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Disentangling the relationship between mathematical learning disability and second-language acquisition

1 Introduction

Several studies have established that the mathematical achievement of language minority students (students whose first language differs from the language of instruction) is poorer than that of native speakers (students whose first language is the academic language of the instruction; Haag et al., 2015; Paetsch & Felbrich, 2016; Vukovic & Lesaux, 2013; Warren & Miller, 2015). However, despite the expanding literature on the mathematical learning of language minority students and of native speakers, very little is known about the relationship between mathematical learning disabilities and second-language acquisition. More detailed research on this topic is important for several reasons: Gonzáles and Artiles (2015) report that Latina/o students in the United States who perform below expectations in literacy tests are often diagnosed as having learning difficulties, which, in turn, often leads to their exclusion from mainstream education. Further, language minority students with low mathematical achievement in Switzerland – and probably also in other countries – often receive special second-language support, but they do not receive support for mathematics because it is assumed that their mathematical problems are caused by their language background. Therefore, it is important to investigate the extent to which the problems of language students with mathematical learning disabilities may be caused by math-related, as opposed to language-related, factors.

This study investigates whether the relationship between selected language variables and mathematical achievement gains is similar for native speakers with mathematical learning disabilities and language minority students with mathematical learning disabilities. The research was conducted by evaluating grade 3 students (students who are in the third year of school after attending kindergarten) over the course of a school year.

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2 The relationship between mathematical learning and language

The mathematical learning process and therefore the mathematical achievement gain are closely linked to language. According to Morgan et al. (2014: 845), "language has a special role in relation to mathematics because the entities of mathematics are not accessible materially." Language is an important tool that gives access to mathematics. However, the language of instruction, the academic language, differs from everyday language (Cummins, 2000) and has its own characteristics and challenges for all students (Schleppegrell, 2004; Snow & Uccelli, 2009). Prediger et al. (2019) categorized the challenges for students on word, sentence, and text, and on discourse level. The challenges can, individually or in combination, affect the mathematical learning process and thus contribute to learning difficulties in mathematics.

On word level, the linguistic structure of the number words has an influence on the acquisition of numbers (e.g., Klein et al., 2013; Miura et al., 1994). Math vocabulary is also an issue at this level. According to Haag et al. (2015) more difficult lexical features in test items, such as a more specialized vocabulary, increase the item difficulty in math tests.

On a sentence and text level, logical relationships, complex prepositional clauses (Jorgensen, 2011), conditional clauses, and complex issues of cohesion are difficult (e.g., Schleppegrell, 2004). Further, Haag et al. (2015) showed that text length and an increased number of noun phrases made comprehension more challenging for third graders. Koponen et al. (2018) found a link between reading competence and mathematical achievement. Students with a very low performance in reading showed low performance in mathematics across all grades.

Language in mathematics classrooms is also important on a discourse level. Language is both a medium of knowledge transfer and discussion, and a tool for thinking (Morek & Heller, 2012). Moschkovich (2015) and Erath et al. (2018) emphasized the importance of students' participation in discourse for developing conceptual understanding. Moschkovich (2015) points out that it is not the use of formal mathematical words that makes a discussion mathematical, but the use of mathematical concepts. Such concepts may also be expressed using informal words and phrases. Nevertheless, it may be assumed that performance in informal language production, regardless of use of mathematical vocabulary, is an important prerequisite for participation in classroom discourse.

In summary, this short review shows that language factors and mathematical learning are closely related on multiple levels.

3 Language performance and mathematical learning of students with mathematical learning disabilities

Although the scientific community has yet to agree on a formal definition of mathematical learning disability (e.g., Nelson & Powell, 2017), and different countries use different diagnostic criteria, studies have identified some characteristics commonly found in students with mathematical learning disabilities: low competence in counting tasks (Desoete et al., 2009; Stock et al., 2010), problems with understanding different aspects of the base-10 number system (e.g., Herzog et al., 2019; Moeller et al., 2011; Vukovic & Siegel, 2010), and dealing with word problems (e.g., Kingsdorf & Krawec, 2014; Peake et al., 2015; Zhang & Xin, 2012). Students with mathematical learning disabilities also have problems with fact retrieval, which can be related to deficits in working memory (e.g., De Weerdt et al., 2012; Geary et al., 2012). This study uses the term "mathematical learning disabilities" to refer to students with below-average mathematical achievement who have the characteristics described in this section (for cut-off criteria, see instruments).

Little is known about the relationship between language performance and mathematical learning in students with mathematical learning disabilities. Most studies to date have investigated the differences between students with and without comorbid reading disabilities.

On word level, the relationship between mathematics vocabulary and mathematical learning disabilities, with and without reading disabilities, has been investigated (Forsyth & Powell, 2017). Fifth graders with mathematical learning disabilities only or with reading disabilities only demonstrated a significantly weaker grasp of mathematics vocabulary than typically achieving students. Students with both mathematical learning disabilities and reading disabilities scored significantly lower than students who had problems only with either reading or mathematics.

On text and sentence levels, research generally focuses on reading disabilities. Mann Koepke and Miller (2013) conclude that 17–66% of students with mathematical learning disabilities also have reading disabilities. Several studies confirm this relationship (Vukovic, 2012; Vukovic & Siegel, 2010).

Peake et al. (2015) examined another factor on sentence level. They found a relationship between arithmetic problem solving and syntactic awareness in a sample of students with reading disabilities and a group of comorbid disabilities (reading and math), but not with students with mathematical learning disabilities only.

In summary, evidence in the literature supports the hypothesis that students with mathematical disabilities often have problems with mathematical vocabulary and reading.

4 Language performance and mathematical learning of language minority students

The achievement gap in mathematics between native speakers and language minority students (see introduction) is often explained by the difference between everyday language and academic language (Schleppegrell, 2004). This relationship is complex. The research of Martinellio (2008) showed that the poor test results of language minority students in grade 4 were caused by several factors: lack of knowledge of the specific context of a word problem, lack of vocabulary (e.g., likely, unlikely, certain), as well as the linguistic complexity of the items. Bochnik (2017) investigated the relationship between mathematical vocabulary, overall language proficiency in German, and mathematical achievement in a sample of German-speaking native speakers and language minority students in grade 3 who had lower mathematical achievement than their peers. The difference between the samples was predicted by overall proficiency in German. But, proficiency in the technical language of mathematics was the strongest predictor when explaining differences in the mathematical achievement of native speakers and language minority students. Vukovic and Lesaux (2013) investigated the relationship between language ability and mathematical cognition in a sample of language minority and native speakers aged 6 to 9. The authors found that language proficiency predicted gains in data analysis, probability, and geometry, but not in arithmetic, which was assessed with a computation test. The authors concluded that language seems to play a limited role in numerical manipulation but may be necessary for forming mathematical concepts and representations. Prediger et al. (2018) showed in a survey of grade 10 students that language proficiency is the background factor with the strongest connection to mathematics achievement of all the social and linguistic background factors. Language proficiency therefore was more strongly interrelated to mathematics achievement than multilingualism, immigrant status, or socioeconomic status.

Finally, Rodriguez et al. (2001) found that the performance rate of solving word problems by culturally and linguistically diverse learners in special bilingual classrooms was lower than that of students with learning disabilities and of students in general classrooms. This occurred even when reading requirements were minimal and the students were capable of solving computation problems.

The authors assume that the issues with problem solving might be because the student had never seen this kind of problem before.

These studies confirm the strong relationship between language proficiency on different levels (word, sentence, and text, discourse) and mathematical learning for all students. Therefore, it is important to heed Moschkovich (2010), who recommends focusing less on differences between monolinguals and bilinguals, and more on their similarities.

5 Research questions

The literature review reveals that little is known about the relationship between mathematical learning and the contributing language factors of students with mathematical learning disabilities, and as far as we have been able to ascertain, there has been no research into this relationship that looks specifically at language minority students with mathematical learning disabilities. Reported findings from the literature for typically developing students lead to the hypothesis that the mathematical learning gains of language minority students with mathematical learning disabilities and those of native speakers with mathematical learning disabilities may be influenced by similar variables. This would also mean that differences between language minority students with and without mathematical learning disabilities could be explained by math-related factors. This study will examine these assumptions by investigating the following research questions in the framework of a year-long study:

- Are the mathematical learning gains of language minority students with mathematical learning disabilities better explained by math-related or language-related factors?
- Are the mathematical learning gains of native speakers and language minority students with mathematical learning disabilities explained by the same or by different factors?

In order to investigate the relationship between math-related and language-related factors of language minority students with mathematical learning disabilities, it is also important to compare language minority students with mathematical learning disabilities and those without to find an answer to the following question:

- What factors explain the differences between the mathematical learning gains of language minority students with mathematical learning disabilities and those without?

It is assumed that the mathematical learning gains of language minority students with and without mathematical learning disabilities are explained by specific math-related factors.

6 Method

6.1 Participants

The participants were 70 third graders from Switzerland (32 classrooms). These students were selected from an initial sample of 888 students from 58 inclusive classrooms which participated voluntarily in a study on inclusive mathematics instruction (for selection criteria, see section “Measures”). All participants had written parental consent. To reduce the chances of influential variables in a highly selective sample (language minority students, students with mathematical learning disabilities), a matched-sample design was chosen.

Two samples were studied separately. The first sample included matched pairs of native and language minority students with mathematical learning disabilities (sample mathematical learning disabilities, $n = 40$; for criteria see below). The second sample consisted of matched pairs of language minority students with or without mathematical learning disabilities (language minority student sample, $n = 42$). Information on the language background of the students was gathered using a two-step procedure. First, data on languages spoken at home (first, second, third language) were collected using a teacher questionnaire, and students with a first language other than German, or with two first languages, were selected. Then, a language-use telephone interview was conducted with 52 of the 62 parents of these students, following the protocol established by Ritterfeld and Lüke (2011). Based on this information, the first-language variable was dummy-coded (German vs other).

Finding a sufficiently large sample of language minority students with mathematical learning disabilities was challenging due to the small population of such students. Therefore, 12 of the students were part of both samples. Because the analyses for each sample were conducted separately, the problem of dependency can be discounted. Mathematical achievement was measured at the end of grade 2 (t1), then eight months later with a post-test (t2), and at the end of grade 3 with a follow-up (t3). Information on selected language and control variables was collected at t1, after selecting the subsamples.

The mathematical learning disability sample consisted of 20 pairs of matched students ($n = 40$, Tab. 1) with below-average mathematical competence at t1 (for

Tab. 1: Demographic characteristics of the sample mathematical learning disabilities and the sample of language minority learners.

	Mathematical learning disabilities sample			**Language minority learner sample**		
	First language German	**Other first language**	**Total**	**With math disabilities**	**Without math disabilities**	**Total**
Students	20	20	40	21	21	42
Classes	16	15	24	17	14	23
Gender						
Boys	7	7	14	10	10	20
Girls	13	13	26	11	11	22

criteria, see below). Each pair comprised one student with German as his or her first language and one student with German as a second language. The pairs were matched by mathematical achievement at pre-test (difference math score ≤ 6 points), IQ (difference IQ score ≤ 8 points), age (difference ≤ 6 month), and gender. A Wilcoxon signed-rank test showed no significant difference between the groups on the basis of the matching criteria (math pretest $U = -1.09$, $p > .05$; IQ: $U = -0.13$, $p > .05$; age: $U = -0.11$, $p > .05$). A significant difference was found for socioeconomic status ($U = -3.36$, $p < .01$). The impact of this variable will be controlled in the analysis. The language minority students spoke the following first languages: Albanian (7), Croatian (1), Portuguese (4), Serbian (2), Tamil (2), Tigrinya (1), Turkish (2), and Urdu (1).

The language minority learner sample consisted of 21 matched pairs of students with German as a second language ($n = 42$, Tab. 1). One student from each pair had below-average mathematical achievement (for criteria, see below), and the other had average or good mathematical achievement (difference math score ≥ 10 points). The matching criteria were IQ (difference IQ score ≤ 8 points), age (difference ≤ 6 month), gender, and first language. A Wilcoxon signed-rank test showed no significant difference between the groups on the basis of the matching criteria IQ and age and socio-economic status (IQ: $U = -0.93$, $p > .05$; age: $U = -0.85$, $p > .05$; SES: $U = -0.80$, $p > .05$). As intended, the groups differed significantly in the math pretest ($U = -4.02$, $p < .001$). The language minority students spoke the following first languages: Albanian (9), Chinese (1), English (1), Italian (1), Croatian (2), Macedonian (2), Portuguese (8), Serbian (2), Tamil (6), Tigrinya (1), Turkish (8), and Urdu (1).

6.2 Measures

This study used highly selective samples with reduced variance. This can lead to the measures having low reliability. Therefore, with one exception, standardized tests and scales from standardized instruments were used. Nevertheless, some scales had to be excluded from the analyses due to low reliability scores. All tests were carried out in German. Sum scores were used for all scales.

6.2.1 Mathematics measures

General mathematical achievement: Standardized math tests to diagnose mathematical learning disability with norms for Germany and Switzerland (t1: Moser Opitz et al., 2020; t2 and t3: Moser Opitz, 2019) were conducted. The tests assess basic mathematical competences (e.g., understanding place value, number decomposition, doubling, halving, addition, subtracting, solving simple word problems). Linguistic requirements are minimal as most information is given with tables and pictures. In addition, the test administrators were allowed to read out the short instructions. The cut-off score for determining mathematical learning disability was set in the pre-test on the basis of the test norms (percentile 16) for all students. Average mathematical achievement was defined as scores that were the mean of the initial sample or higher. Table 2 gives an overview of the number of items and Cronbach's alpha.

Solving word problems: Researcher-designed scale with simple and complex comparison, combine, and exchange problems (details see Tab. 2).

Counting competences (counting forward and backwards by twos and tens) was also tested; however, this scale had to be excluded due to low reliability scores.

6.2.2 Language-related variables

In order to answer the research question, different language-related variables on sentence and on text level (reading comprehension, understanding semantic relationships), as well as on discourse level (verbal fluency) were assessed in an in-depth examination. Most of the scales are subtests of the standardized instrument SET5-10 constructed to diagnose language impairment (Petermann, 2010; number of items and Cronbach's alpha; see Tab. 2).

Tab. 2: Overview on the measures.

	Number of items	α sample mathematical learning disabilities	α sample of language minority students
Math pre-test (t1)	30	.62	.84
Math post-test (t2)	43	.81	.93
Math follow-up (t3)	43	.78	.93
Solving word problems	10	.71	.87
Listening comprehension	12	.71	.61
Verbal fluency	7	.75	.63
Reading comprehension	24	.84	.84
Verbal working memory	28	.84	.84

- Listening comprehension: understanding semantic relationships (Petermann, 2010)
- Verbal fluency (forming a sentence from a given word, e.g. lemon and sour; Petermann, 2010)
- Reading comprehension (subtest of K-ABC, Kaufman & Kaufman, 2009)

Mathematical vocabulary, cases, and plural formation were also examined. However, this data had to be excluded due to low reliability scores.

6.2.3 Control variables

Intelligence was tested with CFT 1 at t1 (Weiß & Osterland, 1997). The student's socioeconomic status was determined using the "books-at-home" index (Paulus, 2009), as measures like free meals are not available in Switzerland, and it was not possible to collect data such as the professional qualification of the mother. The books-at-home index involves showing pictures of five bookshelves with different numbers of books, from which the student is asked to choose the one that is most similar to that at home. To improve reliability, students were polled on three occasions (t1, t2, t3). The average of the responses was used in the analyses. Verbal working memory (repeating nonsense words) was tested in-depth using the Mottier-Test (Gamper et al., 2012). Visual processing speed was also assessed, but the reliability score was very low, and therefore this measure could not be used in the analyses.

Table 3 gives an overview on the descriptives of the two samples.

Tab. 3: Descriptives of math-related, language-related, and control variables of the samples.

	Mathematical learning disabilities sample			Language minority student sample		
	First language German	Second language German	All students	With learning disabilities	Without learning disabilities	All students
	M (SD)			*M (SD)*		
IQ	97.65 (11.70)	97.55 (11.74)	97.70 (11.57)	99.38 (11.66)	98.48 (11.17)	98.39 (11.28)
SES	4.50 (0.76)	2.70 (1.34)	3.6 (1.41)	2.95 (1.40)	3.29 (1.45)	3.12 (1.27)
Age month	102.40 (3.68)	102.30 (3.71)	102.30 (3.6)	103.71 (4.60)	104.76 (4.60)	104.24 (4.70)
Math t1	11.50 (2.19)	11.00 (2.94)	11.25 (2.57)	10.90 (2.55)	24.62 (2.58)	17.76 (7.39)
Math t2	16.85 (6.00)	12.70 (5.44)	14.78 (6.02)	13.52 (5.83)	27.19 (5.31)	20.36 (8.84)
Math t3	18.40 (5.34)	15.25 (5.33)	16.82 (5.50)	16.10 (6.61)	30.95 (4.80)	23.34 (9.45)
Word problems	5.15 (2.62)	3.95 (3.27)	4.55 (2.99)	3.95 (3.04)	6.14 (3.68)	5.05 (3.51)
Reading comprehension	16.50 (2.80)	12.20 (2.75)	15.35 (2.98)	13.67 (2.87)	15.48 (2.06)	14.57 (2.63)
Listening comprehension	9.25 (2.02)	7.40 (2.54)	8.33 (2.45)	7.05 (2.29)	8.14 (2.01)	7.60 (2.20)
Verbal fluency	9.45 (1.43)	6.40 (3.03)	7.93 (2.81)	6.14 (2.71)	6.62 (2.52)	6.38 (2.59)
Verbal working memory	22.90 (4.25)	22.90 (6.55)	22.90 (5.45)	23.67 (5.30)	25.00 (3.58)	24.33 (4.52)

6.3 Data analysis procedures

First, correlations were calculated for both samples. Second, due to the small sample size, which would allow for the inclusion of only few predictors, hierarchical multiple regression analyses were conducted with the dependent variable "mathematical achievement" at the post-test and the follow-up, with separate models. The residuals of these variables were normally distributed for the mathematical learning disabilities sample. This was not the case for the follow-up of the language minority student sample. Because of this and because some students dropped out at t3, the analysis for this sample was carried out with the dependent variable of the post-test only. In order to assess the impact of first language, two models were tested for the language minority student sample. One model put first language in a first step (minimal handicap), followed by math-related variables (math t1, solving word problems), control variables which are known as predictors for mathematical achievement (IQ, socio-economic status, working memory), and finally language-related variables (maximum handicap). In the second model, math t1 was placed in the first step, and first language was put in the last step (maximum handicap). This second model was formulated on the basis of the findings of Vukovic and Lesaux (2013) and Prediger et al. (2018), which show that language proficiency has an impact on mathematical achievement of all students regardless of the language background. For the language minority student sample, two analyses were conducted: one model in which math-related variables were included first, followed by control variables and language-related variables; and a second model with language variables at the first place (minimal handicap). The assumptions of multicollinearity and autocorrelation were not violated.

7 Results

7.1 Correlation analyses

Due to space limitations, only strong correlations are reported (without table). In the students with mathematical learning disabilities sample, a strong correlation ($r = .50^{**}$) was found for math t1 and math t3, and for math t2 and math t3 ($r = .72^{**}$). First language (German/other) was strongly negatively correlated with verbal fluency ($r = -.64^{**}$). In addition, reading comprehension and verbal fluency were strongly correlated ($r = .66^{**}$).

In the language minority student sample, a strong correlation was found between math at t1 and t2 ($r = .78^{**}$). In addition, verbal fluency correlated strongly with reading comprehension ($r = .62^{**}$) and listening comprehension ($r = .54^{**}$).

7.2 Hierarchical multiple regression analysis

7.2.1 Mathematical learning disabilities sample

In the first model for the post-test (Tab. 4), first language was included in step 1. R^2 was .12 ($f = .37$). R^2 rose significantly to $R^2 = .42$ ($\Delta F = 9.40$, $p < .001$) when math-related predictors were inserted with a strong effect size ($f = .85$) according to Cohen (1969). Control variables did not lead to a significant increase of R^2. However, for the language-related variables, R^2 rose significantly to .57 ($\Delta F =$ 3.02, $p < .05$, effect size of the increase of R^2 $f = .39$).

Tab. 4: Hierarchical multiple regression analysis summary in sample mathematical learning disabilities (dependent variable post-test and follow-up mathematics) with first language included as first predictor.

Predictor	Post-test				Follow-up			
	R^2	ΔR^2	ΔF	P	R^2	ΔR^2	ΔF	P
Step 1 First language	.12	.12	5.28	.027	.08	.08	3.49	.07
Step 2 Math-related variables	.42	.30	9.40	.001	.54	.46	18.09	.000
Step 3 Control variables	.44	.02	.37	.778	.58	.04	1.01	.40
Step 4 Language-related variables	.57	.3	3.02	.045	.61	.03	.78	.556

The pattern of development over 12 months (dependent variable follow-up math with post-test math as one of the math-related variables) differed (Tab. 4). The proportion of the explained variance, when including first language in step 1, was .08 ($\Delta F = 3.49$, $p < .1$). The significance threshold was missed and the effect size ($f = .29$) is medium, according to Cohen (1969). R^2 increased to .54 when including the math-related variables ($\Delta F = 18.09$, $p < .001$). The effect size for the increase of $R^2 = .46$ was high, with $f = .92$. Neither the control variables nor the language-related variables explained the additional variance.

By including the math-related predictors in step 1 for the post-test (Tab. 5), R^2 was .37, with a very high effect size ($f = .77$). R^2 increased also significantly ($R^2 = .57$, $\Delta F = 4.25$, $p < .05$) when language variables were included. The effect size of the increase was $f = .47$, which is a high effect (Cohen, 1969). The first language variable as well as the control variables did not lead to a significant increase of R^2.

Tab. 5: Hierarchical multiple regression analysis summary in mathematical learning disabilities sample (dependent variable post-test and follow-up mathematics) with math-related variables included as first predictor.

Predictor	Post-test				Follow-up			
	R^2	ΔR^2	ΔF	p	R^2	ΔR^2	ΔF	p
Step 1 Math-related variables	.37	.37	10.83	.000	.54	.54	21.92	.000
Step 2 Control variables	.39	.02	0.44	.730	.56	.02	.46	.710
Step 3 Language-related variables	.57	.18	4.25	.013	.59	.03	.76	.527
Step 4 First language (German/other)	.57	.00	.10	.751	.61	.02	1.67	.207

When math-related variables were included in step 1 for the follow-up (Tab. 5), R^2 was .54 ($\Delta F = 21.92$, $p < .001$). The effect size of $f = 1.08$ was higher than that in the model with the posttest as the dependent variable ($f = .77$). Neither the control variables nor the language variables resulted in a significant change of R^2.

To sum up, math-related variables explained the highest proportion of the variance for mathematical progress, especially over the longer term. First language was found to have a significant impact only for the post-test, when included in the first step of the model.

7.2.2 Language minority student sample

In the language minority student sample, only two models were tested (for depended variable post-test math, see Section 6.3): one model, which put language-related variables in the first step (Tab. 6), and a second model, which inserted math-related variables in the first step (Tab. 7). Putting language-related variables in the first step resulted in $R^2 = .17$ ($\Delta F = 2.57$, $p = .069$. $f = .44$). R^2 increased to .67 ($\Delta F = 27.38$, $p < .001$) when math-related variables were given a heavy weighting with a very high effect size of R^2 ($f = .71$). No impact of control variables was found.

Tab. 6: Hierarchical multiple regression analysis summary in language minority student sample (dependent variable post-test math) with first language included as first predictor.

Predictor	R^2	ΔR^2	ΔF	p
Step 1 Language-related variables	.17	.17	2.57	.069
Step 2 Math-related variables	.67	.50	27.38	.000
Step 3 Control variables	.70	.03	1.12	.360

Tab. 7: Hierarchical multiple regression analysis summary in the language minority student sample (dependent variable post-test math) with math-related variables as first predictor.

Predictor	R^2	ΔR^2	ΔF	p
Step 1 Math-related variables	.66	.66	37.61	.000
Step 2 Control variables	.69	.03	1.15	.344
Step 3 Language-related variables	.7	.01	.44	.725

A very high proportion of the variance (66%) was explained by including the math-related variables ($\Delta F = 37.61$, $p < .001$; $f = 1.40$). No increase of R^2 occurred when control and language variables were added (Tab. 7).

In conclusion, in this sample, only math-related variables led to an increase of R^2.

8 Discussion

This study aimed to investigate which factors contribute to the mathematical learning gains of native speakers with mathematical learning disabilities and language minority students with mathematical learning disabilities. To answer this question, data on the mathematical achievement and on selected language variables (sentence and text, discourse level) as well as control variables were collected in a sample of students with mathematical learning disabilities (native speakers and language minority students) and a sample of language minority students with and without mathematical learning disabilities.

8.1 Correlation analysis

In the sample of students with mathematical learning disabilities, verbal fluency had the highest negative correlation. This is an interesting result which might have an effect on the discourse level. Verbal fluency was assessed with a measure of day-to-day language (forming a sentence with given words). Proficiency in day-to-day language is important for mathematical learning on discourse level, when expressing mathematical concepts. Moschkovic (2015) and Erath et al. (2018) emphasize the importance of students' participation in the classroom discourse in helping them to develop a conceptual understanding of mathematics. However, if students have problems with verbal fluency in day-to -day vocabulary, this may have an impact on their participation in classroom discourse and, therefore, hinder the mathematical learning process. Evidence from Bochnik (2017) supports this assumption. The study found that proficiency in German had an impact on mathematical achievement. However, no causal relationships can be drawn with the aforementioned correlation results.

8.2 Hierarchical multiple regression analysis

The results show that math-related variables had the highest impact on the mathematical learning gains of participants with learning disabilities in mathematics. A significant increase of R^2 for first language was found only when including this variable as a first step. An effect of language-related variables was found for the post-test. However, this effect disappeared completely at the follow-up, four months later. These findings lead to the conclusion that the slower rate of mathematical progress shown by students with mathematical learning disabilities seems to be caused by specific mathematical problems, rather than by language issues. This result is in line with the findings of Vukovic and Lesaux (2013). Their research with typically developing students shows that the relationship between language ability and mathematical cognition seems to be similar for language minority students and native speakers (see Prediger et al., 2018 for older students). This also seems to be true for students with mathematical learning disabilities, regardless of language background. Geary et al. (2017) found, in a sample of typically achieving students, that domain-general effects (intelligence, working memory, reading) on mathematics achievement remained stable across grades, whereas the overall mathematics-specific effects increased across grades. This could also be due to the cumulative nature of mathematics, and might be even more the case for students with mathematical learning disabilities and explains the absence of any effect from the control variables.

In the language minority student sample, only math-related variables had a significant impact on students' mathematical learning gains. This leads to the conclusion that the influences of language and control variables seem to be similar for language minority students, with or without mathematical learning disabilities.

8.3 Limitations

Some limitations of the study should be mentioned. First, the samples were small due to the fundamental challenge of finding a highly selective sample of language minority students with and without mathematical learning disabilities. The highly selective sample led to rather low alpha coefficients in some scales, even when standardized measures were used. The norms of these tests are based on representative samples of students, and might not be suitable for language minority students. Developing suitable measures and testing existing measures for text equity for language minority students are therefore major objectives for future research.

In addition, the choice of language variables and instruments could be queried. The reading test gives only a global score of reading competence, and using an instrument that assesses different components of reading competence could have resulted in different findings. Further, the test for examining working memory was language-based, and a measure without language requirements (e.g., visual processing) could have led to other results. Visual processing was tested, but had to be excluded from the analyses due to a low reliability score. The small sample size also meant that hierarchical regression analyses had to be performed, and the impact of single predictors, such as reading comprehension or working memory, could not be estimated. Therefore, it was not possible to outline effects on word, sentence and text, and discourse level.

Finally, because of the constraints of the budget, it was not possible to compare language minority students with average math achievement with a similar sample of native speakers. However, it is the first longitudinal study disentangling the relationship between language-related variables and mathematical learning gains.

9 Conclusion

The results of the study confirm the assertion by Moschkovich (2010) that it is important for studies to focus not only on differences between monolinguals and bilinguals, but also on their similarities. Our study provides evidence that one of these similarities is the profile of mathematical learning disabilities, irrespective of the student's first language. Our results indicate that these students need support both in second-language acquisition and in mathematics. Future research into this problem would benefit from studies with bigger sample sizes. A very interesting factor, which has rarely been investigated, is verbal fluency and its influence on participation in classroom discourse. This means that, as suggested by Vukovic (2012), research on mathematical learning disabilities and specifically on language minority students with mathematical learning disabilities has to focus on both numerical and language skills.

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