic as well as pictorial properties. The objects depicted set up a general context (e.g., monkeys eating bananas). Their arrangement in general was supposed to elicit a multiplication or division task by activating the respective operational understanding (vom Hofe, 1998): While two pictures (monkeys and candies) suggested a distributive context, the other two pictures (flowers and fish) were expected to lead to a compositional understanding.

Two pictures – one multiplication (flowers) and one division task (candies) – contained numbers from the multiplication table (12 and 3) that would elicit a whole number answer. The other two pictures were designed such that the answers would include fractions. As well, one of these illustrations was an intended multiplication task (fish; 3 + 3 + 4) and one was an intended division (bananas; 13 and 4). In case of number pairs with common factors (flowers and candies), the illustrated situations supported a dressed-up problem clearly. Thus, we will refer to these situations as relatively obvious compared to the other situations, in which number pairs did not support a direct dressed-up problem. We expected that the number pairs that were designed to elicit whole number answers would lead to more multiplication and division tasks than the less obvious number pairs suggesting rational number answers.

Multiplication and division problems: Children were given 27 multiplication and division problems in the number range up to 100 (Crombach's α = .72). The problems contained dressed-up word problems as well as pure arithmetic problems. This subtest was not timed, so children solved these problems without any time pressure. In all division problems number pairs could be divided without remainder. In all problems products and dividends were two-digit numbers, while factors, divisors, and quotients were single-digit numbers.

Reading skills: A standardized speed test for reading skills was administered (ELFE 1–6, Lenhard & Schneider, 2006). The test consisted of two subscales: sentence comprehension and text comprehension. In the sentence comprehension subtest, children were asked to complete a given sentence to form a meaningful and grammatically correct sentence. Text comprehension required reading a short text and answering one question on information given in the text.

7.3 Results

Word problem production (RQ1): We evaluated whether the produced word problems were correct as intended in the task ("Write a mathematical word problem connected to the picture"). We considered those word problems as correct that described a situation connected to the picture and contained a problem that could be solved mathematically.

To fulfil the criterion "connected to the situation," it was sufficient to adopt the given context (i.e., bananas and monkeys, candies and jars, flowers and vases, fish and fishbowls) in the word problem. Partial adoption (e.g., only one part: "There are 10 fish to sell at the pet shop. How many are there, after 4 are sold?") as well as additional, not given, information (e.g., "If one fish needs 12 g fish food per week, how much fish food do 10 fish need per week?") was also accepted. The same principle applied to children's counterintuitive interpretations of the pictures (e.g., "Here you see 3 duplication machines [=jars]. Every hour, 12 candies are produced by each machine. How many candies do you have in 1 hour, 13 days, and 4 years?").

The criterion "mathematical word problem" was handled rather strictly. Word problems were considered correct if they demanded a mathematical problem-solving process. That could be implemented by formulating a particular question ("There are 4 vases containing 3 flowers each. How many flowers are there?") or a direct operational instruction ("4 monkeys have 12 bananas. Share them equally").

Evaluated as not correct were word problems that just described the situation without any problem or question ("The jars are empty, but there are 12 candies in the air") and those that just described how to perform a (maybe suitable) operation ("Calculate 4 monkey multiplied by 12") or the solution

("4 candies go in each jar"). Moreover, the word problems did not necessarily have to report the amounts given in the illustration, because children might have anticipated that the illustrations would be part of the word problem (e.g., "How many fish are there?"). As the level of linguistic complexity of a word problem does not matter regarding its solvability, more complex word problems (e.g., "Here you see 3 duplication machines. Every hour, 12 candies are produced by each machine. How many candies do you have in 1 hour, 13 days, and 4 years?") were not distinguished from more simple word problems (e.g., "How many monkeys are there?"). Word problems did not need to include arithmetic operations to be considered correct (e.g., "Which mathematical geometrical shape do the jars have?").

In general, most students were able to correctly write a mathematical word problem suited to the picture. Task 1 (Bananas) was solved by 86.4%, task 2 (Candies) by 88.6%, task 3 (Flowers) by 87.2%, and task 4 (Fish) by 84.8% of the students. There were no significant differences regarding the correct solution rates between all tasks ($\chi^2(3) = 3.257$, $p = .354$). Correct solution rates did not differ between pictures showing a distributive (Tasks 1 and 2) or a multiplicative situation (Tasks 3 and 4) ($\chi^2(1) = .425$, $p = .514$). Contrary to our expectations, pictures directly eliciting multiplication or division tasks did not have higher correct solution rates than less obvious pictures showing situations that cannot be translated directly into either a multiplication or a division task ($\chi^2(1) = 2.757$, $p = .097$).

Reading and mathematical performance (RQ2): Correlation analyses between raw sums of correctly written word problems and number of correctly solved multiplication and division (mathematical performance), as well as sentence and text comprehension tasks, were run. While sentence comprehension correlations were low ($r = .194$, $p < .001$), text comprehension ($r = .322$, $p < .001$) and mathematical performance ($r = .292$, $p < .001$) showed low to medium correlations. This result is mirrored in a regression analysis explaining word problem production ($F(3,363) = 19.914$, $p < .001$, adj. $R^2 = .134$). Only mathematical performance ($\beta = .201$, $p < .001$) and text comprehension ($\beta = .239$, $p < .001$) were significant predictors, while sentence comprehension ($\beta = .027$, $p = .623$) had almost no predictive power. Tab. 1 provides an overview of these results.

Operations in the word problems (RQ3): Written word problems can refer to operations that lead to the solution more or less directly. Very clearly encoded operations are typical of dressed-up problems in which cue words or typical structures indicate the targeted operation (Frank & Gürsoy, 2014; Jitendra et al., 2007). Based on the produced word problems, the encoded operations can be compared. In most word problems (86.3%), a specific encoded operation could be assigned. These were the basic operations and counting. The operations

Tab. 1: Relations between predictor variables and word problem production.

Predictor	Correlation (*r*)	Regression (β)
Sentence comprehension	.194**	.027
Text comprehension	.322**	.239**
Mathematical performance	.292**	.201**

Note. ** = $p < .001$.

might be explicitly mentioned ("There are 4 vases and 3 flowers in each vase. Calculate how many flowers there are. Add them all up") or implicitly suggested by cues ("There are 12 candies and they are supposed to be shared equally among the jars"). In some produced word problems, no mathematical operation was encoded (no operation). In these cases, mostly no mathematical problem was formulated (e.g., "Those are vases," "There are 4 monkeys and one of them eats a banana"). In other word problems a mathematical problem was formulated, the specific operation, however, was not explicated (unclear, e.g., "Calculate how many flowers are in one vase"). Fig. 3 shows the shares of these categories for the four tasks.

In general, word problems encoded operations as intended when designing the illustrations. Tasks 1 (Bananas) and 2 (Candy) mostly lead to division encoding, while in task 3 (Flowers) most word problems encoded a multiplication task. However, the picture is rather fuzzy for task 4 (Fish): About one third of the word problems encoded a multiplication or division task and another third was classified as unclear. In most of these word problems, children described the situation and simply asked for the total sum of fish but indicated no particular operation. As depicted in Fig. 3, there was a greater variety in the encoded operations in the less obvious tasks 1 and 4.

Verbalizations of word problems (RQ 4): A mathematical word problem can be decomposed into three main parts (Frank & Gürsoy, 2014): A background story that describes the situational setting of the word problem (e.g., "Lukas, Emma, and Lilli go to the kiosk to buy some candies"), a mathematical problem ("They bought 12 candies. They wanted to share them fairly"), and a mathematical task ("How many candies does everyone get?"). While the mathematical problem refers to the operational relation of the numbers, the mathematical task refers to the formulation of a prompt to solve the mathematical problem. Obviously, the background story is not necessary to solve the mathematical problem. However, the situational background is relevant when the mathematical result has to be evaluated and explained regarding the real-world context (steps 6 and 7) in the modeling cycle (see Fig. 1, Blum & Leiss, 2007). Note that

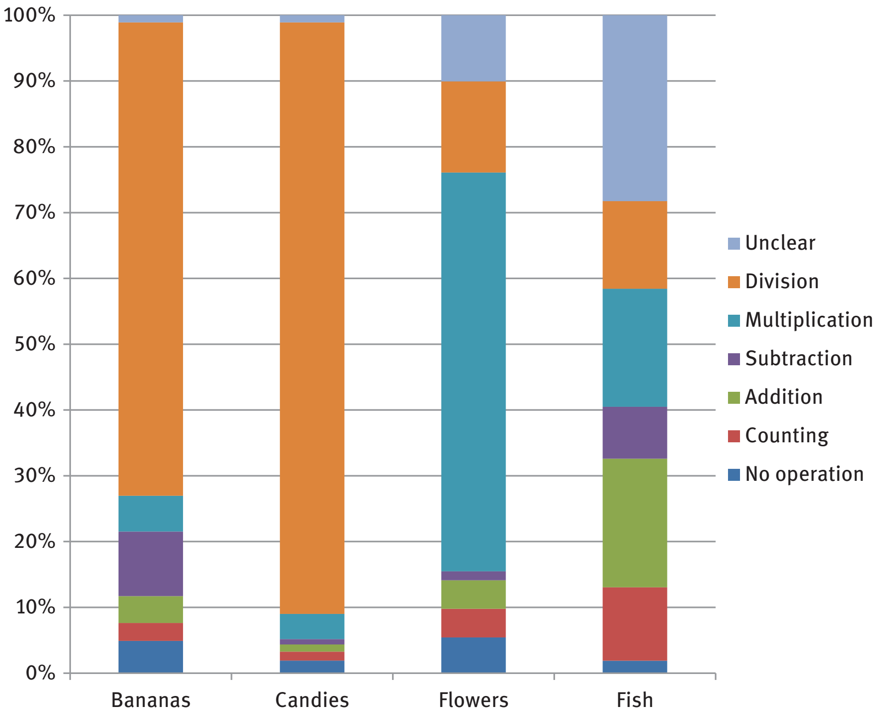

Fig. 3: Operations encoded in the word problems.

the background story has to provide more situational information than describing the items relevant for the mathematical task (e.g., "There are 4 monkeys and 13 bananas"). We claim that a lack of situational contexts is one of the characteristics for "dressed-up" problems, since such word problems do not refer to a realistic context (Verschaffel et al., 2000). Word problems can be solved only if the (numerical) relation of the elements (e.g., candies and children) is specified. This criterion constitutes a mathematical problem. Finally, a word problem is supposed to contain a mathematical task (e.g., "How many candies and jars are there in total?"). As mathematical word problems might have more than one possible task, this part is highly relevant. Obviously, the mathematical problem and the task have to be related in order to be solved. In other words, the mathematical task has to extend the mathematical problem.

In this study, we accepted such background stories that introduced names and specific characters (e.g., parents or zookeepers) or any situation going beyond the mere mention of the depicted items. Any mathematical relation between elements that can be solved by mathematical means was coded as a valid mathematical problem. Finally, mathematical tasks were accepted if they formulated a question that can be solved mathematically in general. Note that mathematical problems and mathematical tasks were evaluated separately and thus did not necessarily have to be related. If the mathematical problem and the mathematical tasks were related, this was evaluated as an independent characteristic of the produced mathematical word problem. In the last step, we checked if the mathematical word problem contained all the information necessary to solve the respective word problem.

Table 2 provides an overview of the percentages of correctly verbalized characteristics as found in the word problems the students produced. In general, there are little differences between the tasks. This indicates that the way children produced word problems was independent of the given picture and refers to children's ideas and skills in general. Children's word problems rarely contained background stories. Chi-square statistics reveal no significant differences in correct response rates between the tasks regarding writing a background story ($\chi^2(3) = 1.894$, $p = .595$), verbalizing a mathematical problem ($\chi^2(3) = 1.232$, $p = .745$), and formulating a mathematical task ($\chi^2(3) = 2.885$, $p = .410$). Only success in matching mathematical problem and mathematical task differed between the tasks ($\chi^2(3) = 27.783$, $p < .001$), obviously due to lower matching rates in the less obvious tasks (bananas and fish).

Tab. 2: Correctly verbalized characteristics of the produced word problems in percent.

	Bananas	Candies	Flowers	Fish
Background story	12,5	14,9	16,0	16,3
Mathematical problem	64,9	63,3	66,6	67,7
Mathematical task	88,6	91,3	92,4	89,4
Match of problem and task	58,4	76,4	71,5	67,7

As they were not explicitly asked to do so and background stories are not obligatory, it cannot be guaranteed that children would produce more background stories if these were made more salient in advance (e.g., by providing an example). Children did not provide mathematical problems in about one third of the word problems. One reason might be that not all students understood the necessity for this integral part of a mathematical word problem.

About one third of the produced word problems did not contain a proper mathematical problem. This result indicates that verbalizing the mathematical problem, which includes describing the relevant context and mentioning the necessary information, was a main obstacle for students in this study.

In contrast, children had fewer difficulties in formulating a mathematical task in their word problems, as the vast majority of produced word problems contained a clearly and explicitly formulated task (e.g., "How many flowers go in each vase?"). One might thus argue that children are rather familiar with the aspect of mathematical task in word problems.

While formulating tasks did not challenge students considerably, matching the task to the mathematical problem was not easy for all students. Notably regarding the less obvious tasks that contained numbers that had no common factors (Bananas and Fish), children more often failed to match the problem and the task. In general, this appears as a second main obstacle to students in writing a mathematical word problem.

8 Discussion

To the best of our knowledge, the vast majority of empirical studies on modeling focus on children's construction of a situation model based on a given text. This study aimed at investigating to what extent students are able to write a word problem to a given situation model. In the context of the cyclic model by Blum and Leiss (2007), this approach means reversing the process of understanding between the real situation and the situation model, which generally implies bidirectional processes (Borromeo Ferri, 2006; Leiss et al., 2010). Although this is not part of the national curriculum standards, children did not show particular difficulties with writing mathematical word problems to given situations, as mirrored throughout in the high correct solution rates of about 85%. The produced word problems mostly were "dressed-up" problems suited to the simple situations, which indicate that students generally know how to encode a word problem directly. Characteristics of the given situations, such as being more multiplicative or more distributive, did not affect correct solution rates significantly. In addition, it had no effect on solution rates if numbers were selected from the multiplication table or not.

In line with previous findings, writing word problems is associated with reading proficiency and mathematical performance (Daroczy et al., 2015; Leiss et al., 2019). Thus, these individual factors affect understanding in both directions. In particular text comprehension has to be highlighted, while sentence comprehension correlated only minimally with writing proficiency. This result strengthens the position that mathematical modeling relies on a comprehensive understanding of the situation and goes beyond cue words or phrases (Thevenot & Barrouillet, 2015). However, correlations and the predictive power of the regression analysis in this study are rather low, which corresponds to the literature on this topic (e.g., Leiss et al., 2019).

Children mostly used the intended operations multiplication and division. Varying the obviousness of the given situation led to more variety in the operations used. This outcome might indicate that fifth graders anticipate a kind of standardized dressed-up problem based on typical situations. This might result from schooling that focusses on this kind of word problems (Verschaffel et al., 2000). How deep children's understanding of the situation in such cases is is doubted (Maaß, 2010). It is rather likely that children produce (and read) such word problems by applying a certain, well-trained verbal frame (Borromeo Ferri, 2006).

Obviously, situations indicating a distribution elicit division more specifically than groupings do for multiplication. A closer look at the visualizations used in classrooms might provide a better understanding of the processes underlying children's use of operations.

Children's focusing on standardized dressed-up problems might also be observed in the few differences between the less obvious (bananas and fish) and more obvious (flowers and candy) situations in the study. First, it has to be noticed that obviousness did not significantly affect performance at all. However, the operations referred to in the word problems were more diverse in the less obvious situations, which indicates that the children's search for a dressed-up problem was hampered by the less obviousness of relatively prime numbers. The less obviousness of tasks 1 and 4 might have resulted in less-well-trained problem-solving processes, in which children used a broader variety of operations. Consequently, not all of these operations could be conceptually supported by the illustrations to the same extent. In this problem-solving process, children did not struggle more with formulating a mathematical problem or a mathematical task than in the more obvious tasks. However, children showed more difficulties in relating the mathematical problem and the task, which could be consequence of their adapted problem-solving process that digressed from the well-trained search for a dressed-up problem.

Children's production of word problems might reflect what they focus on when they encounter word problems in mathematics classrooms. The results show that the mathematical task was particularly salient to the children in this study. The mathematical task is usually the actual question that is asked in word problems. Children obviously are used to focus on that question as they have to answer it and are evaluated based on their answer. However, the mathematical problem, which is often considered as a main aspect of a word problem, was less often given attention. In addition, the children in this study showed considerable difficulties in matching the mathematical problem and task (see Tab. 2). These results suggest that fifth graders pay much attention on the question asked at the cost of understanding the mathematical problem. It would be of great interest to investigate to what extent children are aware of the different parts of a word problem and how they address them.

Such knowledge of children's meta-knowledge of word problems could be useful for instruction. Based on the framework of so-called *genre pedagogy*, producing word problems could help students to understand word problems (Hyland, 2007). The central idea of genre pedagogy is that every discipline has its own typical text types (genres) that have specific properties and follow particular rules. This coherence allows genre-specific teacher scaffolding. A typical "teaching-learning cycle" (Hyland, 2007: 159) covers teaching the purpose and typical use of the genre; identification of key characteristics of the genre as well as possible variations; a joint construction of a typical text type such as a word problem, in which the teacher supports the children by providing appropriate exercises; children's independent construction of such a text; and interrelating the text type to other types (e.g., theorems).

With the teacher scaffolding and co-constructing word problems, children can understand which parts (e.g., given information or targeted result) a word problem has and how these parts can be identified. This instructional method might be a successful addition to existing approaches such as schema-based instruction to promote students' ability to establish a suitable situation model (Frank & Gürsoy, 2014; Hyland, 2007; Jitendra, 2019). Developing such an intervention definitively asks for more detailed and more qualitative research on children's production of word problems in response to given situations.

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