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Blindness and deafness: A window to study the visual and verbal basis of the number sense

1 Introduction

When children acquire numerical skills, they have to learn a variety of specific numerical tools. The most obvious are the numerical codes such as number words (one, two, three, etc.) or Arabic numerals (1, 2, 3, etc.). Other skills will be relatively more abstract: arithmetical facts (i.e., $4 \times 2 = 8$), arithmetical procedures (i.e., borrowing), or arithmetical laws (i.e., $a + b = b + a$). The acquisition of these numerical tools is complex and probably not facilitated by the fact that a numerical expression does not have a single meaning. Indeed, numbers can be used as a kind of label or proper name (i.e., Bus 51). They can also be part of a familiar fixed sequence (i.e., 51 comes immediately after 50 and before 52). They can be used to refer to continuous analogue quantities (i.e., 51,2 grams) (Butterworth, 2005; Fuson, 1988) and, most importantly, they can be used to denote the number of things in a set – the *cardinality* of the set.

Children are able to understand the special meaning of cardinality because they possess a specific and innate capacity for dealing with quantities (Feigenson et al., 2004). Supporting the innate nature of the “number sense,” it has been found, for instance, that fetuses in the last trimester are already able to discriminate auditory numerical quantities (Schleger et al., 2014). A large set of behavioral studies using the classic method of habituation has also revealed sensitivity to small numerosities (e.g., Starkey & Cooper, 1980) in young children. In the study of Starkey and Cooper (1980), for example, slides with a fixed number of 2 dots were repeatedly presented to 4- to 6-month-old infants until their looking time decreased, indicating habituation. At that point, a slide with a deviant number of 3 dots was presented and yielded significantly longer looking times, indicating dishabituation and therefore discrimination between the numerosities 2 and 3. This effect was replicated with newborns (Antell & Keating, 1983) and with various stimuli such as sets of realistic objects (Strauss & Curtis, 1981), targets in motion (Van Loosbroek & Smitsman, 1990; Wynn et al., 2002), two- and three-syllable words (Bijeljac-Babic et al., 1991), or puppet making two or three sequential jumps (Wood & Spelke, 2005; Wynn, 1996). However, it was not observed

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with other numerosities such as 4 and 6 (Starkey & Cooper, 1980). Taken together, these data therefore suggest that infants may possess a concept of small numbers which is dependent on the absolute number of items presented (up to 3 or 4).

Besides their ability to discriminate particularly small numerosities, infants are also able to discriminate large numerosity sets. However, this discrimination ability differs dramatically from that observed with small numerosities: performance no longer depends on the absolute number of items presented but on the numerical ratio that separates the two numerosities to be discriminated. Hence, while 6-month-old infants are able to discriminate numerosities with a 1:2 ratio (4 vs. 8, 8 vs. 16 and 16 vs 32 stimuli) (Brannon et al., 2004; Lipton & Spelke, 2003, 2004; Wood & Spelke, 2005; Xu, 2003; Xu & Spelke, 2000; Xu et al., 2005), they fail to discriminate numerosities with a 2:3 ratio (8 vs. 12 and 16 vs. 24) (Xu & Spelke, 2000). By 9 or 10 months, however, the precision of the representation improves since infants become able to discriminate between 8 and 12 elements (Lipton & Spelke, 2003; Xu & Arriaga, 2007).

Following these two waves of investigations on numerosity discrimination in infancy, it has been suggested that basic numerical abilities could be sustained by two different proto-mathematical systems (e.g., Butterworth, 1999; Carey, 2001; Dehaene, 1992; Feigenson et al., 2004; Xu, 2003; Xu & Spelke, 2000): **(1)** an object-tracking system (e.g., Scholl, 2001; Trick & Pylyshyn, 1994; Uller et al., 1999) which has a clear limit on set size (3 or 4), allows the precise representation of small number of objects, and provides a basis for the verbal and accurate quantification process of counting; **(2)** a large numerosity estimation system, commonly known as the Approximate Number System (ANS), which has no set size limit and allows the approximate representation of large non-symbolic numerosity sets (Xu, 2003; Xu & Spelke, 2000).

Formal mathematics could therefore emerge from these two proto-mathematical systems. A commonly accepted hypothesis is that symbolic number representations acquire their numerical meaning through being mapped onto non-symbolic representations. Accordingly, many visuo-spatial and verbal processes have been linked to the development of complex numerical skills. Among these, we could cite visual working memory (WM; Bull et al., 2011a; LeFevre et al., 2010), visual attention (Anobile et al., 2013), visuo-spatial mental rotation (Reuhkala, 2001), basic visual perception (Lourenco et al., 2012; Tibber et al., 2013), visual movement perception (Sigmundsson et al., 2010), phonological awareness (Alloway et al., 2005; Leather & Henry, 1994; Simmons & Singleton, 2008), and the relative linguistic transparency of the language used (Almoammer et al., 2013; Miller et al., 1995; Miura et al., 1988; Miura & Okamoto, 1989; Miura et al., 1993; Miura et al., 1994).

Interestingly, the relative importance of these visual and verbal processes seems to vary with age and the specific numerical task that is investigated. Several reports, for example, demonstrated that visuo-spatial and verbal processes might be differently engaged in the resolution of specific arithmetic operations. While the resolution of subtraction and addition principally relies on finger-based and visuo-spatial calculation strategies (Siegler & Shrager, 1984), overlearned simple multiplication facts have been associated with verbal memory retrieval (Cooney et al., 1988). Similarly, a recent theoretical framework has developed the idea that different types of spatial information might be engaged in different numerical tasks. In Western populations, people tend to represent number along a left-to-right-oriented mental number line (MNL; Dehaene, 1989, 1992). A compelling demonstration of this strong association between numbers and space resides in the SNARC effect. This effect refers to the observation that responses to small numbers are faster in the left side of space, while responses to large numbers are faster in the right side of space (Dehaene et al., 1990). Because the SNARC effect was observed even when participants crossed their hands, numbers were assumed to be mapped onto an external frame of reference (Dehaene et al., 1993), where small and large numbers facilitate responses in the left and right sides of space irrespective of the hand of response. More importantly, however, while the SNARC effect was assumed to primarily originate from visuo-spatial associations in magnitude comparison tasks, it was assumed to primarily arise from verbal associations in parity judgment tasks (Herrera & Macizo, 2008; van Dijck et al., 2009).

Given the importance of visual and verbal processes in the development of the number concept, it is not surprising to see that visual and verbal deficits can prevent the acquisition of numerical skills. Hence, it has been reported that children presenting non-verbal learning disability frequently show comorbid mathematics learning difficulties (Crollen et al., 2015). Difficulties in arithmetic are also remarkably common in dyslexia, particularly when it comes to retrieving arithmetic facts from semantic long-term memory, as is the case in multiplication (De Smedt & Boets, 2010; Göbel, 2015; Simmons & Singleton, 2008; Träff & Passolunghi, 2015). A possible explanation for this finding is that numerical processing might be influenced by visuo-spatial and phonological processes (Dehaene et al., 2003; De Smedt & Boets, 2010; Geary & Hoard, 2001). However, to date, it is still unknown whether visuo-spatial and verbal processes are mandatory or only co-vary with the development of good numerical skills (Szűcs, 2016).

Evidence from sensory-deprived individuals offers a unique opportunity to address this question. If vision and language play a foundational role in the development of the concept of numbers, then blind and deaf individuals should

present atypical numerical behavior. Alternatively, if the “number sense” can be acquired without vision and without typical verbal input, then blind and deaf individuals should show typical numerical abilities. In this chapter, we will address this question by reviewing recent empirical evidence examining mathematical reasoning in blind and deaf individuals. We will call into question arguments stating that (1) the development of the number concept is scaffolded mainly by visuo-spatial development, and (2) that language becomes integrated only after the concepts are created.

2 Numerical processing in the blind

Although vision has long been considered as critical in the emergence of numerical representations and skills, a growing set of studies on numerical performances following early visual deprivation nevertheless indicate that blind individuals perform as efficiently as their normally seeing peers in various numerical tasks. It was shown, for instance, that early and congenitally blind participants were as good as sighted individuals in number comparison tasks with both small and large numbers (Castronovo & Seron, 2007a; Szűcs & Csépe, 2005). Similar results were found when they had to perform parity judgment task (Castronovo & Seron, 2007a). In addition to that, blind participants have also been shown to perform as accurately as their sighted peers in counting (Crollen et al., 2014) and subitizing tasks (i.e., fast and accurate processing of a small collection of up to three or four elements; Ferrand et al., 2010). Interestingly and more surprisingly, the lack of vision since early age might even have a positive impact on some numerical skills. When submitted to a numerical estimation task, blind individuals indeed demonstrated enhanced abilities as compared to sighted participants, especially when the task involved touch and proprioception (i.e., key press estimation task): they showed greater accuracy in both small (up to 9; Ferrand et al., 2010) and large numerical ranges (up to 64; Castronovo & Seron, 2007b). Moreover, this greater estimation skill in blind individuals was not specific to a particular modality (i.e., tactile), neither to a familiar numerical context (i.e., close to their daily life use of numerical information – a footstep production task). It was indeed found in more unfamiliar contexts requiring verbal, non-tactile processing (i.e., non-word repetition task; Castronovo & Delvenne, 2013). Finally, early blind participants also showed enhanced abilities to perform arithmetic operations: addition, subtraction, and especially multiplication of different complexities (Dormal et al., 2016). In sum, there is plenty of evidence suggesting that early visual

deprivation does not prevent the development of good numerical skills but may even induce greater efficiency in estimation and manipulation of numerical quantities.

So far, it seems clear that blind individuals are capable of developing a good number understanding. However, it is still unknown whether the representation of numerical magnitude in visually deprived individuals shares the same “visuo”-spatial properties as the one of the sighted. The close connection between numbers and space is well established in the literature of numerical cognition and illustrated by the metaphor of the logarithmic and left-to-right-oriented mental number line (MNL; Castronovo & Seron, 2007a; de Hevia et al., 2008). This mental spatial organization of numbers has been supported by the recurrent observation of three main effects: (1) the size effect, which refers to the fact that larger numbers are less easy to discriminate than smaller ones (e.g., 2 vs. 4 is easier than 8 vs. 10; Ashcraft & Stazyk, 1981; Brysbaert, 1995; Cantlon et al., 2009; McCloskey et al., 1991); (2) the distance effect, which refers to the fact that it is easier to discriminate numbers that are distant on the MNL than numbers that are close to each other (e.g., 2 vs. 8 is easier than 2 vs. 4; Cantlon et al., 2009; Moyer & Landauer, 1967); and (3) the Spatial Numerical Associations of Response Codes (SNARC) effect, which refers to the observation that responses to small and large numbers are faster when performed in the left and right sides of space respectively (Fias & Fischer, 2005; Wood et al., 2008). Because the SNARC effect was observed even when participants crossed their hands, numbers were assumed to be mapped onto an external frame of reference, where small and large numbers facilitate responses in the left and right sides of space irrespective of the responding hand (Dehaene et al., 1993; Fias & Fischer, 2005). Supporting the idea that the MNL is oriented from left to right, it was also observed that people tend to overestimate the leftward space on the MNL (Loftus et al., 2008). When asked to perform a numerical bisection task (which consists in estimating, without calculating, the number midway between two others), participants indeed present a leftward bias (also called the “pseudo-neglect” effect): they systematically tend to mis-bisect the numerical interval slightly to the left of its objective midpoint. Finally, the spatial organization of numbers was also found in tasks involving arithmetic operations (Masson & Pesenti, 2014; McCrink et al., 2007). It was indeed suggested that additions and subtractions involve attentional shifts along the MNL: toward the right (i.e., larger numbers) for addition and toward the left (i.e., smaller numbers) for subtractions (Knops et al., 2009a, 2009b; Masson & Pesenti, 2014; McCrink et al., 2007; Pinhas & Fischer, 2008).

Does vision root the construction of the relationship between numbers and space? Interestingly, many studies indicate that visually deprived individuals possess a numerical magnitude representation that shares the same

spatial characteristics as the one of sighted individuals. Congenitally blind participants indeed show similar distance and size effects as sighted individuals when submitted to number comparison (Castronovo & Seron, 2007a; Szűcs & Csépe, 2005) and parity judgment tasks (Castronovo & Seron, 2007a). In addition to that, they both show a pseudo-neglect effect (i.e., a leftward bias) when asked to perform a numerical bisection task (Cattaneo et al., 2011). Both blind and sighted participants also present the SNARC effect when they have to indicate the parity status of a number (odd or even) by means of a manual response with the left or the right hand (Castronovo & Seron, 2007a; Crollen et al., 2013; Szűcs & Csépe, 2005). The same SNARC effect was furthermore observed when blind and sighted participants had to judge whether a presented number was smaller or larger than 5 (Crollen et al., 2013). However, unlike sighted individuals, blind individuals were shown to present a reversed SNARC effect when performing the numerical comparison task with the hands crossed over the body midline (Crollen et al., 2013). They indeed produced faster responses to small numbers in the right space (i.e., with the left hand) and to large numbers in the left space (i.e., with the right hand). Consequently, it was proposed that blindness may shape the frame of reference onto which numbers are represented: while sighted individuals rely on a world-centered (external) representation of space, blind individuals rather use a representation that is body-centered (internal). Importantly, blind participants did not present any reversed SNARC effect when performing the parity judgment task with the hands crossed over the body midline. Because this task is thought to involve verbal-spatial processes instead of visuo-spatial processes (van Dijck et al., 2009; but see Huber et al., 2016 for alternative findings), it was therefore suggested that early visual experience shapes the nature of the visual association between numbers and space but does not influence the verbal one (Crollen et al., 2013). Although blind individuals develop a spatial representation of numerical magnitude similar to the one of sighted individuals, their use of this representation might therefore present some specificities.

Another evidence supporting the idea that blindness shapes some qualitative aspects of the numerical processing can be found in the interactions that occur between numbers and fingers (Crollen et al., 2011, 2014). The latter has been illustrated by the use of the finger counting strategy, a procedure that often accompanies the development of basic arithmetic in the sighted population. Although the use of finger counting was thought to play a functional role in the development of a mature numerical system (Butterworth, 2005), it has been shown that blind children less spontaneously use this strategy while learning counting and arithmetic (Crollen et al., 2011, 2014). Fingers were indeed assumed to permit the assimilation of basic numerical skills

(Andres et al., 2008) and the connection between non-symbolic and symbolic numerosities (Fayol & Seron, 2005). However, Crollen and colleagues (2011; 2014) observed that blind children, compared to their sighted peers, used their fingers less spontaneously and in a less canonical way to count and show quantities, despite similar counting performances. Consequently, it was suggested that visual experience drives the establishment of finger-number interactions.

Differences between blind and sighted individuals were also found in more complex numerical domains like arithmetic skills. As already mentioned, early blind individuals show enhanced abilities to perform addition, subtraction, and multiplication of different complexities (Dormal et al., 2016). In addition to these observations, it was recently suggested that blindness may shape the neural foundations of arithmetic reasoning. While it is widely recognized that mathematical skills are supported by a bilateral fronto-parietal network in sighted and blind individuals (Amalric & Dehaene, 2016, 2019; Amalric et al., 2018; Arsalidou & Taylor, 2011; Dehaene et al., 2003), imaging studies have indeed highlighted that blind, but not sighted, participants recruit some early visual areas in addition to this math-responsive network while calculating (Crollen et al., 2019; Kanjlia et al., 2016). Consequently, it was suggested that the occipital cortex – which typically process visual information – might be cognitively pluripotent (i.e., capable of assuming non-visual cognitive functions; Kanjlia et al., 2016). However, this conclusion does not take into account the computational relation between number and visual processes, nor does it consider the variety of strategies that are used while solving arithmetic operations (Campbell & Timm, 2000; Dehaene & Cohen, 1997; Hecht, 1999). Indeed and as mentioned previously, visuo-spatial procedures are principally used to solve subtractions while retrieval is the dominant method for solving easy and overlearned multiplications (Ashcraft, 1992; Campbell & Xue, 2001). It is therefore possible that early visual deprivation selectively affects the brain organization of visuo-spatial arithmetic operations (subtraction) while keeping intact the neural network involved in the arithmetic operations learned by rote verbal memory (multiplication).

This later assumption was recently supported by a study contrasting the brain activity of blind and sighted participants while performing subtraction vs. multiplication arithmetic operations (Crollen et al., 2019). An enhanced activity of the occipital cortex was indeed observed in the blind while they performed subtraction, but was not observed for the multiplication operations (Crollen et al., 2019). This recent study therefore challenges the idea that the brain is cognitively pluripotent and rather suggests that the recruitment of the occipital cortex in the blind actually relates to its intrinsic computational role (Crollen et al., 2019). It is also interesting to note that the brain results obtained by

Crollen et al. (2019) are reminiscent of the behavioral results observed with the SNARC: when crossing their hands over the body midline, blind individuals indeed showed a reversed SNARC effect in tasks relying on visuo-spatial processes (numerical comparison), but not in tasks involving verbal processes (parity judgment; Crollen et al., 2013).

Altogether, numerous studies clearly indicate that early visual experiences are not essential for the development of numerical cognition. Indeed, evidence has shown that (1) blind individuals perform as efficiently as their sighted peers in various numerical tasks; (2) blind individuals even possess enhanced abilities to perform numerosity estimation and calculation tasks; (3) blind individuals develop a numerical magnitude representation that shares the same spatial properties as sighted individuals. However, blindness was found to shape some qualitative aspects of numerical processing. For instance, it was shown that (1) blindness impacts the reference frame in which the associations between numbers and space occur; (2) the lack of vision reduces the use of finger counting strategies; and (3) blindness shapes the neural foundations of arithmetic operations relying on visuo-spatial processes.

Although blindness was found to have a positive impact on numerical abilities such as estimation and calculation, the mechanisms sub-serving these greater abilities are still unknown. One possible explanation lies in the use of enhanced high-level cognitive processes such as WM (Castronovo & Delvenne, 2013). As WM and numerical abilities are linked to each other (De Smedt et al., 2009; Simmons et al., 2012) and as blind individuals present greater WM skills than sighted participants (Crollen et al., 2011; Hull & Mason, 1995; Swanson & Luxenberg, 2009), numerical skills in blind individuals could indeed potentially be accounted by the use of enhanced WM processes.

Finally, although blind individuals present greater skills in some numerical domains, it is still unknown whether they would present some delays in other abilities not tested so far. It could be worth examining basic knowledge of geometry as this mathematical knowledge is intrinsically linked to visuo-spatial representations. Answering these questions in the future may potentially have important implications for mathematics teaching and mathematics rehabilitation programs.

3 Numerical processing in the deaf

Deaf signers show advantages in various visual domains. They, for example, outperform their hearing peers in the speed of shifting visual attention and visual scanning and in the peripheral detection of motion (Bavelier et al., 2000;

Chinello et al., 2012; Proksch & Bavelier, 2002). These advantages are suggested to be primarily due to their experience with sign language since this includes a highly significant spatial component. Accordingly, it is assumed that the primacy of visual cognition in deaf signers may influence numerical skills, and that the spatial components of sign language may have an impact on some visuo-spatial features of the mental number line (Chinello et al., 2012). Despite this common-sense conception, the visuo-spatial advantages of deaf do not appear to support or enhance deaf students' performance compared to hearing students (Ansell & Pagliaro, 2006; Borgna et al., 2018; Marcelino et al., 2019 for review). Numerous studies have consistently showed that deaf children from preschool onward through their school years into higher education, as well as deaf adults tend to be slower and less accurate in numerical processing compared to their hearing counterparts (Ansell & Pagliaro, 2006; Blatto-Vallee et al., 2007; Bull et al., 2006, 2005, 2018, 2011b; Chinello et al., 2012; Korvorst et al., 2007; Marschark et al., 2013, 2015; Rodríguez-Santos et al., 2014; Zarfaty et al., 2004). Several studies have indeed indicated that deaf pupils experience a delay of 2 to 3.5 years in comparison with hearing children on mathematical achievement tests (Bull et al., 2005; Nunes & Moreno, 2002). Growth curves of deaf students are identified to be much flatter than those for hearing learners (Zarfaty et al., 2004) and differences are often noted in: (1) standardized achievement tests; (2) measurement and number concepts; (3) understanding fractions; (4) computation and reasoning; (5) logical thinking; (6) communication about time; and (7) problem solving (Allen, 1995; Ansell & Pagliaro, 2006; Austin, 1975; Bull et al., 2011b; Marschark & Everhart, 1999; Nunes & Moreno, 2002; Pagliaro & Kritzer, 2013; Rodríguez-Santos et al., 2014; Traxler, 2000; Titus, 1995; Zarfaty et al., 2004). Geometry, in contrast, is indicated as an area of strength (Pagliaro & Kritzer, 2013).

While the competences mentioned above are quite complex, some other research has been performed on rather simple abilities such as subitizing. Deaf individuals could have an advantage in performing subitizing tasks because of their enhanced abilities in some aspects of visual and spatial processing (Bull et al., 2006). Nevertheless, the patterns of results are found to be very similar for both deaf and hearing individuals and this is also true for different presentation formats (symbolic and non-symbolic) (Bull et al., 2006). Basic differences in subitizing skills are therefore not believed to be the roots of the mathematical difficulties observed in the deaf (Bull et al., 2006).

Besides subitizing, the accuracy to discriminate quantities is restricted by the ratio difference of the quantities being compared. The closer this ratio is to 1, the more difficult the discrimination of the magnitude will be. Hearing as well as deaf individuals show a distance effect when they are asked to make

magnitude judgments, but not when they have to make physical size judgments (Bull et al., 2018). A similar distance effect is found when sign language is used as representation mode in both deaf and hearing adults. This finding implicates that the signed numbers automatically activate information about magnitude for both groups (Bull et al., 2006, 2005). Furthermore, similar size and distance effects are seen in symbolic and non-symbolic tasks for deaf and hearing participants, which demonstrates that deaf individuals have no deficits in building abstract symbolic and non-symbolic numerical representations. Nonetheless, slower reaction times were observed for the symbolic task in deaf individuals. This suggests that both groups have similar quantity representations, but that deaf participants might experience a delay in accessing representations from symbolic codes (Rodriguez-Santos et al., 2014). This conclusion has also been reached in number-to-position tasks, requiring participants to estimate a number's position on a 0–100 number line (Borgna et al., 2018; Bull et al., 2011). Deaf students have consistently made less accurate number-line estimations (Borgna et al., 2018; Bull et al., 2011b) than their hearing peers. This accuracy difference has been found at very young age before much exposure to formal education has taken place (Bull et al., 2018). Deaf individuals indeed appear to be more accurate in arithmetic estimation tasks involving non-symbolic stimuli (Masataka, 2006). In contrast, tasks requiring symbolic processing appear to be more challenging for deaf individuals than for their hearing peers, as this relies more on linguistic skills (Masataka, 2006; Rodriguez-Santos et al., 2014). A reduced accuracy in estimation for deaf participants may therefore be apparent only when number meaning has to be accessed from symbols (Masataka, 2006). This assumption has been called the “access deficit hypothesis” and was first proposed to explain difficulties of children who present mathematical learning disabilities (Rousselle & Noël, 2007). According to Masataka (2006), the difference in performance between deaf and hearing adults might be related to the variability in WM architecture, which is due to the difference of languages both groups acquired. Our WM is traditionally divided into two major domains, namely, a verbal and a visuo-spatial domain. The existence of a sign-based rehearsal loop mechanism that is parallel to the speech-based rehearsal loop is provided in adults who acquire a sign language as their first language, which could thus possibly account for their superior capacity to execute non-symbolic arithmetic (Masataka, 2006).

The ability to compare numbers and to judge whether numbers are odd or even also represents a basic numerical skill. When performing such tasks, both deaf and hearing individuals show faster responses to low numbers with the left hand and to high numbers with the right hand (i.e., SNARC effect). This has been demonstrated with Arabic digits as well as with sign language numerals

(Bull et al., 2006; Chinello et al., 2012) and could therefore indicate that, just like Arabic digits, sign language number signs may be directly mapped into an underlying left-to-right-oriented representation of magnitude (Bull et al., 2006). However, the speed of deaf participants making the SNARC decision in congruent condition (low numbers, left response box) was similar to the speed of hearing participants making the SNARC decision in the incongruent condition (low numbers, right response box) (Bull et al., 2005; Chinello et al., 2012; Iversen et al., 2004). This, again, demonstrates that the processing of numbers may be slower in deaf individuals. In 2007, Korvorst and colleagues presented number triplets (in Arabic digits or in sign language) to deaf and hearing adults who had to determine if the middle number was the numerical mean of the two outer numbers. Hearing individuals appeared to be faster in confirming valid bisection. In the sign language mode, deaf individuals had similar performances as hearing individuals (Korvorst et al., 2007). When asked to estimate as quickly as possible the midpoint of a series of numerical intervals that are presented in ascending and descending order, deaf and hearing participants were equally accurate in their estimations and were significantly biased toward lower numbers (Cattaneo et al., 2014). Nevertheless, the underestimation bias in deaf persons was smaller than in hearing when using a descending order, indicating that the decisions of the hearing individuals fall more systematically to the left (i.e., were more underestimated) than those of deaf participants (Cattaneo et al., 2016).

Finally, Nuerk, Iversen, and Willmes performed a study in 2004 in which they observed that hearing individuals respond faster in an even-right/odd-left condition than for the reverse parity-response box condition. This Markedness Association of Response Codes (MARC) effect has been interpreted as a linguistic markedness congruency effect since “even” and “right” are believed to be the linguistically marked antonyms of “odd” and “left” (Nuerk, H., Iversen, W., & Willmes, K. 2004). The effect also appeared to be stronger for written words than for Arabic numerals, which might reflect a stronger access to verbal-linguistic concepts via verbal stimuli, as suggested by the authors (Hines, 1990). An inversed MARC effect has interestingly been shown in deaf individuals, with native signers responding faster with the left-handed side to even numbers, and responding faster to the odd numbers with the right-handed side (Iversen et al., 2004). This result suggests that the structure of the sign language may influence number representations in a specific way.

To conclude, deaf and hearing individuals show SNARC, distance, and size effects that are normally associated with a representation of magnitude on a visual-analog MNL. However, deaf participants have slower response times when making comparative judgments, which indicates that their numerical representation of magnitude information is not distinct from that of hearing individuals, but

that they might process basic numerical information in a less efficient way (Bull et al., 2005; Chinello et al., 2012; Iversen et al., 2004; Rodriguez-Santos et al., 2014). While deaf individuals seem to use a left-to-right-oriented mental number line, it is still not known whether the associations between numbers and space occur in external coordinates or whether deafness, like blindness, shapes the reference frame in which these associations occur.

Therefore, future research should be conducted to: (1) establish a wider base of studies about cognitive abilities among deaf students; (2) determine the specific cognitive mechanisms that are slower in development in deaf individuals and causing a lag in mathematical achievement; (3) assess early representations of number that do not involve counting in younger children to clarify the status of the early abilities in number representation; (4) evaluate if there is an effect of home language, the medium of instruction, and the test language on children's mathematical performance; (5) study the aspects of sign language contributing to mathematical learning; (6) clarify how the human mind spatially represents abstract concepts and the extent to which differences are related to visual characteristics or linguistic values; (7) determine whether poorer acuity of numerical estimation is distinguishable from any language component associated with the task; (8) investigate further the influence of assistive hearing devices on child development and academic functioning; (9) identify differences among deaf individuals and how to accommodate for their needs (Ansell & Pagliaro, 2006; Borgna et al., 2018; Bull et al., 2011b; Cattaneo et al., 2017; Chinello et al., 2012; Gottardis et al., 2011; Korvorst et al., 2007; Marschark et al., 2015; Marcelino et al., 2019; Rinaldi, Merabet, Vecchi & Cattaneo, 2018; Zarfaty et al., 2004).

In summary, future research is necessary to better understand the factors that contribute to the academic achievements for deaf students across various subject areas for both theoretical and practical reasons. This would enhance the scientific understanding of cognitive, social, and linguistic functioning in deaf individuals as well as it would help to develop educational materials, methods, and interventions to support deaf learners in their academic achievement (Marschark et al., 2015).

4 Discussion

The representation of abstract concepts such as numbers has been proposed to originate from sensorimotor interactions within the world around us (Bonato et al., 2012; Winter et al., 2015). Hence, if a normal sensorimotor experience is strictly mandatory in order to represent numbers, we should expect that sensory

deprivation would have an impact on the development of this representation. The present chapter examined this question by reviewing experimental data on numerical performances in blind and deaf individuals. From a quantitative point of view, it is interesting to note that blindness does not prevent the emergence of good numerical skills while deafness, in contrast, seems to delay these acquisitions. These observations are at odd with the hypothesis suggesting that mathematical representations are rooted in visuo-spatial thinking and develop through visual experience (Burr & Ross, 2008; Ross & Burr, 2010). They nevertheless support the idea that language plays an important role in learning the meaning of numbers (Spaepen et al., 2011). Recent years have seen a surge in empirical studies examining the role of language in accounting for cross-cultural disparities in children's number understanding and arithmetic competence (Fuson & Kwon, 1992; Göbel et al., 2014; Krinzinger et al., 2011; Wang et al., 2008). It has, for example, been suggested that the superior arithmetic performance of Chinese and other Asian students could be explained by the relative linguistic transparency of the Asian counting systems (Fuson & Kwon, 1992; Miller et al., 2005) which gives a clear and consistent representation of the base-ten system. While comparisons across different auditory languages have been made, examining numerical competences in deaf individuals will additionally allow to compare auditory and visuo-manual languages.

Two hypotheses may account for the existence of good numerical skills in blind individuals. The first one assumes that blind individuals learn mathematics by compensating their visual lack through other modalities. In this case, the same numerical performances in blind and sighted individuals would arise from different neural correlates (e.g., areas involved in auditory or tactile processing). The second hypothesis assumes that mathematical activity is in fact based on highly abstract representations which are amodal rather than primarily visual. In this case, the same mental representation of numbers would be accessed indifferently from visual, auditory, or tactile inputs (Piazza et al., 2006; Riggs et al., 2006; Tokita et al., 2013). In the present chapter, we demonstrated that the reality may probably lie in-between these two main hypotheses. When solving arithmetic operations, congenitally blind adults were indeed shown to activate a number-related network very similar to the one observed in sighted subjects (Crollen et al., 2019; Kanjlia et al., 2016). These findings show that numerical thinking can develop in the absence of visual experience and is rooted in typical number-related brain circuits, therefore lending support to the second hypothesis. However, an additional activity of the occipital cortex was also demonstrated but only when blind participants had to perform subtraction operations (not when they had to perform multiplications) (Crollen et al., 2019). This additional activity was not observed in the sighted and probably reflects the use of compensating

strategies to perform a numerical task assumed to primarily rely on visuo-spatial processes.

In the literature, two main visuo-motor functions are assumed to be associated with the representation of numbers. On the one hand, following the recurrent observation that small numbers are preferentially associated with the left side of space while large numbers are preferentially associated with the right side of space (i.e., SNARC effect; Dehaene et al., 1993), numbers were assumed to interact with space. On the other hand, following the observation that children often use their fingers to learn the counting sequence and basic arithmetic operations, numbers were assumed to interact with finger movements (Butterworth, 1999). Interestingly, these two interactions are assumed to take place in the parietal cortex, a brain area which is part of the dorsal visual pathway. Consistently with the idea that blind individuals use alternative strategies to develop their understanding of numbers, we demonstrated that blind participants present a reversed SNARC effect when performing a numerical comparison task with their hands crossed over the body midline (Crollen et al., 2013). Importantly, they did not show a reversed SNARC effect in a parity judgment task (Crollen et al., 2013), suggesting that early visual experience drives the development of the visuo-spatial representation of numbers but do not shape the verbal associations that occur between numbers and space. We also demonstrated that the finger counting strategy was not often used by blind participants while counting and calculating (Crollen et al., 2011, 2014). Together, these observations lend some support to the idea that visual deprivation may promote the development of strategies that allow blind individuals to understand the number concept without relying on visuo-spatial processes.

Several studies already suggested that deaf individuals tend to be slower and less accurate with regard to numerical processing than normally hearing controls (Bull et al., 2011b; Epstein et al., 1994; Rodriguez et al., 2014). Deaf children may also be delayed in developing mathematic skills compared to their normally hearing peers (Gottardis et al., 2011). Interestingly, we showed the opposite dissociation as the one observed in the blind. The delay deaf individuals present in numerical development seems indeed to be more pronounced with symbolic tasks than with non-symbolic tasks. The study of this question should, however, be further investigated in the future. While it has already been demonstrated that deaf individuals represent numerical information along a left-to-right-oriented mental number line (Bull et al., 2005; Chinello et al., 2012; Iversen et al., 2004), the spatial frame of reference they preferentially use to map numbers onto space is still unknown. Moreover, to our knowledge, the spatial frame of reference onto which numbers are represented in deaf has so far never been compared across visuo-spatial and verbal-spatial tasks.

Furthermore, several studies have indicated that WM functioning is correlated with both symbolic and non-symbolic approximation, which points out that the individual variation in our WM could predict the mathematical achievement beyond the effect of approximation skills (Bull et al., 2018). However, symbolic approximation skills appear to correlate with mathematic ability beyond the effect of WM capacity. This might indicate unique contributions from both domain-specific and domain-general abilities (Bull et al., 2018). It has been stated that individuals with hearing loss seem to suffer from difficulties in verbal short-term memory, WM, and executive functioning (Bull et al., 2018; Marcelino et al., 2019 for review). On the other hand, it is suggested that deaf native signers have a better visuospatial WM than hearing individuals (Proksch & Bavelier, 2002).

To be able to better evaluate the respective contribution of visual vs. verbal processes in the development of the number concept, future studies should also examine the brain plasticity phenomenon following deafness. It has already been demonstrated that the temporal “auditory” cortex of deaf individuals changes its functional tuning to support visual or tactile functions (Fine et al., 2005; Finney et al., 2003; Finney & Dobkins, 2001; Nishimura et al., 1999; Petitto et al., 2000; Sadato et al., 2004; Shibata, 2007). However, it is still unknown whether the temporal “auditory” cortex of the deaf can be activated by higher cognitive function such as arithmetic and whether this activation is, as already observed in the blind, operation-specific (observed for multiplication, but not for subtraction in this case). Studying the neural correlates of numerical processes in deaf and comparing this to what has already been observed in the blind will provide a thorough understanding of the development of numerical competencies without vision or audition and give rare insights about the role of experience on the cerebral development of high cognitive functions. This question is really important to understand the principles of brain architecture and its reorganization under sensory deprivation. It will hopefully yield important novel insights into how the brain develops and whether this development is malleable or resistant to atypical sensory experiences.

Beyond this theoretical question, we also argue that a better understanding of the mechanisms underlying number understanding after visual and auditory deprivation plays a critical role in better characterizing what does dyscalculia look like in blind and deaf individuals. An advance in the understanding of this issue is timely since clinicians are currently lacking standardized norms to evaluate the numerical abilities of sensory-deprived individuals. Better understanding number development in these populations will therefore constitute a starting point for elaborating programs that stimulate numerical learning mechanisms in blind and deaf children presenting numerical difficulties. As poor mathematical skills are associated with employment difficulties, developing further such field

of research therefore holds the promises to have a substantial fundamental impact, but also some applied, social, and societal implications.

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