

# Contents

## Preface — v

<b>1</b>	<b>Introduction and Statement of Main Results — 1</b>
1.1	First Main Result: Absolute and Relative Boundary Conditions — 3
1.2	Other Problems Involving Tangential and Normal Components of Harmonic Forms — 11
1.3	Boundary Value Problems for Hodge-Dirac Operators — 21
1.4	Dirichlet, Neumann, Transmission, Poincaré, and Robin-Type Boundary Problems — 24
1.5	Structure of the Monograph — 43
<b>2</b>	<b>Geometric Concepts and Tools — 49</b>
2.1	Differential Geometric Preliminaries — 49
2.2	Elements of Geometric Measure Theory — 67
2.3	Sharp Integration by Parts Formulas for Differential Forms in Ahlfors Regular Domains — 91
2.4	Tangential and Normal Differential Forms on Ahlfors Regular Sets — 96
<b>3</b>	<b>Harmonic Layer Potentials Associated with the Hodge-de Rham Formalism on UR Domains — 109</b>
3.1	A Fundamental Solution for the Hodge-Laplacian — 109
3.2	Layer Potentials for the Hodge-Laplacian in the Hodge-de Rham Formalism — 117
3.3	Fredholm Theory for Layer Potentials in the Hodge-de Rham Formalism — 128
<b>4</b>	<b>Harmonic Layer Potentials Associated with the Levi-Civita Connection on UR Domains — 139</b>
4.1	The Definition and Mapping Properties of the Double Layer — 140
4.2	The Double Layer on UR Subdomains of Smooth Manifolds — 169
4.3	Compactness of the Double Layer on Regular SKT Domains — 173
<b>5</b>	<b>Dirichlet and Neumann Boundary Value Problems for the Hodge-Laplacian on Regular SKT Domains — 185</b>
5.1	Functional Analytic Properties for Harmonic Layer Potentials in UR Domains — 186
5.2	Invertibility Results for Layer Potentials Associated with the Levi-Civita Connection — 196
5.3	Solving the Dirichlet, Neumann, Transmission, Poincaré, and Robin Boundary Value Problems — 204

<b>6</b>	<b>Fatou Theorems and Integral Representations for the Hodge-Laplacian on Regular SKT Domains — 231</b>
6.1	Convergence of Families of Singular Integral Operators — 231
6.2	A Fatou Theorem for the Hodge-Laplacian in Regular SKT Domains — 250
6.3	Spaces of Harmonic Fields and Green Type Formulas — 261
<b>7</b>	<b>Solvability of Boundary Problems for the Hodge-Laplacian in the Hodge-de Rham Formalism — 275</b>
7.1	Preparatory Results — 275
7.2	Solvability Results — 288
<b>8</b>	<b>Additional Results and Applications — 315</b>
8.1	de Rham Cohomology on Regular SKT Surfaces — 315
8.2	Maxwell's Equations in Regular SKT Domains — 336
8.3	Dirichlet-to-Neumann Operators for the Hodge-Laplacian in Regular SKT Domains — 339
8.4	Fatou Type Results with Additional Constraints or Regularity Conditions — 347
8.5	Weak Tangential and Normal Traces in Regular SKT Domains with Friedrichs Property — 352
8.6	The Hodge-Poisson Kernel and the Hodge-Harmonic Measure — 367
<b>9</b>	<b>Further Tools from Differential Geometry, Harmonic Analysis, Geometric Measure Theory, Functional Analysis, Partial Differential Equations, and Clifford Analysis — 371</b>
9.1	Connections and Covariant Derivatives on Vector Bundles — 371
9.2	The Extension of the Levi-Civita Connection to Differential Forms — 381
9.3	The Bochner-Laplacian and Weintzenböck's Formula — 386
9.4	Sobolev Spaces on Boundaries of Ahlfors Regular Domains: The Euclidean Setting — 393
9.5	Sobolev Spaces on Boundaries of Ahlfors Regular Domains: The Manifold Setting — 408
9.6	Integrating by Parts on the Boundaries of Ahlfors Regular Domains — 417
9.7	A Global Sobolev Regularity Result — 444
9.8	The PV Harmonic Double Layer on a UR Domain — 446
9.9	Calderón-Zygmund Theory on UR Domains on Manifolds — 451
9.10	The Fredholmness and Invertibility of Elliptic Differential Operators — 474
9.11	Compact and Close-to-Compact Singular Integral Operators — 482
9.12	A Sharp Divergence Theorem — 490

- 9.13      Clifford Analysis Rudiments — 493
- 9.14      Spectral Theory for Unbounded Linear Operators  
                    Subject to Cancellations — 496

**Bibliography — 501**

**Index — 507**

