## **Contents**

## Preface — VII

| 1.1<br>1.2 | Preliminaries —— <b>1</b><br>Basic Concepts of the Ill-Posed Problems Theory —— <b>14</b> |
|------------|---|
| 1.2        | Basic Concepts of the Ill-Posed Problems Theory —— <b>14</b>                              |
|            |   |
| 2          | Regularization Methods For Linear Equations —— 20   |
| 2.1        | Linear Equations in Hilbert Space —— 21   |
| 2.2        | Regularization Algorithms in Banach Space —— 37   |
| 2.3        | Necessity of Sourcewise Representation in Banach Space —— 45                              |
| 2.4        | Ill-Posed Cauchy Problem in Banach Space —— 74  |
| 2.5        | Logarithmic Convergence of Solution Methods for Cauchy Problems —— 77                     |
| 2.6        | Necessary Conditions for Logarithmic Estimates —— 87                                      |
| 2.7        | Linear Ill-Posed Problems Under Random Noise —— <b>96</b>                                 |
| 3          | Finite Difference Methods —— 101  |
| 3.1        | Scalar Cauchy Problem —— 102  |
| 3.2        | Finite Difference Regularization Methods in Banach Space —— 110                           |
| 3.3        | Power Rate of Convergence Estimates —— 126  |
| 3.4        | Necessity of the Source Condition —— 134  |
| 4          | Iterative Regularization Methods —— 144   |
| 4.1        | Parametric Approximation —— 145   |
| 4.2        | Regularized Gauss-Newton Methods —— 153   |
| 4.3        | Necessity of Sourcewise Representation in Hilbert Space —— 163                            |
| 4.4        | Regularized Newton-Kantorovich Methods —— 171   |
| 4.5        | Necessity of Sourcewise Representation in Banach Space —— 179                             |
| 4.6        | Iteratively Regularized Gradient Method —— 184  |
| 4.7        | Regularized Iterative Methods Under Large Noise —— 191                                    |
| 4.8        | Iteratively Regularized Methods Under Random Noise —— 204                                 |
| 4.9        | Convexity of Tikhonov Functional —— <b>211</b>  |
| 5          | Finite-Dimensional Iterative Processes —— 226   |
| 5.1        | Adaptive Choice of Projectors —— 227  |
| 5.2        | A Priori Choice of Projectors —— 239  |
| 5.3        | Class of Stable Iterative Methods —— 242  |
| 5.4        | A Posteriori Estimates —— <b>251</b>  |
| 5.5        | Conditionally Well-Posed Inverse Problems —— <b>254</b>                                   |

| 6       | Variational Inequalities and Optimization Problems — 269          |
|---------|---|
| 6.1     | Browder–Tikhonov Regularization —— 269                            |
| 6.2     | Simple Iteration Method and Newton-Kantorovich Type Method —— 277 |
| 6.3     | Iterative Regularization Methods for Noisy Data —— 284            |
| 6.4     | Variational Inequalities on a Ball —— 288                         |
| 6.5     | Tikhonov Regularization of Ill-Posed Optimization Problems —— 296 |
| 6.6     | Purely Data-Driven Regularization Algorithms —— 303               |
| Bibliog | raphy —— 315  |
| Index – | <del></del>   |