Notations

The following is a (non-exhaustive) list of notations used throughout the book. Other notations will be presented at the first time used.

- $\mathbb R$ is the field of real numbers, $\mathbb R_+$ the real non-negative numbers, $\mathbb R_{++}$ the real positive numbers, \mathbb{R}_{-} the real non-positive numbers, and \mathbb{R}_{-} the real negative numbers.
- 2. For $x, y \in \mathbb{R}$, $\min(x, y) := x \land y$, $\max(x, y) := x \lor y$.
- 3. $\Delta^{N}(\gamma) := \{(x_1, \dots, x_N) \in \mathbb{R}_+^N, \sum_{i=1}^N x_i = \gamma\}.$ 4. $\underline{\Delta}^{N}(\gamma) := \{(x_1, \dots, x_N) \in \mathbb{R}_+^N, \sum_{i=1}^N x_i \leq \gamma\}.$
- $\mathbb{M}_+(N,J) := \mathbb{R}_+^N \otimes \mathbb{R}_+^J$. It is the set of $N \times J$ matrices of non-negative real numbers. Likewise, $\mathbb{M}'(N,J) := \mathbb{R}^N \otimes \mathbb{R}^J$ is the set of $N \times J$ matrices of real numbers.
- 6. For $\vec{\mathbf{M}} = \{m_{i,j}\} \in \mathbb{M}_+(N,J) \text{ and } \vec{\mathbf{P}} = \{p_{i,j}\} \in \mathbb{M}'(N,J), \vec{\mathbf{P}} : \vec{\mathbf{M}} := \operatorname{tr}(\vec{\mathbf{M}}\vec{\mathbf{P}}^t) =$ $\sum_{i=1}^{N} \sum_{j=1}^{J} p_{i,j} m_{i,j}$.
- (X, \mathcal{B}) is a compact measure space, and \mathcal{B} is the Borel σ -algebra on X.
- 8. $\mathcal{M}(X)$ is the set of Borel measures on X. $\mathcal{M}_+ \subset \mathcal{M}$ is the set of non-negative measures, and \mathcal{M}_1 are the *probability measures*, namely $\mu(X) = 1$.