Sarah R. Powell, Samantha E. Bos, and Xin Lin

The assessment of mathematics vocabulary in the elementary and middle school grades

1 Introduction

Students use academic language, which involves vocabulary, grammatical structures, and linguistic functions, to learn knowledge and perform tasks in a specific discipline (e.g., mathematics; Cummins, 2000). Understanding these discipline-specific ways of using language requires deep knowledge of discipline-specific content and a keen understanding connecting academic language to learning (Fang, 2012). Therefore, not surprisingly, academic language has been shown to be closely related to academic performance (Kleemans et al., 2018) and a significant predictor of academic achievement (Townsend et al., 2012). Mathematics, a challenging discipline for many students (Berch & Mazzocco, 2007), also develops academic language specific to the discipline, which is often referred to as mathematics language. Mathematics language is used to express mathematical ideas and to define mathematical concepts, and it can facilitate connections among different representations of mathematical ideas (Bruner, 1966).

In this Introduction, we provide a definition of mathematics vocabulary and discuss the importance of understanding mathematics vocabulary. Then, we review why and how students experience difficulty with mathematics vocabulary. In the rest of the chapter, we describe the development and testing of several measures of mathematics vocabulary. These measures could be used by educators to understand which mathematics vocabulary cause difficulty for students and could be a focus of mathematics instruction.

1.1 Definition of mathematics vocabulary

Mathematics vocabulary, a key component of mathematics language (Moschkovich, 2015; Simpson & Cole, 2015), includes terms routinely used in mathematics instruction, textbooks, and assessments (Monroe & Orme, 2002; Moschkovich, 2013). In this chapter, we define *mathematics vocabulary* as terms used to describe specific mathematical concepts or procedures. We use the word "term" because a large proportion of mathematics vocabulary includes more than one word (e.g., *greater than*,

https://doi.org/10.1515/9783110661941-016

minus sign, *quarter past*, *rectangular prism*, *unequal shares*). In the mathematics learning progression (Browning & Beauford, 2011; Purpura & Logan, 2015), mathematics vocabulary first emerges as number terms (e.g., *one*, *three*), terms representing quantities (e.g., *more*, *less*), and terms representing spatial relations (e.g., *above*, *below*). Some of the earliest learning of mathematics occurs through learning mathematics vocabulary (Purpura et al., 2017).

As the complexity of mathematics skills increases by grade level, mathematics vocabulary becomes accumulatively complex with students expected to understand hundreds of different mathematics vocabulary terms by middle school (Powell et al., 2017). Mastering foundational mathematics vocabulary may be necessary for understanding advanced mathematics vocabulary and concepts. For example, students need to master the term *multiple* to understand the term *least common multiple*, which describes part of the procedure for identifying common denominators when adding or subtracting fractions. Given the complexity and accumulative nature of mathematics vocabulary, an explicit focus on vocabulary has become a point of interest in mathematics education (Browning & Beauford, 2011; Riccomini et al., 2015). In the next section, we discuss mathematics standards and research practices that highlight the importance of mathematics vocabulary.

1.2 Importance of mathematics vocabulary

Mathematics practice standards in the United States (U.S.) highlight the use of mathematics vocabulary as a medium to learn and perform mathematics. For example, within the Curriculum Focal Points of National Council of Teachers of Mathematics in the U.S. (2006), students are expected to "develop vocabulary to describe" various attributes of shapes (p. 31) or use "language" to compare quantities (p. 11). Similarly, use of mathematics vocabulary as a medium to learn mathematics is also shown in the mathematics standards used in the U.S. (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Specifically, mathematics standards suggest that students be able to use clear vocabulary to communicate precisely to others, explain how to solve problems, construct viable arguments, and critique the mathematic reasoning of others. For example, it is outlined that students should use "language to describe" (p. 42) or "describe their physical world using . . . vocabulary" (p. 9).

In addition to a focus in mathematical standards in the U.S., mathematics vocabulary is important because of its association to mathematics performance. For example, Powell and Nelson (2017) noted a significant correlation between the mathematics vocabulary and mathematics fluency (i.e., fluency with mathematics facts such as $4 + 7$ or $36 \div 6$) scores of U.S. first-grade students.

Similarly, Powell et al. (2017) identified significant correlations between a test of mathematics vocabulary and mathematics computation (i.e., addition, subtraction, multiplication, or division or multi-digit numbers involving algorithms) for both third- and fifth-grade U.S. students. Peng and Lin (2019) noted an analogous pattern with Chinese fourth-grade students with significant correlations on measures of mathematics vocabulary and mathematics fluency, computation, and word problems. Besides correlational findings, Fuchs et al. (2015) further demonstrated that mathematics vocabulary may partially or fully explain the relation between general cognitive skills and students' word-problem solving performance.

Consistent with the focus of mathematics vocabulary in U.S. practice standards for mathematics and more recent research demonstrating an association between mathematics-vocabulary knowledge and mathematics performance, an increasing number of researchers have started to focus on the importance of instruction related to mathematics vocabulary (Harmon et al., 2005; Livers & Elmore, 2018; Monroe & Orme, 2002; Riccomini et al., 2015). Monroe and Panchyshyn (1995) explained instruction needs to occur for four categories of mathematics vocabulary. First, students need to learn technical terms that include terms specific to mathematics (e.g., *decagon*). Second, students need instruction on subtechnical terms. Subtechnical terms have multiple meanings, one of which is mathematics related (e.g., *cube*). Third, students require instruction on symbolic terms (e.g., *minus sign*). Fourth, educators need to ensure students understand general terms. These are non-mathematics terms (e.g., *measure*) used in the mathematics classroom.

Many researchers and educators have provided suggestions for teaching mathematics vocabulary including using explicit instruction (Bay-Williams & Livers, 2009; Monroe & Orme, 2002), mnemonic strategies (Riccomini et al., 2015), and graphic organizers with the definitions, characteristics, examples, and nonexamples of a mathematics vocabulary term (Bruun et al., 2015). However, in classrooms, mathematics vocabulary instruction is not often prioritized. Specifically, except for the instruction of definitions in textbook, educators provide students with few opportunities to explicitly learn mathematics vocabulary (Monroe & Orme, 2002). In addition, educators of mathematics often use informal language in the classroom – for example, *diamond* for *rhombus*, *bottom number* for *denominator*, and *line* for *fraction bar* (Karp et al., 2014; Rubenstein & Thompson, 2002). Because students do not receive enough mathematics vocabulary instruction, understanding mathematics vocabulary can be difficult.

1.3 Difficulty of understanding mathematics vocabulary

As described by Barrow (2014), mathematics is not a universal language, and all students should be considered learners of mathematics vocabulary and language. Not understanding academic language may prohibit students from engaging fully in the mathematics classroom (Ernst-Slavit & Mason, 2011; Schleppegrell, 2012). Mathematics vocabulary may be difficult for many students because of the complexity of the vocabulary. Rubenstein and Thompson (2002) listed 11 difficulties students may encounter when learning mathematics vocabulary terms: (1) when used in mathematics context, some common English terms have alternative meanings (e.g., *expression*, *face*); (2) some terms have similar but more precise meanings (e.g., *area*, *average*); (3) some terms involve technical terms specific to mathematics (e.g., *parallelogram*, *integer*); (4) some terms have more than one mathematical definition, such as *cube* as a solid figure versus to *cube* a number; (5) some terms used in mathematics have different technical meanings when used in other disciplines (e.g., *prism* is a solid figure in mathematics versus *prism* is an object that refracts light in science); (6) some mathematical terms have homophones or homographs, such as *pi* versus *pie*; (7) some related mathematical terms have distinct meanings, but are easily confused (e.g., *divisor* and *dividend*); (8) the translation of a single mathematical term into another language may have multiple ways, which may cause confusion – for example, the Spanish word *tabla* can be translated to the data *table*, but not the *table* we eat from (this would be *mesa*); (9) the spelling of terms is not regular (e.g., *half* vs. *halves*); (10) some mathematical terms can be verbalized in more than one way (e.g., *one-quarter* and *one-fourth*); and (11) the use of informal language in many classrooms, as discussed earlier, make the learning of mathematics vocabulary more difficult.

Given the difficulty of understanding mathematics vocabulary, research on mathematics-vocabulary instruction is in need. Only one experimental study (Petersen-Brown et al., 2019), according to our knowledge, has specifically focused on the instruction of mathematics vocabulary. Although their study showed the effectiveness of mathematics-vocabulary instruction for third and fourth graders, their study only included instruction about eight mathematics-vocabulary terms. Future experimental research involving more grade-level important mathematics vocabulary is in need. However, before conducting experimental work, it is necessary to understand how to develop a measure of mathematics vocabulary. Understanding such process could help determine important mathematics vocabulary terms to be involved in mathematics-vocabulary intervention. Such measures should assist in determining the efficacy of any mathematics-vocabulary intervention in addition to providing an understanding of the baseline levels of mathematics vocabulary of students and variability in mathematics-vocabulary performance. In the next section, we describe our efforts at designing several mathematics-vocabulary measures at different grade levels.

2 Development of mathematics-vocabulary measures

Across the last few years, we developed a series of mathematics-vocabulary measures for use at grade 1 (ages 6–7), 3 (ages 8–9), 5 (ages 10–11), 7 (ages 12–13), and 8 (ages 13–14). We utilized a similar development process across the measures, and we describe this process in the following paragraphs to demonstrate our development framework and to aid any researchers or educators who want to develop their own mathematics-vocabulary measures in English or another language.

2.1 Determine grade-level mathematics vocabulary

First, we compiled lists of grade-specific mathematics vocabulary. Unfortunately, educators in the U.S. do not have access to a common list of mathematics vocabulary at each grade level, so we accessed several mathematics textbooks at a single grade level (e.g., first-grade mathematics textbooks) and created our own grade-level lists of mathematics vocabulary. Our list of textbooks included *enVision MATH*, *Everyday Mathematics*, *Go Math!*, and *Houghton Mifflin Math*. We reviewed glossaries in the textbooks and created a database of mathematics-vocabulary terms and definitions. We accessed two or three textbooks at each grade level from kindergarten through eighth grade. Our complete database contained 1,220 mathematics-vocabulary terms with many terms appearing at several grade levels. Fig. 1 displays the counts of mathematics-vocabulary terms within each grade level.

In the early elementary grades (i.e., kindergarten through second grade), glossaries featured approximately 150 different terms. In the third grade, we noted a large increase in mathematics-vocabulary expectations, and we attributed this to the introduction of multiplication, division, and fractions in U.S. classrooms in the third grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). We identified another substantial increase from fifth grade to sixth grade as U.S. students enter middle school, and instruction about algebra and rational numbers becomes a core focus of mathematics instruction.

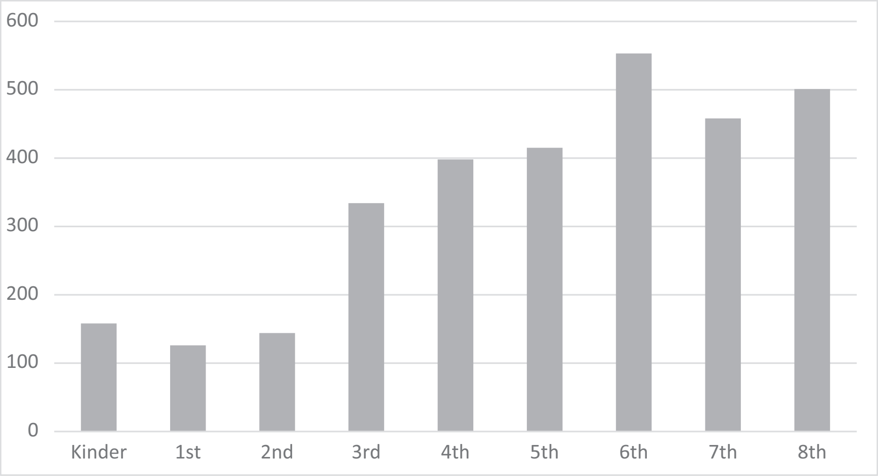

Fig. 1: Mathematics terms featured within grade-level textbooks.

2.2 Focus on important mathematics vocabulary

After we compiled the vocabulary database, we determined it was important to streamline the mathematics vocabulary that would be included in any measure. That is, we could not possibly create a test with 553 separate mathematics-vocabulary terms identified from sixth-grade glossaries; no student would take that test, and no educator would want to grade that test.

Our process for streamlining involved the following. We awarded 1 or 2 points if a term appeared in one or two additional glossaries within a specific grade level. We believed a term gained importance if multiple textbooks written by different author teams used the same term. We identified many textbook-specific terms (e.g., *break apart a ten*, *count back*, *in all*) that did not appear in more than one textbook. Then, we awarded 1 to 3 points if the term appeared in glossaries at other grade levels. For example, during the first development of the first-grade measure, a term earned points for also appearing in kindergarten, second grade, and third grade glossaries. We noted the use of some terms (e.g., *half past* or *T-chart*) occurred only at a single grade level. Other terms (e.g., *greater than*, *length*, *number line*, *octagon*) were utilized across several grade levels, which indicated such terms had carried greater weight because students would hear, see, and use these terms across multiple years. Third, we consulted mathematics standards used across the U.S. (National Governors Association Center for Best Practices & Council of

Chief State School Officers, 2010) and determined whether standards explicitly used terms. For example, at first grade, *addend*, *quarters*, and *trapezoid* are each explicitly named within the first-grade mathematics standards. A term earned 1 point if it is specifically mentioned within standards. In sum, we included the terms with the highest point values.

For the development of a middle school measure for use in seventh and eighth grades, we further tightened our method for streamlining vocabulary terms (Hughes et al., 2020). After confirming terms were featured across glossaries and grade levels and within standards, we had approximately 200 terms remaining. We placed the terms on an online survey and asked middle school educators to select the 50 most important mathematics-vocabulary terms for their grade level. For our measure, we included approximately 70 terms that multiple educators categorized as important. We noted consistency across teachers; for example, 53 of 53 educators agreed *expression* was important with 52 teachers coming to agreement on *distributive property* and 50 educators stating *equation* was an important middle school mathematics vocabulary term.

To avoid basement or ceiling effects, we included terms introduced before and beyond the target grade levels. We did this based on grade level of introduction of the term in the textbook glossaries. For example, at third grade, we included many terms (e.g., *circle*, *odd*) introduced before the third grade, some as early as kindergarten, to ensure students who experienced difficulty with third-grade terms could answer some questions about more familiar terms. At fifth grade, we included several terms introduced beyond the fifth grade (e.g., *positive integer*, *slope*) to ensure a distribution of mathematics-vocabulary scores and limit ceiling effects. For this reason, it was important to collect terms from textbook glossaries across the elementary and middle school grades to be able to understand the grade level of introduction of a term, in how many grades the term appeared, and the grade level of disappearance of a term.

2.3 Develop questions to assess mathematics-vocabulary knowledge

We designed each of our measures for educators to use in the general education classroom. Therefore, we planned for a paper-and-pencil task in which students would read prompts and respond via writing. For each selected term, we created three levels of questions: recall, comprehension, and use in complex tasks (Haladyna & Rodriguez, 2013). Figure 2 displays sample questions. Often, our recall questions involved matching a letter to a term. Comprehension questions

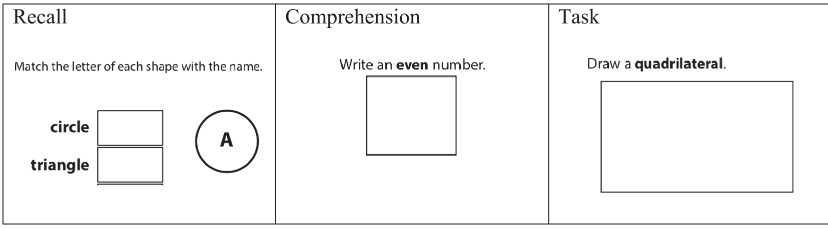

Fig. 2: Examples of different questions on third-grade measure.

encouraged the students to provide a quick response. Task questions asked students to draw something. After generating the levels of questions, we selected the level of question that would be easiest for student response, but we also considered balancing levels so that students answered a variety of questions.

When creating the test forms, we started each test with an easier question as the first problem. We then grouped similar items together (e.g., questions about *even* and *odd* numbers or *numerator* and *denominator* appeared next to one another), but we distributed terms by mathematical domain. For all grade levels except the first grade, examiners read a set of directions and then provided time (e.g., 20 min or 30 min) for students to work. Examiners read no questions or terms aloud. Our decision to not read the test aloud meant that students had to use reading to interpret the mathematics-vocabulary term, just as students would do in a textbook, on a test, or on a computer screen. At the first grade, because of the limited reading experiences of the students, examiners read each prompt and answer choices aloud and permitted students to respond to the question before moving on to read the next prompt aloud.

3 Student performance on mathematics-vocabulary measures

In the U.S., we administered mathematics-vocabulary measures to students at different grade levels. In the following sections, we describe five different studies. In the first three studies, we investigated the use of mathematics-vocabulary measures at understanding the mathematics-vocabulary knowledge of students in grade 1 (Powell & Nelson, 2017), in grades 3 and 5 (Powell et al., 2017), and grades 7 and 8 (Hughes et al., 2020). In the fourth study (Forsyth & Powell, 2017), we compared the mathematics-vocabulary scores of fifth-grade students with and without mathematics or reading difficulty. In the fifth study

(Powell et al., 2020), we compared mathematics-vocabulary scores of third-grade students with and without mathematics difficulty who also categorized as dual-language learners or native English speakers. Tab. 1 presents an overview of the data from each study.

3.1 Elementary students' mathematics vocabulary knowledge

3.1.1 Grade 1

Powell and Nelson (2017) investigated the mathematics-vocabulary knowledge of first-grade students and also explored the relationship between general English vocabulary, mathematics fluency, and mathematics vocabulary. Students showed wide variability in mathematics-vocabulary performance. Findings suggested a significant and positive relationship between both general English vocabulary and mathematics vocabulary as well as mathematics fluency and mathematics vocabulary.

Powell and Nelson (2017) explored whether students struggled with a particular type of mathematics vocabulary. Few clear patterns emerged regarding which mathematics-vocabulary terms caused the most difficulty for students. According to categories suggested by Monroe and Panchyshyn (1995), students had higher accuracy rates in identifying general mathematics-vocabulary terms (91.1%) than symbolic terms (54.5%), technical terms (42.0%), or subtechnical terms (56.4%). Student accuracy varied based on whether the vocabulary term was introduced in kindergarten, first or second grade. That is, students demonstrated a 67.1% accuracy rate with terms introduced in kindergarten textbooks, followed by an accuracy of 48.8% of terms introduced in the first grade. As expected, accuracy on terms introduced in the second grade was 29.2%. Powell and Nelson (2017) tested the measure near the final weeks of the first grade when students should have mastered kindergarten and first-grade mathematics vocabulary. This finding suggested students may already struggle with mathematics-vocabulary terms as early as the first grade.

3.1.2 Grades 3 and 5

Powell et al. (2017) continued to explore the mathematics vocabulary knowledge of elementary-aged students, examining trends in the knowledge base of third- and fifth-grade students. The authors analyzed the mathematics vocabulary knowledge, general English vocabulary, and mathematics computation knowledge

Tab. 1: Descriptions of Mathematics-Vocabulary Assessments.

Study	Grade	n	Comparison	Measure	Maximum terms	M	SD	Range	Correlations or comparison
Powell & Nelson (2017)	1	104	All students	Mathematics Vocabulary – Grade 1	64	36.3	8.1	15–55	r = .697 with general English vocabulary r = .586 with mathematics fluency
Powell et al. (2017)	3	65	All students	Mathematics Vocabulary – Grades 3 and 5	129	35.6	14.0	6–68	r = .606 with general English vocabulary r = .669 with mathematics computation
	5	128	All students			57.5	20.6	5–100	r = .659 with general English vocabulary r = .626 with mathematics computation
Hughes et al. 2020	7	338	All students	Mathematics Vocabulary – Grades 7 and 8	69	45.0	12.6	NR	d = 0.22 (8th grade > 7th grade)
	8	153	All students			48.7	11.2	NR	
Forsyth & Powell (2017)	5	70	No difficulty	Mathematics Vocabulary – Grades 3 and 5	129	68.1	15.1	NR	g = 1.37 (typical > MD) g = 1.40 (typical > RD) g = 3.21 (typical > MD + RD)
		16	MD			46.0	19.4	NR	g = -0.03 (MD = RD) g = 1.41 (MD > MD + RD)
		18	RD			46.6	16.1	NR	g = 1.67 (RD > MD + RD)
		10	MD + RD			21.9	10.4	NR	

Powell et al. (2020)	3	242	DLL; no difficulty	Mathematics Vocabulary – Grade 3 (revised)	45	14.1	5.7	0–42	$d = 0.78$ (no difficulty non-DLL > DLL)
		545	Non-DLL; no difficulty			19.1	7.0	0–44	
		36	DLL; EQD			10.6	5.3	3–28	$d = 0.54$ (EQD non-DLL > DLL)
		89	Non-DLL; EQD			13.6	5.9	3–40	
		101	DLL; WPD			9.0	4.7	0–23	$d = 0.32$ (WPD non-DLL = DLL)
		34	Non- DLL; WPD			10.4	3.5	4–20	
		136	DLL; EQ + WPD			7.0	4.3	0–21	$d = 0.24$ (EQ + WPD non-DLL = DLL)
		75	Non- DLL; EQ + WPD			8.0	4.5	0–26	

of both third- and fifth-grade students. Similar to the findings of Powell and Nelson (2017), Powell et al. (2017) determined general English vocabulary and mathematics computation knowledge were both significant predictors of mathematics-vocabulary knowledge in third- and fifth-grade students, although the strength of the relationships varied depending on the students' level of mathematics vocabulary. The relation was significantly stronger in third-grade students. That is, third-grade students demonstrated greater dependence on general English vocabulary and computation compared to fifth-grade students.

Furthermore, the authors examined whether the influence of general vocabulary and mathematics computation on mathematics vocabulary varied for students with different levels of mathematics vocabulary. The findings showed that general English vocabulary was a more accurate predictor of mathematics vocabulary for students with lower mathematics-vocabulary scores, but mathematics computation served as a stronger predictor for students with higher mathematics-vocabulary scores in the third-grade sample. In the fifth-grade sample, however, mathematics computation was a stronger predictor for students with lower mathematics-vocabulary performance but not for students with the higher mathematics-vocabulary performance; general English vocabulary was a significant predictor across all performance levels of mathematics vocabulary. This reversal in the trend lines between grades could reflect the growing development of a mathematical lexicon in the third grade and therefore a greater dependence on general English vocabulary compared to fifth-grade students.

3.2 Secondary students' mathematics knowledge

3.2.1 Grades 7 and 8

Hughes et al. (2020) measured mathematics vocabulary of both seventh- and eighth-grade students. The authors determined the mathematics-vocabulary measure was well targeted for middle school students and measured mathematics vocabulary with high validity and high reliability. The differential performance between seventh- and eighth-grade students provided evidence to suggest that a mathematics-vocabulary measure can detect differences in the growth of mathematics vocabulary from one grade level to the next. Also notable was the wide variability in students' scores. The average score of both versions of the mathematics-vocabulary measure showed that students knew only two-thirds of vocabulary deemed essential by middle school educators, textbooks, and standards.

3.3 Mathematical vocabulary knowledge of students with difficulties

3.3.1 Students experiencing mathematics and reading difficulty

Forsyth and Powell (2017) examined the impact of mathematics and reading difficulties on the mathematics-vocabulary knowledge of fifth-grade students. Using the same measure of mathematics vocabulary as Powell et al. (2017), Forsyth and Powell (2017) examined the mathematics-vocabulary scores of students who experienced a reading-only difficulty (RD-only), a mathematics-only difficulty (MD-only), or comorbid reading and mathematics difficulty (MDRD), determined by performing below cut-off benchmarks on a test of general English vocabulary, a test of mathematics computation, or both assessments, respectively. The authors compared scores to typically developing students. Typically developing students demonstrated significantly higher performance over students with RD-only, MD-only, and MDRD, with the largest effects in comparison to students with MDRD. Students with RD-only and MD-only did not differ significantly on mathematics-vocabulary scores, but both groups of students had significantly higher mathematical-vocabulary scores than students with MDRD.

When Forsyth and Powell (2017) examined the impact of the year the term was introduced to students upon student knowledge of that term, the same pattern held, in which typically developing students outperformed students with MD-only, RD-only, and MDRD; students with MD-only and RD-only did not significantly differ; and students with MD-only and students with RD-only outperformed students with MDRD, with the greatest contrast between typically developing students and students with MDRD. This pattern did not hold for mathematics-vocabulary introduced in fifth-grade and sixth-grade textbooks and not included in textbook glossaries. Specifically, when mathematics-vocabulary were terms introduced in fifth-grade or not included in textbook glossaries, typical students significantly outperformed all other difficulty groups, with no significant differences among MD-only, RD-only, and MDRD groups. Given that mathematics-vocabulary introduced in sixth-grade was difficult for all groups, there was no significant group difference among typically developing and other disability groups.

3.3.2 Dual-language learners

Powell et al. (2020) assessed the mathematics-vocabulary performance of third-grade students to determine if performance differences existed among dual-language learners and native English speakers with and without MD. Powell et al. (2020) categorized students based on performance on measures of equation solving and word problems. This categorization included: equation-only difficulty (EQD; i.e., performing below 27th percentile on equation solving), word-problem-only difficulty (WPD; i.e., performing below 28th percentile on word-problem solving), or word-problem and equation difficulty (EQ + WPD). In addition, the study included students without equation or word-problem difficulty.

The Grade 3 measure of mathematics vocabulary was a revised version of the mathematics-vocabulary measure used in grades 3 and 5 (Powell et al., 2017). The revised assessment did not contain terms introduced in grades 4, 5, or 6. Students with no MD who were native English speakers outperformed students with no MD who were dual-language learners. The same was true for students with equation-only difficulty in which native English speakers outperformed dual-language learners. This pattern, however, did not hold for students with word-problem difficulty or combined difficulty. Instead, Powell et al. (2020) identified no significant differences between native English speakers and dual-language learners with word-problem-only difficulty or combined difficulty.

In addition, Powell et al. (2020) noted differences in students' mathematics vocabulary knowledge between different types of mathematics difficulty. For dual-language learners, students with no MD outperformed students with any type of MD (i.e., equation-only difficulty, word-problem-only difficulty, and combined difficulty). Although there was not a significant difference in the mathematics vocabulary between dual-language learners with equation-only difficulty and word-problem-only difficulty, both groups significantly outperformed dual-language learners with the comorbid difficulty. Across native English speakers, students without a MD outperformed their peers with any form of MD. Native English speakers with equation-only difficulty significantly outperformed native English speakers with combined difficulty. Similar to the findings of Forsyth and Powell (2017), students with comorbid difficulties demonstrated the weakest mathematics vocabulary. It is noteworthy that students without any mathematics difficulty, on average, did not score above 50% on the third-grade mathematics-vocabulary measure, suggesting many students struggle answering questions about mathematics vocabulary.

4 Conclusion

In this chapter, we described five studies in which students in the U.S. answered questions about mathematics vocabulary. All five studies above demonstrated wide variability in students' understanding of mathematics vocabulary. That is, within a grade level, the mathematics-vocabulary scores of students ranged from very low – in some cases, zero – to well above average. Across studies, the average mathematics-vocabulary score was at or below 67% of all terms on a measure. This indicated that all students have room for improvement on measures of mathematics vocabulary. Some students answered fewer than 10% of items on a mathematics-vocabulary measure even when the terms were introduced in previous grades and students should have experienced multiple opportunities to interact with such terms.

In a few studies, we noted significant correlations between mathematics-vocabulary scores and general English vocabulary as well as mathematics performance on measures of fluency or computation. The connection between mathematics and general English vocabulary is important, but our analyses did not explore whether greater mathematics vocabulary led to greater English vocabulary or vice versa. Future research should investigate this relationship. Similarly, we would ask researchers to conduct more research on the connection between mathematics-vocabulary knowledge and the understanding of mathematics concepts. We noted several significant correlations between mathematics vocabulary and fluency or computation, but it would be important for both researchers and educators to understand whether increased mathematics-vocabulary knowledge contributes to improved understanding of a concept or procedure. That is, how important is mathematics vocabulary on the pathway to learning mathematics?

Furthermore, students experiencing mathematics difficulty scored significantly below peers without mathematics difficulty. In the study by Forsyth and Powell (2017), the average score for students with both mathematics and reading difficulty was markedly below-average scores of students experiencing difficulty in only mathematics or reading. Such results should help educators understand which students in a classroom require more or less mathematics-vocabulary support. In the study by Powell et al. (2020), we observed differing average scores for dual-language learners and non-dual-language learners when the students experienced no mathematics difficulty or only equations difficulty. When students experienced word-problem difficulty, the difference between non-dual- and dual-language learners faded. These results should inform discussions about mathematics language support for dual-language learners.

Even with the need to conduct future research, we suggest all educators should use mathematics-vocabulary measures to understand the mathematics-vocabulary profiles of their students. This knowledge could inform mathematics instruction. Based on our findings that many students demonstrated low mathematics-vocabulary scores, we also suggest that educators provide explicit instruction on mathematics vocabulary to help all students develop a deep lexicon related to mathematics vocabulary. Such explicit instruction may be essential for students experiencing mathematics difficulty to ensure these students have access to the mathematics curriculum throughout their education.

References

Barrow, Melissa (2014): Even math requires learning academic language. *Phi Delta Kappan Magazine* 95 (6), 35–38.

Bay-Williams, Jennifer & Livers, Stefanie (2009): Supporting math vocabulary acquisition. *Teaching Children Mathematics* 16 (4), 238–245.

Berch, Dan & Mazzocco, Michelle (eds.) (2007): *Why is Math so Hard for some Children? The Nature and Origins of Mathematical Learning Difficulties and Disabilities*. Baltimore, MD: Brookes.

Browning, Sandra & Beauford, Judith (2011): Language and number values: The influence of number names on children's understanding of place values. *Investigations in Mathematics Learning* 4 (2), 1–24.

Bruner, Jerome (1966): *Toward a Theory of Instruction*. Cambridge, MA: Harvard University Press.

Bruun, Faye, Diaz, Joan & Dykes, Valerie (2015): The language of mathematics. *Teaching Children Mathematics* 21 (9), 530–536.

Cummins, Jim. (2000): Academic language learning, transformative pedagogy, and information technology: Towards a critical balance. *TESOL Quarterly* 34 (3), 537–548.

Ernst-Slavit, Gisela & Mason, Michele (2011): Words that hold us up:" Teacher talk and academic language in five upper elementary classrooms. *Linguistics and Education* 22 (4), 430–440.

Fang, Zhihui (2012): Language correlates of disciplinary literacy. *Topics in Language Disorders* 32 (1), 19–34.

Forsyth, Suzanne & Powell, Sarah (2017): Differences in the mathematics-vocabulary knowledge of fifth-grade students with and without learning difficulties. *Learning Disabilities Research and Practice* 32 (4), 231–245.

Fuchs, Lynn, Fuchs, Douglas, Compton, Donald, Hamlett, Carol & Wang, Amber (2015): Is word-problem solving a form of text comprehension?. *Scientific Studies of Reading* 19 (3), 204–223.

Haladyna, Thomas & Rodriguez, Michael (2013): *Developing and Validating Test Items*. New York, NY: Routledge.

Harmon, Janis, Hedrick, Wanda & Wood, Karen (2005): Research on vocabulary instruction in the content areas: Implications for struggling readers. *Reading & Writing Quarterly* 21 (3), 261–280.

Hughes, Elizabeth, Powell, Sarah & Lee, Joo-Young (2020) – this is no longer in press: Development and psychometric report of a middle school mathematics vocabulary measure. *Assessment for Effective Intervention*. 34 (2), 417–447.

Karp, Karen, Bush, Sarah & Dougherty, Barbara (2014): 13 rules that expire. *Teaching Children Mathematics* 21 (1), 18–25.

Kleemans, Tijs, Segers, Eliane & Verhoeven, Ludo (2018): Role of linguistic skills in fifth-grade mathematics. *Journal of Experimental Child Psychology* 167, 404–413.

Livers, Stefanie & Elmore, Patricia (2018): Attending to precision: Vocabulary support in middle school mathematics classrooms. *Reading & Writing Quarterly* 34 (2), 160–173.

Monroe, Eula & Orme, Michelle (2002): Developing mathematical vocabulary. *Preventing School Failure: Alternative Education for Children and Youth* 46 (3), 139–142.

Monroe, Eula & Panchyshyn, Robert (1995): Vocabulary considerations for teaching mathematics. *Childhood Education* 72 (2), 80–83.

Moschkovich, Judit (2013): Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education* 6 (1), 45–57.

Moschkovich, Judit (2015): Scaffolding student participation in mathematical practices. *ZDM: Mathematics Education* 47 (7), 1067–1078.

National Council of Teachers of Mathematics (2006): *Curriculum Focal Points for Pre-Kindergarten through Grade 8 Mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers (2010): *Common Core State Standards Mathematics*. Washington, DC: Author.

Peng, Peng & Lin, Xin (2019): The relation between mathematics vocabulary and mathematics performance among fourth graders. *Learning and Individual Differences* 69, 11–21.

Petersen-Brown, Shawna, Lundberg, Ashlee, Ray, Jannine, Paz, Iwalani Dela, Riss, Carrington & Panahon, Carlos (2019): Applying spaced practice in the schools to teach math vocabulary. *Psychology in the Schools* 56 (6), 977–991.

Powell, Sarah, Berry, Katherine & Tran, Le (2020): Performance differences on a measure of mathematics vocabulary for English learners and non-English learners with and without mathematics difficulty. *Reading and Writing Quarterly: Overcoming Learning Difficulties* 36 (2), 124–141.

Powell, Sarah, Driver, Melissa, Roberts, Greg & Fall, Anna-Maria (2017): An analysis of the mathematics vocabulary knowledge of third-and fifth-grade students: Connections to general vocabulary and mathematics computation. *Learning and Individual Differences* 57, 22–32.

Powell, Sarah & Nelson, Gena (2017): An investigation of the mathematics-vocabulary knowledge of first-grade students. *The Elementary School Journal* 117 (4), 664–686.

Purpura, David & Logan, Jessica (2015): The nonlinear relations of the approximate number system and mathematical language to early mathematics development. *Developmental Psychology* 51 (12), 1717–1724.

Purpura, David, Napoli, Amy, Wehrspann, Elizabeth & Gold, Zachary (2017): Causal connections between mathematical language and mathematical knowledge: A dialogic reading intervention. *Journal of Research on Educational Effectiveness* 10 (1), 116–137.

Riccomini, Paul, Smith, Gregory, Hughes, Elizabeth & Fries, Karen (2015): The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading and Writing Quarterly* 31 (3), 235–253.
Rubenstein, Rheta & Thompson, Denisse (2002): Understanding and supporting children's mathematical vocabulary development. *Teaching Children Mathematics* 9 (2), 107–113.
Schleppegrell, Mary (2012): Academic language in teaching and learning: Introduction to the special issue. *The Elementary School Journal* 112 (3), 409–418.
Simpson, Amber & Cole, Mikel (2015): More than words: A literature review of language of mathematics research. *Educational Review* 67 (3), 369–384.
Townsend, Dianna, Filippini, Alexis, Collins, Penelope & Biancarosa, Gina (2012): Evidence for the importance of academic word knowledge for the academic achievement of diverse middle school students. *The Elementary School Journal* 112 (3), 497–518.