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Culture and language: How do these influence arithmetic?

International comparisons such as those carried out by TIMSS and PISA (e.g., Mullis et al., 2016a, b; OECD, 2016) tend to show considerably better arithmetical performance by children in some countries than in others. The position of countries can vary over time, but one consistent finding is that children from countries in the Far East, such as Japan, Korea, Singapore, and China, tend to perform better in arithmetic than do children in most parts of Europe and America.

Stevenson et al. (1993) looked at performance in different subjects. They found that Japanese and Korean children outperformed American children to a greater extent in mathematics than in reading. This may be in part because of specific difficulties with regard to reading that are posed by East Asian writing systems; but it is also likely that the results reflect a special focus on mathematics in East Asian countries.

1 Why do some countries seem to perform better than others?

There are a number of reasons why some countries may perform better than others. These include (1) suitability and comparability of the tests for different national or cultural groups; (2) the social and economic situation of the countries; (3) cultural attitudes toward mathematics; (4) teaching methods, and the emphasis placed on mathematics in school; (5) mathematical experience in out-of-school contexts; and (6) influences of language and in particular the counting system.

(1) *Suitability and comparability of tests*

Considering the emphasis that has been placed on international comparisons, there has been relatively little study of how suitable particular tests may be in different international contexts. Yet this may well be quite important. First of all, children in some countries and cultural groups are in general more accustomed than others to being tested and more familiar with the conventions of testing (e.g., the concept of being questioned in order to find out what they know, rather than to find out the answer). While those countries that participate

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in international comparisons are unlikely to have many children who are totally inexperienced with regard to testing, there are certainly likely to be differences in the extent of testing experience that they have.

There is also the issue of context. Children are likely to perform better in a context that is familiar to them, as will be discussed in section (5); and contextual familiarity of any problem may vary between countries, locations within countries, and cultural groups.

More generally, it is difficult to standardize a test reliably for all countries in which it may be used, which creates problems for any international comparison. Kreiner and Christenson (2014) pointed out that the PISA results are not totally reliable, and can fluctuate significantly according to which test questions are used. The Rasch measurement model used by PISA is valid only if the questions are equally difficult for each country. This appears not always to be true: especially for reading, but also for mathematics.

(2) *The economic and political situation of a country*

The very poorest countries, where a high proportion of people have little or no secondary school education, are rarely included in international comparisons. However, even among those countries that are included in such comparisons, social and economic deprivation and political violence and insecurity are likely to be associated with reduced performance. The lowest-achieving countries for Grade 8 Mathematics in the TIMSS 2011 and 2015 studies (Mullis et al., 2016a, b) were mostly countries in the Middle East that are listed on the ISI Register of Developing Countries, and which were in many cases also experiencing political turmoil at the time when the testing took place. The lowest-achieving country of all was Ghana, which was also the only country in southern or western Africa to be included, and, while it is one of the more stable and prosperous countries in Africa, it still shares some of the economic problems of the region.

It should be noted at this point that although the use of international comparisons may give the impression that countries are largely homogeneous, there can be much variation within a country, which may have links to regional economic differences. For example, in the TIMSS comparisons, Massachusetts was very much above the average in mathematics achievement, while Alabama was somewhat below it. This makes a general rating for the United States unreliable. The difference is likely to reflect the economic situation of the two states: Massachusetts comes first or second and Alabama 45th in recent rankings of the 50 states in terms of GDP per capita. In China, it is well known that rural areas are much poorer economically than urban areas, but almost all testing for international comparisons has been in urban areas, and thus may not reflect the whole country.

(3) *Cultural attitudes toward mathematics*
Some cultures seem to value mathematics more highly than others, which may affect performance. There appears to be a tendency for East Asian cultures to place a higher value on academic performance in general, and mathematics and related subjects in particular, than do many other countries (Askew et al., 2010; Stevenson et al., 2000). This may lead to greater attention to, and practice in, mathematics.

The relationships between emotions and attitudes toward mathematics, mathematical performance, and national achievement in mathematics appear to be complex, though *within* any country positive attitudes toward mathematics are linked to better performance, and in particular mathematics anxiety is negatively related to performance (Foley et al., 2017; Lee, 2009). Children in high-achieving Asian countries, such as Korea and Japan, tended to demonstrate high mathematics anxiety; while those in high-achieving Western European countries, such as Finland, the Netherlands, Liechtenstein, and Switzerland, tended to demonstrate low mathematics anxiety. This may be because the high importance given to mathematics in East Asian countries makes failure more threatening. It may also reflect some cultural differences in the social acceptability of expressing anxiety about academic subjects.

(4) *Teaching methods, and amount of time devoted to arithmetic in school*
Since international comparisons have come into extensive use, there have been many proposals that countries with medium or low positions in the international league tables for school mathematics performance should emulate the teaching methods of higher scorers, such as the Pacific Rim countries. While this sounds plausible, it is important to exercise caution (Jerrim, 2011; Sturman, 2015). For one thing as stated above, factors such as the suitability of the measures used, and the economic situation of a country, could be what influences performance, as could the level of emphasis put on mathematics within a culture. For another, even as regards teaching methods there are usually many differences between mathematics instruction in different countries, and it may be difficult to isolate which factors are causing differences in mathematics performance.

For example, there has been considerable emphasis recently on the "Mathematics Mastery" approach of Singapore and other East Asian countries, and some UK schools have begun to introduce aspects of this approach. It can be difficult, however, to tease apart the numerous aspects of this approach that might contribute to better mathematical performance (Jerrim & Vignoles, 2015). For example, the Mathematics Mastery approach breaks different parts of the mathematics curriculum into units with clearly defined goals. It has a narrower, but deeper, focus than some other primary curricula such as that of the UK, aiming to teach a smaller number of topics within arithmetic in depth rather than a larger number more superficially. It also aims to ensure that all pupils have mastered each unit before going on to another one. While the approach is sometimes misinterpreted as involving whole-class teaching, with all children expected to succeed so that no allowances or individualized interventions are given to weaker pupils, in fact teachers are expected to look at the children's work, and to intervene immediately with individuals' misconceptions before moving on. This, of course, places high demands on the teacher.

Here we may possibly find another difference between East Asian countries and many others: the status of and requirements for the teaching profession in many East Asian cities may influence performance as much as any specific aspects of the curriculum. Teaching tends to be regarded as a high-status profession, which requires high academic qualifications for entry, and to involve extensive continued professional development (Ma, 1999).

Another reason for international differences in arithmetic may be the sheer amount of time devoted to it in different countries. In the UK, and certainly in England, primary school children study a wider variety of subjects than children in some other countries, resulting in less time devoted specifically to mathematics. Within mathematics, children study a wide variety of topics: not only arithmetic, but shape and space; measurement; recording data; applying mathematical knowledge to real-world problems, and so on. This could lead to English children being less good at arithmetic, but better at some other aspects of mathematics, than those in some other countries. One international comparison did indicate that English children were worse at arithmetical calculation, but better at applying mathematics to real-world problems, than those in most other European countries.

Children in Pacific Rim countries, at least the urban children who are most commonly included in international comparisons, spend more time in academic pursuits, both in school and in homework, than those in many other countries. Mathematics comprises a higher proportion of that time than it does in many other countries. Thus, the sheer amount of time devoted to mathematics may explain at least part of the superior performance in mathematics by children in these countries.

(5) Children and indeed adults may learn mathematical techniques, strategies, and concepts within a particular context, and may not transfer them to other contexts. People may not always transfer what they learn in school mathematics to real-world non-school contexts and vice versa.

Carraher et al. (1985) studied Brazilian child street traders aged between 9 and 15 years. All attended school, though many attended somewhat irregularly. They were given the same arithmetic problems in three different contexts: (1) a "street" context, where the researchers approached them as customers and asked them about prices and change; (2) a "word problem" context where they were given school-type word problems dealing with prices and change in hypothetical vending situations; and (3) a numerical context, where they were given the problems in the form of written sums. The children performed much better in the street context than on word problems and much better on word problems than on written sums. They solved almost all – 98% – of the problems correctly in the street context. Seventy-four percent of the same problems were solved correctly when presented in the form of word problems; but only 37% were solved correctly when presented in the form of written sums. By contrast, middle-class children, who attended school regularly but had no street market experience, performed better in a numerical context than a market-type context.

Other studies of the effects of schooling versus street trading experience were carried out by Saxe (1985, 1990; Saxe & Esmonde, 2005). Saxe studied the arithmetical strategies of Oksapmin children in Papua New Guinea. Some were street vendors with little or no schooling; some attended school but had no vending experience; and some had both types of experience. They were all given word problems based on the prices and profits for selling sweets. Those with more schooling relied more on written numbers and place value notation. Those with little or no schooling referred more to the specific features of the currency.

Among children with equal amounts of schooling, children with vending experience used more derived fact strategies. Those without vending experience relied more on well-learned, school-taught algorithms. Those with more schooling relied more on written numbers and place value notation. Those with little or no schooling referred more to the specific features of the currency.

Even apart from school-taught arithmetic, different cultures may have different preoccupations. For instance, in many cultures, including the UK and white Australia, age is an important preoccupation, whereas it is much less important, for example, to Aboriginal Australians. On the other hand, navigation and the estimation of distance and direction are very important in Aboriginal Australian culture: far more than they are for most white Australians. Presumably for these reasons, Kearins (1991) found that Australian Aboriginal children were better than non-Aboriginal children at estimating direction, which was traditionally very important in this group. On the other hand, non-Aboriginal Australian children were better than Aboriginal children at estimating age, which is very important in Western culture but much less so in Aboriginal culture.

Posner (1982) found that the Dioula, a mercantile group of people on the Ivory Coast, learned to use rather complex calculation strategies for trading and selling purposes. Even those merchants who had never been to school were adept at calculation. Baoule people in the same region, who were farmers rather than merchants, did not demonstrate such high-level calculation abilities. These findings indicate that groups that require sophisticated calculation strategies are likely to develop them, with or without schooling.

The extent and nature of use of technology may also influence arithmetical performance. Ever since calculators became widely used, there have been concerns that they may interfere with children's learning to calculate and/or result in an unthinking approach. Obviously much will depend on *how* calculators are used; but on the whole, studies of the effects of calculator use have suggested that these are surprisingly weak: the use of a calculator as such does not have a large effect on the development of arithmetical calculation or reasoning.

Technological aids to arithmetic did not begin with calculators. In Pacific Rim countries, many people still use the abacus, which involves the use of beads on strings, where the strings represent place value. Experienced calculators can become very fast and accurate, and even use a "mental abacus": visualizing operations on an abacus, even when there is none present (Hatano et al., 1977; Stigler, 1984). However, even highly expert abacus calculators do not always transfer their abacus skills to other arithmetical contexts. The use of the abacus alone is unlikely to explain the superior arithmetical skills of people in Pacific Rim countries.

(6) For centuries, there has been much debate as to the role of number words and numerals in arithmetic. Could we do arithmetic without words? Locke (1690) argued that small numbers can be represented without words by showing numbers of fingers, but words are needed to keep track of larger numbers. According to this theory, speakers of languages without number words would be restricted to the understanding of numbers that can be represented through fingers.

Most languages have number words at least up to 10. However, a few Native American and Native Australian languages (e.g., Warlpiri in Australia) have only words corresponding to "one, two, three, many." Some languages with somewhat more extensive counting systems have limits on how far one can count; for example, some of the languages of Papua New Guinea count by pointing to body parts and use the names of these body parts for their counts (Butterworth, 1999; Lancy, 1983). In the Kewa language "1" is represented by the right little finger, and "34" by the nose. The upper limit of the Kewa counting system is 68, while that of the Oksapmin system is 19. It is arguable that there is an upper limit on the counting sequence in a language; then this may interfere with arithmetic and quantity representation beyond that number. It may also limit the ability to understand the key mathematical concept of infinity:

Pica et al. (2004) studied 55 Munduruku-speaking participants. Munduruku is a language spoken by approximately 7,000 indigenous people in the state of

Para, Brazil. It only includes the words for the numbers one to five. The first test was to name the numbers for sets including from 1 to 15 points. The second test involved showing participants two clouds of dots and asking them to judge which of the two sets was more numerous. The third test involved approximate computation. Participants were shown short video clips illustrating simple operations. For example, approximately 20 seeds fall into a box, and then approximately 30 more were added. The participants were asked whether the total was more or less than another set (e.g., of approximately 40 seeds). The fourth test involved exact computation. The participants were again shown video clips, and were asked to give the result of a precise mathematical operation, for example, 6 seeds minus 4 seeds.

Results showed that participants could not carry out arithmetic operations with quantities above 5. For example, they could not calculate 6–4 or 7–7 accurately. However, they could do approximate arithmetic just as well as French controls! The researchers concluded that numerical approximation ability is a basic cognitive ability that is common to all human beings, and which may be independent of language.

It is possible that even these findings underestimate the mathematical abilities of speakers of languages with limited counting systems. In this study, most of the exact number tasks involved subtraction, which is usually found to be more difficult than addition.

Another study was carried out by Butterworth et al. (2008), involving studied child speakers of two Aboriginal Australian languages, Anindilyakwa and Warlpiri, and compared them with English speakers. These languages do not have number words beyond three. Nonetheless, they showed some capacity for *exact* nonverbal arithmetic.

In this study, participants were given four tasks:

(1) Memory for sets of counters. Children had to reproduce sets comprising two, three, four, five, six, eight, or nine randomly placed counters.
(2) Cross-modal matching. Children had to match numbers of counters with numbers of times that a block was tapped (numerosities ranged from 1 to 7).
(3) Nonverbal addition. Children watched an experimenter put one or more counters under a cover onto a mat; and then add more counters. They were asked to "make your mat like hers." Sums included 2 + 1, 3 + 1, 4 + 1, 1 + 2, 1 + 3, 1 + 4, 3 + 3, 4 + 2, and 5 + 3.
(4) Sharing. Children shared sets of play-dough disks among three toy bears. The trials comprised 6, 9, 7, and 10.

There were effects on performance of both age (6- to 7-year-olds performed better than 4- to 5-year-olds) and of set size (children performed better on problems involving smaller numbers than larger numbers). However, there was little or no effect of language. Speakers of the languages with the limited counting systems performed as well on these tasks as English-speaking Australian controls.

2 Transparency of counting systems

As early as 1798, Edgeworth and Edgeworth (1798) pointed out that English speakers may be at a disadvantage compared with speakers of some other languages due to the relatively irregular English counting system.

Transparency of a counting system involves two major components: (i) The regularity of the spoken number system: the degree to which it gives a clear and consistent representation of the base system (usually base 10) used in the language; and (ii) The degree and consistency of conformity between the spoken and the written number system. In practice, these usually amount to the same thing, as most languages use the same (Arabic) written counting system.

East Asian counting systems are more transparent than most others. Instead of "eleven, twelve, thirteen" and so on, such counting systems use the equivalent of "ten-one, ten-two, ten-three" and so on. Instead of "twenty, twenty-one, . . . thirty, thirty-one . . .," they use the equivalent of "two-tens, two-tens-one, . . . three-tens, three-tens-one . . .."

It is sometimes suggested that the relative transparency of Asian counting systems is a major contributory factor to the superior performance of Pacific Rim children in most aspects of arithmetic. Learning number names may be easier in systems where new numbers may be inferred rather than having to be learned by rote. One might also expect that the concept of place value would be easier to comprehend and use in a regular counting system.

There are indeed results that suggest that users of regular counting systems find it easier to count, even before they start formal schooling. Miller et al. (1995) studied counting in Chinese and American four- and five-year-olds. The two groups performed similarly in learning to count up to 12, but the Chinese children were about a year ahead of the American children in the further development and counting of higher numbers.

There is also evidence that primary school children who use transparent counting systems find it easier to represent two-digit numbers than children who use less transparent counting systems. Miura, Okamoto, and colleagues studied six-year-old children of different nationalities (Miura et al., 1988; Okamoto, 2015). Three groups used regular counting systems; Japanese, Korean, and Chinese. Three groups used less regular counting systems – American, French, and

Swedish. The children were given tasks involving representation of two-digit numbers with base ten blocks (unit blocks and tens blocks; the latter being blocks with ten segments shown on them).

The users of transparent counting systems were far more likely to represent the tens and units by means of the blocks: typically representing 42 by four tens blocks and two unit blocks. The American, French, and Swedish children tended to attempt to represent the numbers as collections of units, such as by representing the number 42 as 42 unit blocks. The researchers concluded that the users of transparent counting systems find it easier to represent multi-digit numbers and that this leads to better arithmetical performance.

Several studies have supported this view, but some have not, and in general it seems likely that the effects of using a transparent counting system are specific to some aspects of arithmetic, rather than affecting all aspects. Some of the studies have involved representing numbers on empty number lines. Siegler and Mu (2008) found that Chinese kindergarten children performed better than American children on mental number line estimation tasks involving a number line spanning from 1 to 100. Laski and Yu (2014) found that Chinese children performed better on such tasks than Chinese-American children, who in turn performed better than monolingual English-speaking American children. This could indicate either that the *extent* to which children use a transparent counting system has a significant effect on their arithmetic (for Chinese-American children, the effect may be diluted by their use of English as well as Chinese) or, perhaps more likely, that both linguistic and educational factors are important to children's number representation.

On the other hand, Muldoon et al. (2011) did not find any such differences in number line performance between Chinese and Scottish four- and five-year-olds, despite the fact that that the Chinese children performed better than the Scottish children on a standardized arithmetic test.

Mark and Dowker (2015) studied children in Chinese and English medium schools in Hong Kong. They found that those in the Chinese medium school did perform somewhat better at a standardized arithmetic test, and at backward and forward counting, but only younger children (6 to 7), and not older children (8 to 9), showed group differences in reading and comparing two-digit numbers.

However, it is difficult to draw firm conclusions about the implications of these results, because there are so many other cultural and educational differences between Asian and Western children (Towse & Saxton, 1998).

The Welsh language can offer important insights here. The main counting system used for school mathematics is, like the counting systems used in Pacific Rim countries, completely regular (Roberts, 2000). The number words are easily constructed by knowing the numbers 1 to 10 and the rule for combining them.

For example, eleven in Welsh is “un deg un” (one ten one), twelve is “un deg dau” (one ten two), and twenty two is “dau ddeg dau” (two tens two).

Wales provides an unusual opportunity for research on linguistic influences on mathematics, since it is a region in which languages with both transparent and non-transparent counting systems are used in schools. In Wales, children receive either English or Welsh medium schooling within the same country, educational system, curriculum, and cultural environment. About 80% of children in Wales, like those elsewhere in the UK, receive their school education in English, but 20% attend Welsh medium schools, where they study in Welsh. Children whose parental language is English may still receive their education from age 4 entirely in Welsh. This makes it possible to compare children, who are following the same National Curriculum, and where the *only* educational difference is in the language used.

Maclean and Whitburn (1996) studied children in their first year of school, and found that those in Welsh medium schools performed better than those in English medium schools on certain numerical measures. In particular, they could count higher. However, comprehension and use of multi-digit numbers were hard to assess in their study, as most of the children were six years old or under, and had not been much exposed to oral and written representations of tens and units.

Dowker et al. (2008) carried out a study investigating the performance of numerical tasks by Welsh children who had just begun dealing with such representations (6-year-olds) and those who had greater experience (8-year-olds).

They investigated the performance of numerical tasks by 30 Welsh children who had just begun dealing with such representations (6-year-olds) and 30 who had greater experience (8-year-olds). One third of the children in each age group spoke Welsh both at home at school; one third spoke English at home but were educated at a Welsh medium school; and one third spoke English both at home and at school.

The children were given three standardized tests: the British Abilities Scales (BAS) Basic Number Skills test (2nd edition), which measures written calculation; the WISC Arithmetic subtest, which measures mental arithmetical reasoning, especially word problem solving; and the WISC Block Design subtest, which measures nonverbal reasoning.

They were also given a Number Comparison task, based on that used by Donlan and Gourlay (1999). In the Number Comparison task, 24 pairs of two-digit numbers were presented to children in a flip booklet. All participants were required to read each pair of numbers aloud before pointing to which was the bigger.

The groups did not differ in Block Design scores. They also did not differ in terms of overall arithmetical reasoning or calculation ability. A two-factor analysis of variance with School and Age as factors was applied to the WISC Arithmetic and

BAS Number Skills scores. No statistically significant differences were found between schools or age groups on the scaled score on either test. This suggests that the counting system on its own does not appear to have an impact on global arithmetical ability in otherwise culturally and educationally similar groups. But might the nature of the counting system have an effect on some more specific aspects of arithmetic?

There were indeed group differences in more specific areas of arithmetical ability – notably in the ability to read and judge number pairs, as shown by the Number Comparison Task. The composite Comparison Error score was found to show highly significant differences in a two-way analysis of variance between schools and between age groups. Not surprisingly, eight-year-olds performed better than six-year-olds. Children who spoke Welsh both at home and at school performed better than those who spoke Welsh only at school, who in turn performed better than those who spoke English both at home and at school. These results suggest that the transparency of the counting system does not have a global effect on arithmetical performance when other aspects of education and culture are kept constant, and is therefore unlikely to be the sole or main reason for superior performance by children in Pacific Rim countries. However, it does appear to have *specific* effects on performance in particular aspects of arithmetic.

This appears to be supported by other studies of Welsh children. Dowker and Roberts (2015) studied children in English and Welsh medium schools in Wales. The study found a trend for children in Welsh medium schools to be more accurate and quicker on number line tasks, but the difference did not quite reach significance. However, the Welsh medium pupils did show significantly lower standard deviations than the English medium pupils, indicating more consistency and lower variability in performance.

Some languages have *less* transparent counting systems than English: in particular those such as German, Dutch, and Arabic, where the tens and units in multi-digit numbers are inverted in speech. For example, in German the written number “24” is spoken as “vier und zwanzig” (four and twenty). While this does not seem to have broad negative effects on arithmetic as a whole (Germany and the Netherlands usually do relatively well in international comparisons), it does seem to affect specific aspects of numerical abilities. Children who use such counting systems are less accurate in placing numbers on empty number lines children who use counting systems with little or no inversion (e.g., Dowker & Nuerk, 2016; Bahnmueller et al., 2018; Göbel et al., 2011; Helmreich et al., 2011; Klein et al., 2013; Lonneman et al., 2016; Moeller et al., 2015). Krinzinger et al. (2011) compared German, Austrian, French, Flemish, and Walloon second-grade children on several numerical tasks. The first two groups had such inversion effects in their language; the others did not. Results showed that inversion had a clear effect on writing Arabic numerals to dictation, but not on reading and recognizing them, or on calculations. Once again, we see specific but not global effects of the level of transparency of the counting system.

3 Conclusions

There are numerous ways in which culture may affect arithmetic: ranging from the effects of learning in different contexts, to the effects of counting in different languages. Many of these affect specific aspects of arithmetic but do not affect mathematical ability globally, or lead to strikingly different levels of performance in different contexts.

In education, it is important to remember that a child's apparent weakness in mathematics in one context does not necessarily mean that they will not be able to carry out apparently similar mathematical tasks in another context. An ideal would be to find ways of helping children to transfer knowledge and skills from one context to others, but that is often remarkably difficult.

It is important to bear in mind, when teaching children mathematics, that the language that they speak and the counting system that they use may influence how easy or difficult they find it to acquire and understand certain aspects of place value. However, the effects of the counting system do not apply to every aspect of mathematics; and even speakers of languages with very limited counting systems can acquire many number concepts and skills.

The many cultural variations that we find should not obscure the fact there is a universal potential for arithmetical reasoning: indeed the variations themselves demonstrate this potential. Arithmetical reasoning can develop in a very wide variety of contexts, not only in a conventional school context, but at home; in school; in games; in shopping, budgeting, and other financial contexts; in jobs ranging from street trading to carpentry; in cooking, sewing, and other domestic activities; and in dealing with measurement in many situations.

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