

Contents

Acknowledgement — VII

Preface — IX

Part I: Continuous-time

1 Introduction to signals and systems — 3

- 1.1 Signals — 3
 - 1.1.1 Signals and their characteristics — 3
 - 1.1.2 Important signals in continuous-time — 5
 - 1.1.3 Important signals in discrete-time — 7
 - 1.1.4 Discrete signals vs. continuous signals — 8
 - 1.1.5 Operations with signals: energy and power — 9
 - 1.1.6 Operations with signals: convolutions — 11
- 1.2 Systems — 12
- 1.3 Characterisation of Linear Systems — 17

2 Continuous-time linear systems and the Laplace transform — 21

- 2.1 The impulse response and the properties of linear systems — 21
- 2.2 Exponentials and LTIS — 21
 - 2.2.1 The Laplace transform — 22
 - 2.2.2 Existence and main properties of the Laplace transform — 24
- 2.3 The differintegrator — 27
- 2.4 Impulse and step responses of causal LS — 30
 - 2.4.1 The general initial value theorem — 30
 - 2.4.2 The general final value theorem — 31
 - 2.4.3 Transfer function series representation — 31
- 2.5 Transfer function series representation for commensurate LS — 35

3 Fractional commensurate linear systems: time responses — 39

- 3.1 Impulse and step responses: the Mittag-Leffler function — 39
 - 3.1.1 In terms of the Mittag-Leffler function — 41
 - 3.1.2 By integer/fractional decomposition — 41
- 3.2 Stability — 45
 - 3.2.1 Knowing the pseudo-poles — 45
 - 3.2.2 Routh-Hurwitz criterion for an integer TF — 46
 - 3.2.3 Special cases of the Routh-Hurwitz criterion for an integer TF — 49
 - 3.2.4 Routh-Hurwitz criterion for a fractional commensurate TF — 50

3.2.5	Special cases of the Routh-Hurwitz criterion for a fractional commensurate TF —	53
3.3	Causal periodic impulse response —	56
3.4	Initial conditions —	57
3.4.1	Special cases —	59
4	The fractional commensurate linear systems. Frequency responses —	61
4.1	Steady-state behaviour: the frequency response —	61
4.2	Bode diagrams of typical explicit systems —	64
4.2.1	The differintegrator —	64
4.2.2	One pseudo-pole or one pseudo-zero —	64
4.2.3	Complex conjugate pair of pseudo-poles or pseudo-zeroes —	67
4.3	Implicit systems —	69
4.3.1	Implicit fractional order zero or pole —	70
4.3.2	Fractional PID —	70
4.3.3	Fractional lead compensator —	72
4.3.4	Fractional lag compensator —	74
4.4	Approximations to transfer functions based upon a frequency response —	75
4.4.1	Oustaloup's CRONE approximation —	75
4.4.2	The Carlson's approximation —	76
4.4.3	The Matsuda's approximation —	76
4.5	System modelling and identification from a frequency response —	77
4.6	Filters —	79
4.6.1	Generalities. Ideal filters —	79
4.6.2	Prototypes of integer order filters —	82
4.6.3	Transformations of frequencies —	85
4.6.4	Fractional filters —	85
5	State-space representation —	87
5.1	General considerations —	87
5.2	Standard form of the equations —	88
5.3	State transition operator —	90
5.4	Canonical representations of SISO systems —	92
6	Feedback representation —	95
6.1	Why feedback? —	95
6.2	On controllers —	96
6.3	The Nyquist stability criterion —	98
6.3.1	Drawing the Nyquist diagram —	100
6.3.2	Gain and phase margins —	102

7 On fractional derivatives — 105

- 7.1 Introduction — 105
- 7.2 Some historical considerations — 105
- 7.2.1 Returning to the differintegrator — 106
- 7.3 Fractional incremental ratio approach — 108
- 7.3.1 The derivative operators and their inverses — 108
- 7.3.2 Fractional incremental ratio — 109
- 7.3.3 Some examples — 112
- 7.3.4 Classic Riemann-Liouville and Caputo derivatives — 113
- 7.4 On complex order derivatives — 113

Part II: Discrete-time**8 Discrete-time linear systems. Difference equations — 119**

- 8.1 On time scales — 119
- 8.2 Systems on uniform time scales: difference equations — 120
- 8.3 Exponentials as eigenfunctions — 120
- 8.3.1 Determination of the impulse response from the difference equation — 122
- 8.3.2 From the impulse response to the difference equation — 123
- 8.4 The frequency response — 124
- 8.5 Classification of systems — 126
- 8.6 Singular systems — 128

9 Z transform. Transient responses — 131

- 9.1 Z transform — 131
- 9.1.1 Introduction — 131
- 9.1.2 Definition — 131
- 9.2 Main Properties of the ZT — 133
- 9.3 Signals whose transforms are simple fractions — 136
- 9.4 Inversion of the ZT — 137
- 9.4.1 Inversion Integral — 137
- 9.4.2 Inversion by decomposition into a polynomial and partial fractions — 138
- 9.4.3 Inversion by series expansion — 140
- 9.4.4 Step response — 141
- 9.4.5 Response to a causal sinusoid — 142
- 9.5 Stability and Jury criterion — 145
- 9.6 Initial Conditions — 148
- 9.7 Initial and final value theorems — 150
- 9.8 Continuous to discrete conversion ($s \rightarrow z$) — 151
- 9.8.1 Some considerations — 151

- 9.8.2 Approximation of derivatives — **152**
- 9.8.3 Tustin rule or bilinear transformation — **154**
- 9.8.4 Direct conversion of poles and zeros into poles and zeros — **156**
- 9.8.5 Invariant response s to z conversion — **157**
- 9.8.6 Conversion from partial fraction decomposition — **159**

10 Discrete-time derivatives and transforms — 163

- 10.1 Introduction: difference vs. differential — **163**
- 10.2 Derivatives and inverses — **163**
 - 10.2.1 Nabla and delta derivatives — **163**
 - 10.2.2 Existence of fractional derivatives — **166**
 - 10.2.3 Properties — **166**
 - 10.2.4 The nabla and delta exponentials — **168**
- 10.3 Suitable transforms — **172**
 - 10.3.1 The nabla transform — **172**
 - 10.3.2 Main properties of the NLT — **173**
 - 10.3.3 Examples — **174**
 - 10.3.4 Existence of NLT — **176**
 - 10.3.5 Unicity of the transform — **176**
 - 10.3.6 Initial value theorem — **177**
 - 10.3.7 Final value theorem — **177**
 - 10.3.8 The delta transform and the correlation — **178**
 - 10.3.9 Backward compatibility — **179**
- 10.4 Discrete-time fractional linear systems — **180**
 - 10.4.1 Polynomial case — **181**
 - 10.4.2 Proper fraction case — **181**
 - 10.4.3 The anti-causal case — **185**
 - 10.4.4 Inversion without partial fractions — **185**
 - 10.4.5 Some stability issues — **188**
 - 10.4.6 A correspondence principle — **188**
 - 10.4.7 On initial conditions — **191**
- 10.5 The Fourier transform and the frequency response — **192**

Part III: Advanced topics

11 Fractional stochastic processes and two-sided derivatives — 197

- 11.1 Stochastic input — **197**
- 11.2 Two-sided derivatives — **199**
 - 11.2.1 Derivative of white noise — **199**
 - 11.2.2 GL type two-sided derivative — **199**
 - 11.2.3 The general two-sided fractional derivative — **200**
 - 11.2.4 The integer order cases — **203**

11.2.5	Other properties and regularisations —	203
11.2.6	Feller derivative —	205
11.3	The fractional Brownian motion —	206
11.3.1	Definition —	206
11.3.2	Properties —	208
12	Fractional delay discrete-time linear systems —	213
12.1	Introduction —	213
12.2	Fractional delays —	213
12.3	Impulse responses —	214
12.4	Scale conversion —	215
12.5	Linear prediction —	216
13	Fractional derivatives with variable orders —	221
13.1	Introduction —	221
13.2	Past proposals —	221
13.3	An approach to VO derivatives based on the GL derivative —	224
13.4	Variable order linear systems —	227
13.5	The VO Mittag-Leffler function —	228
13.6	Variable order two-sided derivatives —	230

Appendixes

A	On distributions —	237
A.1	Introduction —	237
A.2	The axioms —	238
B	The Gamma function and binomial coefficients —	240
C	The continuous-Time Fourier Transform —	243
C.1	Definition —	243
C.2	Differentiation and integration in the context of FT —	244
D	The discrete-time Fourier Transform —	247
D.1	Definition —	247
D.2	Existence —	249
E	Partial fraction decomposition without derivations —	252
E.1	Simplification when there are no conjugate pairs of poles —	252
E.2	Simplification for conjugate pairs of poles on the imaginary axis —	254
E.3	Simplification for conjugate pairs of poles not on the imaginary axis —	256

F The Mittag-Leffler function — 258

F.1 Definition — **258**

F.2 Computation of the MLF — **264**

Bibliography — 267

Further Reading — 273

Index — 279

E1 Erratum to Chapter 7 — 283