Susanne Prediger and Ángela Uribe

Exploiting the epistemic role of multilingual resources in superdiverse mathematics classrooms: Design principles and insights into students' learning processes

1 Introduction

Many language policy documents worldwide have called for treating students' multiple languages as resources in subject matter classrooms (Beacco et al., 2015). Many qualitative observation studies (Adler, 2001; Barwell et al., 2016; Planas, 2018) and some quantitative intervention studies in mathematics education (Schüler-Meyer et al., 2019) have also shown that teaching approaches that activate students' multilingual resources can be beneficial.

However, most of these studies refer to classrooms with shared bilingualism, which means teachers and students share at least two languages. In contrast, Meyer et al. (2016) describe European schools as superdiverse language contexts in which more than five non-shared languages can be present in a classroom with only the language of instruction being shared. This is exemplified in the German-language context: In German schools, 30% of all students are multilingual, with most of them being second- to fourth-generation children of immigrant families. Typical classes in urban areas have five to seven languages, with only German being shared by all students and English learned as a foreign language. Teachers might speak one immigrant language, but not all. Usually, using home languages is allowed in small group work, but not encouraged and built upon.

The prevalence of non-shared multilingualism raises the question of how teaching approaches for activating multilingual resources can be transferred from classrooms with shared bilingualism to superdiverse classrooms with non-shared multilingualism.

In this paper, we argue that this transfer is possible when the focus is not only on the communicative role of multiple languages (i.e., so that students can

Acknowledgment: This paper was developed in the MuM-Multi 2: Fostering Language in Multilingual Mathematics Classrooms project, funded from 2017–2020 by the German Federal Ministry of Education and Research (BMBF; grant 01JM1703A to S. Prediger and A. Redder). We thank our colleagues for the interdisciplinary collaboration.

https://doi.org/10.1515/9783110661941-005

engage more easily in discussions about mathematics), but also on the *epistemic role of multiple languages* (Prediger et al., 2019). Section 2 of this paper presents the theoretical background of this argumentation. In Section 3, we introduce design principles and the design elements used to realize those. Sections 4 and 5 present a case study from an ongoing design research project in Grade 7, in which we explore how to adapt the design principles of multilingual mathematics classrooms for the mathematical topic of covariation. Thus, we pursue the following research question for a topic-specific case study on covariation: *What are the benefits and limitations of activating non-shared multilingual resources in a superdiverse mathematics classroom?*

The current paper is an expanded version of an unpublished conference paper on this topic (Uribe & Prediger, 2020). Whereas the empirical part is mainly preserved, the theoretical part is significantly enhanced.

2 Theoretical background

2.1 Existing research on building upon students' multilingual resources and its epistemic role

As the majority of the societies in the world are multilingual, many students are obliged to learn mathematics in languages of instruction that do not correspond to their home languages. The call for instructional approaches that build upon students' home languages as resources for mathematics learning (Barwell, 2009; Planas, 2018) has its sources in early language policy discourses (Ruíz, 1984), and it is now widely acknowledged in many educational policies (e.g., for the Council of Europe: Beacco et al., (2015)).

Many qualitative and quantitative studies have identified the benefits of activating multilingual students' home languages as resources for mathematics learning (Barwell et al., 2016; Planas 2018). These benefits include (1) higher engagement in classrooms discussions, (2) strengthened identities, (3) better connections to students' everyday school experiences, (4) better support in literacy development, and, ideally, (5) higher mathematical achievements (quantitative empirical evidence for efficacy has been provided in only few controlled trials, e.g., in Schüler-Meyer et al., 2019).

Many researchers in these studies have traced back the possible effects of using multiple languages in code-switching practices, with a strong focus on the *communicative role* of students' home languages. The hypothesized relationship was that being encouraged to use *all* languages strengthened their ability

to participate in communication (1), which then results in the other possible benefits (2–5).

In recent years, researchers have also increasingly emphasized a second role of multilingual resources: the *epistemic role of languages as tools for thinking and knowledge construction*, especially for students' meaning-making processes (Barwell, 2018; Planas, 2018; Prediger et al., 2019). This epistemic role goes far beyond early ideas of simplified communication by code-switching: It has value whereby it emphasizes that connecting languages unconsciously or deliberately can reveal epistemic and didactic potentials for students' processes of meaning-making. For example, Prediger et al. (2019) show how discussing different conceptualizations of fractions in German and Turkish could enrich multilingual students' multifaceted understanding of the part–whole relationship. In these cases, multiperspectivity on mathematics concepts is strengthened by connecting a variety of language-specific nuances (Planas, 2018; Prediger et al., 2019). These benefits cannot be explained by code-switching in complementary bilingual modes (languages do not only complement each other in moments of insufficiency), but require a *connective bilingual mode* in which the individuals deliberatively or unconsciously connect their languages mentally or in external communication. Hence, the *connective bilingual mode* has been presented to realize ideas of translanguaging (Li, 2011) for mathematics learning. The relevance of the *connective bilingual mode* with its added epistemic value for knowledge construction is also the reason, why we speak of activating *multilingual* resources, in other words, the full linguistic repertoire rather than just activating *home languages* (which might be restricted to a complementary bilingual mode rather than a connective mode).

The distinction between the *communicative* and *epistemic role of languages* is crucial for superdiverse classrooms: The communicative role of multilingual resources can mainly be exploited in classrooms with *shared bilingualism*, when teachers and students share at least two languages to be used for communication. In contrast, the epistemic role of multilingual resources can also be exploited in *superdiverse language contexts* with non-shared multilingualism, even for monolingual students in these classrooms, as we will show in this paper.

2.2 The epistemic role of multilingual resources in the literacy engagement framework

The possible connection of the five potential benefits identified in Section 2.1 can be explained through Cummin's (2015) literacy engagement framework, especially in its form in Fig. 1, which was adapted to mathematics learning by including the grey boxes (Uribe & Prediger, 2020).

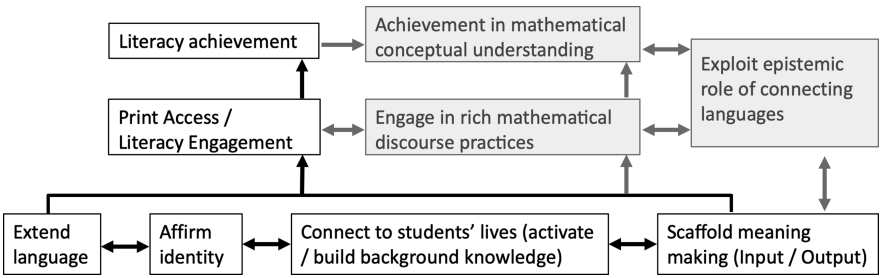

Fig. 1: Literacy engagement framework (Cummins 2015, p. 240), adapted for mathematics by adding grey boxes.

When we *extend* the students' *language* resources by allowing the use of all languages, registers, and representations, we are then not only able to strengthen students' *identities* and *connect* to students' everyday *experiences*, but we also enhance their *meaning-making* processes. This opens the base for extending students' language repertoires with more elaborated language, which can contribute to increased participation in language activities (referred to as print access and *literacy engagement* by Cummins, 2015) and students' language learning in the long run (referred to as *literacy achievement*).

S*caffolded meaning-making* can contribute to *exploiting the epistemic role* of connecting languages (Schüler-Meyer et al., 2019; Barwell, 2018), which can strengthen students' access to joint language activities and, in particular, *engage students in rich mathematical discourse practices*. This can lead to *higher achievement in mathematical conceptual understanding* (Erath et al., 2018).

Exploiting epistemic potentials of language comparisons for enhancing students' conceptual understanding is of particular importance when students are in the process of learning to understand highly compacted concepts (such as functional relationships; see Prediger & Zindel, 2017) by unfolding them in joint discourse practices. This involves de-composing the condensed language (e.g., de-composing nominalizations or condensed adjectives into verbal forms) and connecting it again to the more compacted language. Planas (2019: 27) provides an example of translanguaging practices from a classroom with shared bilingualism in which the connection of nominalized expressions in one language with de-composed expressions in the second language supported students' meaning-making processes when learning about equations.

Although these possible impacts have been shown to *potentially exist*, *realizing* them deliberately in teaching-learning situations is nevertheless a challenging task for instructional designers and teachers. Realization is easier in classrooms with shared bilingualism, but the following section will present design principles

and design elements for realizations in mathematics classrooms with multiple languages where all (but one) languages are not shared.

Since most of the existing studies stem from language contexts of shared multi- and bilingualism (e.g., California, Ireland, South Africa), Meyer et al. (2016) called for more research to explore how existing teaching approaches for activating multilingual resources can be transferred to superdiverse classrooms with non-shared multilingualism. As the empirical part of the paper will show, the bilingual connective mode is crucial as it can also be activated internally when only one speaker of a certain home language is present.

3 Design principles and design elements for exploiting multilingual resources epistemically

Exploiting the epistemic role of multilingual resources can best take place in teaching-learning arrangements that are based on the most relevant design principles for language-responsive teaching-learning arrangements (which will eventually be applied in monolingual contexts; see Erath et al., 2021 for an overview on the research base and Schüler-Meyer et al., 2019 for their adaptation to bi- and multilingual classrooms):

DP1 Engaging students in rich discourse practices
DP2 Macro-scaffolding along two coordinated learning trajectories
DP3 Connecting multiple representations, languages, and registers
DP4 Variation for initiating reflection

DP1 means that teachers should demand and support discourse practices so that *all* students can participate in explaining meanings, arguing, and discussing divergent ideas. DP2 means that the language learning opportunities should be sequenced in parallel to the mathematical learning opportunities starting from students' everyday resources, establishing a common meaning-related language before introducing formal, technical language. Both design principles are crucial and unchanged in both multilingual and monolingual language-responsive teaching-learning arrangements.

In a multilingual setting, DP3, *connecting multiple representations and registers*, is enriched by also connecting languages (Prediger et al., 2016), preferably in the connective bilingual mode of each individual, but perhaps only with inner

speech if no partner with the same languages is present. To initiate this connection, we use the following design elements as examples:

- *Recurrent explicit and implicit prompts to use two languages* (DE3.1) are required in contexts that are usually monolingual before students dare to activate multilingual resources (Meyer et al., 2016). Also, students from monolingual German families are encouraged to use their foreign language, English, in order to treat all students equally as multilinguals. Explicit prompts are required to overcome the usually established monolingual norms (at least in most German schools), but always respect the principle of voluntary language use. Once new multilingual norms are established, this design element is no longer necessary.
- *Graphical representations* (DE3.2), such as the double number line, facilitate students' explanations and descriptions of mathematical relationships by activating the whole range of multimodal resources: Gestures, deictic means, and embodied language are more important meaning-making resources than well-elaborated technical phrases in the home language (Barwell, 2018).
- *Activities to make sense of crucial meaning-related phrases* (DE3.3). Explaining (not only translating) crucial meaning-related phrases in two languages is a key activity to mobilize the epistemic function of multiple languages. This activity usually elicits slightly different nuances of meaning and by this consolidates the conceptual understanding (Prediger et al., 2019).

The last design principle, the variation principle, DP4, means that language-related or mathematical reflection can be initiated by slightly varying some language pieces (form, function, etc.) and comparing them explicitly (Erath et al., 2021). In the multilingual setting, this specifically involves the comparison of expressions and conceptualizations for the same mathematical concept in different languages. It can be realized, for example, by the following design elements:

- *Activities for comparing phrase variations with different meanings* (DE4.1). Concept cartoons or varied word problems can initiate the students' comparison of phrases that look very similar but have different meanings. This can raise students' language awareness of small details (Dröse & Prediger, 2020).
- *Activities for comparing multiple phrases with same meaning* (DE4.2). When students provide synonymous phrases for the same key meaning-related phrase (in the language of instruction and other languages; see DE3.3), the teacher does not necessarily need to be able to translate it: Sometimes it is also interesting to compare the syntactic form. Multilingualism becomes an occasion to reflect on language and on the different words in use in terms

of conceptual relevance: When students translate German lexical means into different languages in their language repertoires, they can reflect on the resulting variations that emerge through the translation process.

In the following case studies, we illustrate how the design elements DE3.3 and DE4.2 initiate a rich mathematical reflection and moments of conceptual insight.

4 Methodological framework of the case study

4.1 Research context of the case study

4.1.1 Embedding in larger design research project

The case study presented here belongs to the larger project MuM-Multi 2 being carried out in a German-language context. The investigation of transfer possibilities between contexts of shared and non-shared multilingualism has been conducted using a design research methodology that iteratively combines the design of teaching-learning arrangements with the qualitative study of initiated teaching-learning processes (Gravemeijer & Cobb, 2006). The design elements and design principles in Section 3 were a first outcome of the design research project on the practical side. Here, we provide some empirical insights into the initiated learning processes.

4.1.2 Mathematical topic in view: Covariational reasoning

The mathematical topic in view is covariation, a core idea for functional relationships that is fundamental to students' mathematical learning: Using functional relationships, we can investigate how one quantity varies with another one as a dynamic phenomenon (Thompson & Carlson, 2017). Students first encounter covariation in the coupling of two quantities that vary simultaneously in proportional reasoning: "The more apples I buy, the higher the total price." In the first step of the learning trajectory, qualitative ideas about covariation are to be developed on the double number line, which proved to be a powerful visualization to activate students' intuitive resources (van Galen et al., 2008) and a tool that supports verbalizing and imagining quantities' values that vary smoothly or continuously. In a later step of the learning trajectory, qualitative covariation is quantified using fixed scalars, and the double number line is then used to engage students in reasoning flexibly up and down (van Galen et al., 2008).

With respect to the epistemic function of languages, covariation is an interesting exemplary topic, as it can be expressed in different degrees of compaction and reification (see Fig. 2).

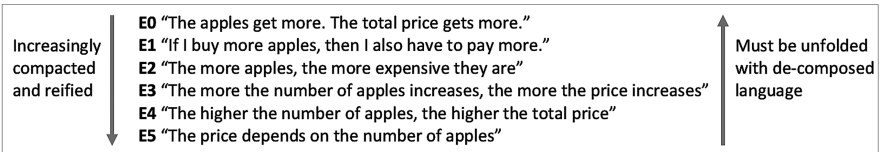

Fig. 2: Context-embedded expressions for covariation with different degrees of compaction.

For our case study, we hypothesize that different expressions for covariation can also provide multiple approaches for understanding this compacted concept. We explore whether this kind of epistemic support can also be provided in non-shared multilingualism.

4.1.3 Teaching learning arrangement in view

The language-responsive teaching-learning arrangement on covariational reasoning was designed based on the design principles and design elements in Section 3. The learning trajectory starts from students' intuitive mathematical resources and language resource (DP2) on informal qualitative covariation. By engaging the students in rich discourse practices of explaining meanings (DP1), the aim is to develop proportional reasoning across different contexts. The double number line, introduced as the continuously used graphical representation, is used to elicit students' multiple multimodal resources (DE3.2).

Fig. 3 shows two of the initial tasks of the learning trajectory, which first are treated in language-homogeneous small groups and then discussed in the whole class. Task 1 introduces the double number line by eliciting its intuitive use. The estimation prompt leads the students into their first thinking about covariation. Task 2 asks students to express their own ideas in two languages (DE3.3); Tarkan's model shows that this is expected to result in establishing a new norm (DE3.1). Students from monolingual German families are encouraged to use their foreign language, English, in order to treat all students equally as multilinguals (DE3.1). Whereas the pre-formulated answer articulates the correspondence approach, the later whole-class discussion of estimation strategies addresses covariation ideas (DE4.1). The whole-class discussion is designed to compare multiple phrases with same meanings in different languages (DE4.2).

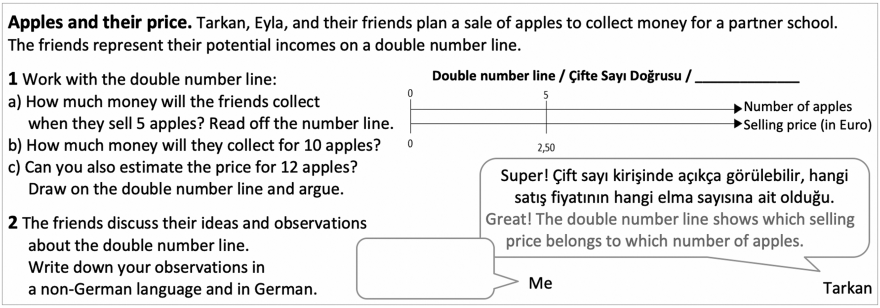

Fig. 3: Initial tasks for activating multilingual resources on covariation.

4.2 Methods of data gathering

The video data of the case study presented here stems from the first lesson of the design experiment Cycle 3 (encompassing four lessons in total), in which the intervention in a regular Grade 7 classroom was video-recorded and partly transcribed.

All students speak German (and English as their common foreign language learned in school); additional home languages are Turkish (spoken by 12 students), Polish (4 students), Arabic (2 students), Albanian (1 student), and Kurdish (1 student). The episode analyzed for this paper stems from a whole-class discussion collecting students' work on Task 2 (shown in Fig. 3).

4.3 Methods of data analysis

The transcript was qualitatively analyzed with respect to students' multiple language use, their conceptions of covariance, and the processes of compacting and unfolding using more or less condensed language. The analytic scheme was deductively derived from Prediger and Zindel (2017) and then inductively adapted to the other learning content. For the final coding scheme the codes E0–E5 were assigned for the expressions (see Fig. 2) used by the students and the teacher in order to identify communicative and epistemic benefits and limitations of work in a non-shared multilingual context. The codes were assigned based on the following criteria:

E0: Variation of quantities is described separately without expressing the relationship between them.
E1: Covariation of both quantities is expressed in complete sub-clauses with verbs connected by the conjunctions "if . . . then . . . ."
E2: Covariation of both quantities is expressed in compacted adverbs, connected, for instance, by "the more apples . . . the more . . ."
E3: Quantities are addressed explicitly by nominalizations (e.g., "the value," "the amount"), and the covariation is expressed by verbs (e.g., "increase") connected by "if . . . then . . ." or "the . . . the . . . ."
E4: Quantities are addressed explicitly using nominalizations (e.g., "the value," "the amount"), and the covariation is compacted into adjectives "the . . . the higher the price," connected by "the . . . the . . . ."
E5: Quantities are addressed explicitly by nominalizations (e.g., "the value," "the amount"), and the covariation is not addressed explicitly but compacted into a phrase such as "depends on."

5 Empirical insights into benefits and limitations of exploiting the epistemic role of multilingual resources

5.1 Qualitative analysis of three sequences

We analyzed a transcript of the whole-class discussion on students' multilingual writings about covariation on the double number line (Task 2 in Fig. 3) in order to show how the students and the teacher made sense of covariation by connecting multiple phrases (DE3.3).

Sequence 1: Separate variations in two languages
The whole-class discussion starts with a conversation on Baydar's German-Turkish writing product (Fig. 4) which is projected onto the whiteboard.

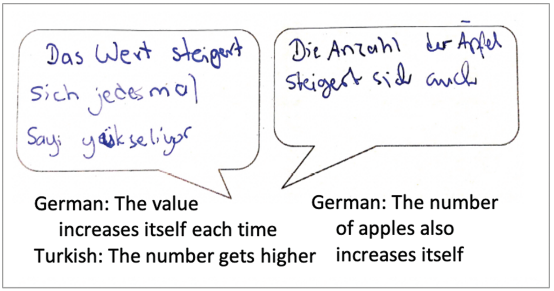

Fig. 4: Baydar's writing product (translated).

89	Teacher	Baydar has written something very nice: that the value increases each time. What is written here? [*points to Baydar's Turkish sentence*]
90	Fatma	[*translates*] That the number gets higher.
91	Teacher	That the number gets higher. And then he says here that the number of apples also increases itself.
92	Fatma	That is the same.
93	Teacher	Is it the same?
94	Shenay	No, that is "number" and that is "value."
95	Fatma	I see! Yes.
97	Shenay	The one means that the value [meaning the price] increases itself each time and, yes, that the value increases. And the other one, that the apples, though the number increases.
98	Teacher	Thus, what does he mean by the value?
99	Shenay	Eh, the selling price.
100	Teacher	The selling price. Ok, he sees that the selling price changes. And what else does he observe?
101	Thilo	That the selling price changes and that the number changes. So, that five apples, eh, ten apples, cost 10 €.

The teacher chooses what Baydar wrote because it describes the increase of both quantities separately in two speech bubbles (included as expression **E0** in the analytic graph in Fig. 5). She asks other Turkish-speaking students to translate the Turkish part for the class (Turn 89), which affirms their identity as competent multilinguals (Fig. 1). Fatma translates (Turn 90) and states that both speech bubbles describe the same thing (Turn 92), which is not true for the Turkish sentence.

As the teacher cannot read the Turkish, she misses this subtlety and continues working with the German text. Shenay distinguishes the value (the selling price) from the number of apples (Turn 94). She uses re-voicing to repair the language slip, changing "increase itself" to "increase" without further comments (a productive practice).

As the teacher's focus is first on striving for precision by turning "value" into "selling price" (Turn 98–101), the students continue articulating two separate statements of variation, without combining them into a statement of covariation until Turn 101. When Thilo tries to explain the two separate variations in Turn 101, he expresses the correspondence, but not yet the covariation approach (still coded as **E0, Corr**).

Sequence 1 shows how students' interest can be attracted to understanding students' bilingual writing, even those students who do not share the language. It also shows a limitation of not sharing a language with the students, as the teacher missed the opportunity to compare the Turkish "number" with the German "value."

Sequence 2: German expressions for covariation

In the next turns, the two separate variations are condensed into a covariation, and the direction of the dependency is questioned:

103	Dennis	That's normal, though! If the value increases itself, then more apples are there. Otherwise, it makes no sense.	E1/E3
105	Teacher	But what depends on what?	
106	Dennis	The, well, the price depends at [*sic*] the number of apples.	E5
107	Teacher	The price depends on the number of apples.	
108	Dennis	Yes.	
109	Teacher	That is more precise, isn't it?	
110	Dennis	Somewhat.	
111	Teacher	How would you articulate the sentence?	
112	Shenay	Em, the more number of apples, the more the price increases.	E3
113	Teacher	The more apples, the higher the number of apples, the . . .	E4
114	Lale	. . . more expensive	
115	Shenay	The higher increases the price.	
116	Lale	Eh, I would say, the more apples, the more expensive they are.	E2
117	Teacher	The more apples, the more expensive.	

Dennis articulates an if-then sentence that first combines the two variations in a covariation (coded as **E1** in Turn 103). Since he uses condensed verbs for expressing the change, the utterance is also coded as **E3**.

The teacher reacts to his inverse direction of dependency by asking a question, but this question is phrased in a much more condensed structure, coded as **E5** (Turn 105). Dennis corrects himself by using the highly condensed expression **E5** that he picks up from the teacher (Turn 106). In order to activate the design principle DP4, variation for initiating reflection, the teacher calls for further ways of articulating the covariance (Turn 111). The collection of utterances (Turns 111–117) reveals four different ways of expressing the covariance (**E5**, **E3**,

E4, and **E2**). These four phrases successively de-compose the highly compacted phrase of the teacher (**E5**).

This sequence shows nicely how the call for further phrases can add multiple modes of expression to the covariation concept in view.

Sequence 3: Multilingual expressions for covariation

So far, three students contributed utterances in German only. In order to activate *all* students to recapitulate the discussion, the teacher calls for finding further expressions in *other* languages:

121	Teacher	Further possibilities, a further idea how to construct the sentence? Who has an idea in Turkish, Polish, or Arabic, how could you construct the sentence in these languages? [. . .] Think about it for a minute. And also think [*directed to the students of monolingual German-speaking families*], how would you express that in English?	
122	Lale	We have one!	
123	Teacher	Yes, you have one. We will allow some more time for the others. Write it into the speech bubbles. [*6-second break*]. You can also use dictionaries but only for selected words, no direct translation.	
124	Teacher	[*After a 5-minute break*] Now, do you want to read the sentence in Turkish and what it means?	
125	Fatma	"Elmalarin sayisi yükselirse fiyati da yükselir." [Translated from Turkish: If the number of apples raised, then the price would also raise.]	E1
126	Teacher	Have you also written "the . . . the" or have you rephrased it?	
127	Lale	We have simply – directly written it.	
128	Teacher	What would it mean exactly, what you have written?	
129	Lale	If the number of apples gets higher, then the price also gets higher.	E1/
130	Teacher	Super, that is also creative. How is it for you?	E3
131	Thilo	We have only written "the more apples" [English in original] and then we wanted to write, eh, "more," eh, the more? [*Student searches for "the . . . the" – "je . . . desto" in German*]	E2
132	Teacher	"Expensive"	
133	Thilo	Yes	
134	Teacher	Well, well, well. And you [*to Dennis*]?	
135	Dennis	We don't have it completely in Polish, but I can try.	
136	Teacher	Yes, so that we can see how Polish . . .	

137	Dennis	"Ile masz jabłko, tak będzie" [Translated from Polish: So many apples as you have, so . . . it will be] and the rest I do not know, there is a word missing.	
138	Teacher	Which word is missing?	
139	Dennis	Eh, expensive	
140	Shenay	Shall I google that quickly?	Corr
141	Dennis	Yeah, do it.	
. . .			
149	Shenay	"Kostownie" [Translated from Polish: valuable]	
150	Dennis	"Ile masz jabłko, tak będzie kostownie." [So many apples as you have, so valuable it will be.]	
151	Teacher	[. . .] How would this sentence be in German?	E4
152	Dennis	This would be, eh. I have so many apples. The more apples I have, the more evaluable it'll be.	
153	Shenay	Yes, you cannot translate everything directly.	
. . .			
155	Qaiss	"Lil- mazīd minat-tuffāḥ kallama kān ʿalayka ʾan tadfaʿa ʾakṯar" [Translated from Arabic: For more apples, all you have to do is pay more]	E1
. . .			
157	Teacher	When you buy more apples, then you have to pay more. Thus, when the number increases, then you have to spend more money.	E1/ E3

In this sequence, the classroom transforms into a *translanguaging space* in which all students feel free to deal with multiple languages for meaning-making in mathematics, regardless of their proficiency level. Even if the conversation takes place mainly in their shared language, German, they individually make use of their multilingual repertoires in an epistemic function that supports a consolidated meaning making of covariation. In Turn 125, Fatma brings in an important sentence variation in Turkish (**E1**). After Lale first declares it to be a literal translation (Turn 127, **E1/E3**), the teachers' repeated questions result in a more accurate translation (Turn 129) that reveals the variation from "the . . . the" to "if . . . then" and a denominalization, both supporting the unpacking of the condensed concept.

The group of foreign English-language speakers searches for the translation of "je . . . desto" in German ("the . . . the" in English) in Turn 131 (**E2**). The teacher provides vocabulary support for "valuable" without noticing this obstacle. (Later, the students use a more de-composed version: As they still do not know "the . . . the," they construct a longer sentence.)

Dennis and three other Polish-speaking students are also unfamiliar with the use of Polish within a mathematical discourse and feel uncomfortable using it (Turn 135). The teacher encourages them several times, and Dennis reads his incomplete sentence (Turn 137). With dictionary support of the non-Polish-speaking peer Shenay (Turns 141–147), he completes their Polish sentence, showing the students' engagement and identity strengthening. Dennis' Polish sentence

reveals an interesting perspective, the correspondence approach, assigning one price for each number of apples (Turn 150). Again, the students' limitations in a language serve as catalysts for multiple perspectives and for enriching the learning situation. However, Dennis' translation back to German (Turn 152, **E4**) does not articulate the correspondence approach anymore, so the teacher cannot exploit it. This shows again the limitations of exploiting non-shared languages. A similar situation occurs with the Arabic sentence in Turn 155, which is only partially exploited (**E1**).

5.2 Overview of three sequences

A summary of the interpretations in the analytic graph in Fig. 5 shows the course of expressions brought in by students and the teacher in Sequences 1–3. During the 70 lines of transcript, the students articulate all six expressions (**E0–E5** as shown in Fig. 2) several times and connect them by moving forward and backward. Sequence 1 started with a misconception **E0** and the teacher funneled the most compacted expression **E1**. Many class discussions stop at this point.

The created translanguaging space, however, allowed the students to connect the highly compacted **E5** to four other versions, which allowed them a multiperspective meaning-making. This was possible for the students even in languages that are not shared in the whole group.

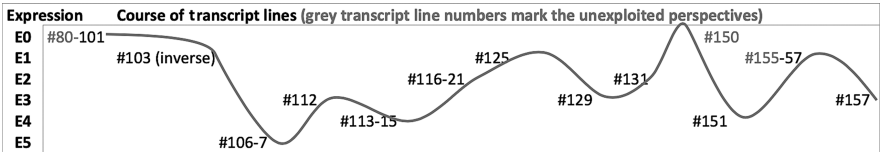

Fig. 5: Analytic graph of the process of connecting multilingual expressions in Sequences 1–3.

6 Discussion and outlook

Instructional strategies for activating multilingual resources for mathematics learning have been shown to be beneficial not only for overcoming communicative obstacles, but also for epistemic purposes, especially by providing multiple perspectives for meaning-making (Cummins, 2015; Planas, 2019; Prediger et al., 2019). The current case study investigates whether these results from

classrooms with shared bilingualism can also be transferred to classrooms with non-shared multilingualism.

Qualitative analysis shows that teachers can also activate students' home and foreign languages and conduct productive discussions about multiple languages (design principles DP3 and DP4 when the conversation itself is in German; by striving for preciseness in different languages, DE4.1), and by comparing different expressions in German and other languages (DE3.3 and DE4.2), the students in the case study were able to realize that the covariation statement "the more *x*, the more *y*" can be expressed in different degrees of compaction. They benefited mathematically by unfolding a highly compacted mathematical concept. Although the macro-scaffolding principle suggests starting with less compacted expressions and connecting them to more compacted ones, the learning is also deepened when different degrees of compaction are mutually connected and explicitly compared. Thus, mobilizing students' multiple language not only increases their agency by being addressed as competent multilingual speakers, but also strengthens the students' mathematical reflection.

The case study also shows that it is not always necessary to share the languages: Help for Polish expressions was provided by a non-Polish speaker offering to google a missing word. The analysis shows how the "bilingual connective mode" can also be established by individual mental connections that are only partially accessible to the non-shared space.

However, the case study also reveals various examples of limitations where the teacher was unable to react or include some students' statements, especially when she cannot exploit subtle differences because they are not covered by the students' translations back to the shared language of instruction.

In future research, the methodological limitations of this case study (being bound to a specific classroom and languages, with a limited scope and small sample) should be overcome by extending the investigations to further cases, mathematical topics, and language contexts.

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