

Contents

Preface — V

1 Prerequisites for calculus — 1

- 1.1 Overview of calculus — 1
- 1.2 Sets and numbers — 6
 - 1.2.1 Sets — 6
 - 1.2.2 Numbers — 8
 - 1.2.3 The least upper bound property — 9
 - 1.2.4 The extended real number system — 11
 - 1.2.5 Intervals — 12
- 1.3 Functions — 14
 - 1.3.1 Definition of a function — 14
 - 1.3.2 Graph of a function — 17
 - 1.3.3 Some basic functions and their graphs — 18
 - 1.3.4 Building new functions — 20
 - 1.3.5 Fundamental elementary functions — 31
 - 1.3.6 Properties of functions — 32
- 1.4 Exercises — 37

2 Limits and continuity — 41

- 2.1 Rates of change and derivatives — 41
- 2.2 Limits of a function — 42
 - 2.2.1 Definition of a limit — 42
 - 2.2.2 Properties of limits of functions — 49
 - 2.2.3 Limit laws — 50
 - 2.2.4 One-sided limits — 55
 - 2.2.5 Limits involving infinity and asymptotes — 59
- 2.3 Limits of sequences — 68
 - 2.3.1 Definitions and properties — 68
 - 2.3.2 Subsequences — 77
- 2.4 Squeeze theorem and Cauchy's theorem — 78
- 2.5 Infinitesimal functions and asymptotic functions — 86
- 2.6 Continuous and discontinuous functions — 91
 - 2.6.1 Continuity and discontinuity — 91
 - 2.6.2 Continuous functions — 94
 - 2.6.3 Theorems on continuous functions — 99
 - 2.6.4 Uniform continuity — 107
- 2.7 Some proofs in Chapter 2 — 108
- 2.8 Exercises — 114

3 The derivative — 121

- 3.1 Derivative of a function at a point — **121**
 - 3.1.1 Instantaneous rates of change and derivatives revisited — **121**
 - 3.1.2 One-sided derivatives — **128**
 - 3.1.3 A function may fail to have a derivative at a point — **129**
- 3.2 Derivative as a function — **133**
 - 3.2.1 Graphing the derivative of a function — **134**
 - 3.2.2 Derivatives of some basic functions — **135**
- 3.3 Derivative laws — **139**
- 3.4 Derivative of an inverse function — **143**
- 3.5 Differentiating a composite function – the chain rule — **147**
- 3.6 Derivatives of higher orders — **152**
- 3.7 Implicit differentiation — **154**
- 3.8 Functions defined by parametric and polar equations — **159**
 - 3.8.1 Functions defined by parametric equations — **159**
 - 3.8.2 Polar curves — **163**
- 3.9 Related rates of change — **166**
- 3.10 The tangent line approximation and the differential — **167**
 - 3.10.1 Linearization — **167**
 - 3.10.2 Differentials — **170**
- 3.11 Derivative rules – summary — **174**
- 3.12 Exercises — **175**

4 Applications of the derivative — 181

- 4.1 Extreme values and the candidate theorem — **181**
- 4.2 The mean value theorem — **188**
- 4.3 Monotonic functions and the first derivative test — **196**
 - 4.3.1 Monotonic functions — **196**
 - 4.3.2 The first derivative test — **199**
- 4.4 Extended mean value theorem and the L'Hôpital rules — **201**
 - 4.4.1 Extended mean value theorem — **201**
 - 4.4.2 The indeterminate forms $\frac{0}{0}$, $\infty - \infty$, $\frac{\infty}{\infty}$, and $0 \times \infty$ — **203**
- 4.5 Taylor's theorem — **209**
 - 4.5.1 The error analysis for the linear approximation — **209**
 - 4.5.2 The quadratic approximation — **210**
 - 4.5.3 Taylor's theorem — **214**
- 4.6 Concave functions and the second derivative test — **219**
 - 4.6.1 Concave functions — **219**
 - 4.6.2 The second derivative test — **225**
- 4.7 Extreme values of functions revisited — **227**
- 4.8 Curve sketching — **231**
- 4.9 Solving equations numerically — **234**

4.9.1	Decimal search —	234
4.9.2	Newton's method —	236
4.10	Curvatures and the differential of the arc length —	238
4.11	Exercises —	243
5	The definite integral —	249
5.1	Definite integrals and properties —	249
5.1.1	Introduction —	249
5.1.2	Properties of the definite integral —	259
5.1.3	Interpreting $\int_a^b f(x) dx$ in terms of area —	265
5.1.4	Interpreting $\int_a^b v(t) dt$ as a distance or displacement —	267
5.2	The fundamental theorem of calculus —	267
5.3	Numerical integration —	275
5.3.1	Trapezoidal rule —	276
5.3.2	Simpson's rule —	277
5.4	Exercises —	279
6	Techniques for integration and improper integrals —	285
6.1	Indefinite integrals —	285
6.1.1	Definition of indefinite integrals and basic antiderivatives —	285
6.1.2	Differential equations —	289
6.1.3	Substitution in indefinite integrals —	293
6.1.4	Further results using integration by substitution —	297
6.1.5	Integration by parts —	300
6.1.6	Partial fractions in integration —	304
6.1.7	Rationalizing substitutions —	312
6.2	Substitution in definite integrals —	313
6.3	Integration by parts in definite integrals —	317
6.4	Improper integrals —	318
6.4.1	Improper integrals of the first kind —	318
6.4.2	Improper integrals of the second kind —	322
6.5	Exercises —	326
7	Applications of the definite integral —	333
7.1	Areas, volumes, and arc lengths —	333
7.1.1	The area of the region between two curves —	333
7.1.2	Volumes of solids —	337
7.1.3	Arc length —	339
7.2	Applications in other disciplines —	344
7.2.1	Displacement and distance —	344
7.2.2	Work done by a force —	345

7.2.3	Fluid pressure —	346
7.2.4	Center of mass —	347
7.2.5	Probability —	349
7.3	Exercises —	350
8	Infinite series, sequences, and approximations —	355
8.1	Infinite sequences —	355
8.2	Infinite series —	357
8.2.1	Definition of infinite series —	357
8.2.2	Properties of convergent series —	359
8.3	Tests for convergence —	363
8.3.1	Series with nonnegative terms —	363
8.3.2	Series with negative and positive terms —	371
8.4	Power series and Taylor series —	375
8.4.1	Power series —	375
8.4.2	Working with power series —	381
8.4.3	Taylor series —	383
8.4.4	Applications of power series —	391
8.5	Fourier series —	393
8.5.1	Fourier series expansion with period 2π —	394
8.5.2	Fourier cosine and sine series with period 2π —	399
8.5.3	The Fourier series expansion with period $2l$ —	400
8.5.4	Fourier series with complex terms —	403
8.6	Exercises —	404
Index —		411